**Mathematics for All**

**Problems of cultural selectivity**

**and unequal distribution of mathematical education**

**and future perspectives**

**on mathematics teaching for the majority**

Report and papers presented in theme group I,

‘Mathematics for All’

at the 5th International Congress on Mathematical Education,

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**Preface**

This resource document consists of twenty-two papers prepared by authors from all regions and

presented at the Fifth International Congress on Mathematical Education (ICME 5). Over 2000

mathematics educators from sixty-nine countries gathered in Adelaide, Australia, in August 1984,

to discuss problems in their field. This document is one outcome. Its purpose is to continue the

dialogue to assist nations in their search for a mathematics programme for all students.

*Mathematics for All is* the first document in mathematics education in Unesco’s Science and

Technology Education Document Series. This, coupled with Unesco’s publications *Studies in*

*Mathematics Education* and *New Trends in Mathematics Teaching,* was initiated to encourage an

international exchange of ideas and information.

Unesco expresses its appreciation to the editors, Peter Damerow, Mervyn Dunkley, Bienvenido

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Education for preparing the manuscript, and to the ICME 5 Programme Committee for permitting

Unesco to produce this report.

The views expressed in the text are those of the authors and not necessarily those of Unesco,

the editors, or of ICME 5.

We welcome comments on the contents of this document. Please send them to: Mathematics

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Introduction:

Report on the Work of Theme Group I

«Mathematics for All» at ICME 5

**1. Introduction**

Many factors have brought about a change in the overall

situation of mathematics education. These include the

move to universal elementary education in developing

countries, the move to universal secondary education in

industrialised countries (where there have also been growing

demands for mathematical competence in an increasingly

technologically and scientifically oriented world) and

from the experience gained with worldwide curriculum

developments such as the new mathematics movement.

The tacit assumption, that what can be gained from mathematics

can be gained equally in every culture and

independently of the character of the school institution and

the individual dispositions and the social situations of the

learner, turned out to be invalid. New and urgent questions

have been raised. Probably the most important ones are:

- What kind of mathematics curriculum is adequate to the

needs of the majority?

- What modifications to the curriculum or alternative curricula

are needed for special groups of learners?

- How should these curricula be structured?

- How could they be implemented?

A lot of work has already been done all over the world in

attempts to answer these questions or to contribute to special

aspects of the problem.

- ICME 4 yielded several presentations of results concerning

universal basic education, the relationship of

mathematics to its applications, the relation between

mathematics and language, women and mathematics,

and the problems of teaching mathematics to special

groups of students whose needs and whose situations

do not fit into the general framework of traditional

mathematics education.

-The Second International Mathematics Study of the

International Association for the Evaluation of

Educational Achievement (IEA) dealt much more than

the first one with the similarities and differences of the

mathematics curriculum in different countries, and the

different conditions which determine the overall outcome

in mathematical achievement. The IEA collected

data on both the selectivity of mathematics and the differences

between countries in the way they produce

yield levels of mathematical qualification. Although final

reports on the Second International Mathematics Study

are not yet available, preliminary analyses of the data

have already produced useful results.

- In several countries national studies have been concerned

with the evaluation of the mathematics education

system. An important recent example is the Report of

the Committee of Enquiry into the Teaching of

Mathematics in Schools in England and Wales (commonly

known as the Cockcroft Report) in 1982.

- Last, but not least, there are many detailed studies, projects

and proposals from different countries dealing with

special aspects such as:

- teaching the disadvantaged;

- teaching the talented;

- teaching mathematics to non-mathematicians;

- teaching mathematics in the context of real life situations;

- teaching mathematics under atypical conditions, etc.

At ICME 5, papers were presented on a variety of topics

related to the theme Mathematics for All. Taken as a whole,

these contribute to a better understanding of the problems

of teaching mathematics successfully, not only to very able

students, but teaching worthwhile mathematics successfully

to all in a range of diverse cultures and circumstances.

**2. Summary of Papers Presented to the Theme Group**

The first group of papers dealt with general aspects of the

theme Mathematics for All.

Jean-Claude Martin, Rector of the Academy of Bordeaux

in France, analysed in his paper, A *Necessary Renewal of*

*Mathematics Education,* the special selectivity of mathematical

education as a result of symbolism and mathematical

language. The teaching of mathematics seems to have

been designed to produce future mathematicians despite

the fact that only a very small percentage of students reach

university level. This general character of mathematical

education causes avoidable, system-related failures in

mathematical learning and often results in a strong aversion

to mathematics. Martin proposed a general reorientation

of mathematical education aiming at a mathematics

which is a useful tool for the majority of students. The teaching

of mathematics as a means of solving multidisciplinary

problems by using modelling methods should restore

student interest, show mathematics as being useful, enrich

students knowledge of related subjects and so enable

them better to memorise mathematical formulas and

methods, encourage logical reasoning and allow more students

access to a higher level of mathematics.

Bienvenido F. Nebres in his paper, *The Problem of*

*Universal Mathematics Education in Developing Countries,*

discussed the same problem of the lack of fit between the

goals of mathematical education and the needs of the majority

in the special circumstances of the situation in developing

countries. He offered a conceptual framework for discussing

the specific cultural dimensions of the problem in

these countries by using the distinction between vertical

and horizontal relationships, i.e. the relationships between

corresponding institutions in different societies and the rel

a t i o n ships between social or cultural institutions within the

same country. The history of social and cultural institutions

in developing countries is that their establishment and growth

has been guided more by vertical relationships, i.e. an

adaption of a similar type of institution from the mother colonial

country, rather than by horizontal relationships. The result

is a special lack of fit between mathematical education

and the needs of the majority of the people.

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There is a tremendous need for researchers in mathematics

education in developing countries to look at the

actual life of urban workers, rural farmers and merchants

and to identify the mathematics in daily life that is needed

and used by people. Then it is necessary to compare this

needed mathematics with what is provided in the curriculum

and to search for a better fit between the two. A cultural

shift must be brought about in these countries.

Mathematical educators, together with other educators and

other leaders of society, should take up the need for the

social and cultural institutions to be better integrated with

one another and to develop together in a more organic

manner than in the past.

In a joint paper *Mathematics for All: Conclusions Drawn*

*from the Experiences of the New Mathematics Movement,*

Peter Damerow of the Max Planck Institute for Human

Development and Education in Berlin, West Germany, and

Ian Westbury of the University of Illinois at

Urbana-Champaign, United States, examined the problem

of designing a mathematics curriculum which genuinely

meets the diverse needs of all students in a country. They

argue that, by continuing to ignore the needs of all except

a small minority of students, the curricula developed within

the new mathematics movement proved to be no more

satisfactory than their predecessors. Traditionally, mathematics

curricula were developed for an elite group of students

who were expected to specialise in the subject, and

to study mathematics subsequently at higher levels in a

tertiary institution. As education has become increasingly

universal, however, students of lesser ability, and with

more modest vocational aspirations and daily life requirements,

have entered the school system in greater numbers.

A major problem results when these students are

exposed to a curriculum designed for potential specialists.

This same type of traditional curriculum has frequently

been transferred to developing and third world countries,

where, because of different cultural and social conditions,

its inappropriateness for general mathematical education

has only been compounded. So called reforms such as

new mathematics did little to resolve the major problems in

that they merely attempted to replace one specialist curriculum

by another.

The question addressed by Damerow and Westbury is

how to cater both for the elite and also for the wider group

of students for whom mathematics should be grounded in

real world problem solving and daily life applications. One

suggestion is that the majority would achieve a mathematical

«Literacy» through the use of mathematics in other

subjects such as science, economics, while school mathematics

would remain essentially and deliberately for specialists.

This is effectively to retain the status quo.

Alternatively, mathematics must be kept as a fundamental

part of the school curriculum, but ways of teaching it effectively

to the majority must be found. The majority of students

will be users of mathematics. Damerow and

Westbury concluded that a mathematics program which is

truly for all must seek to overcome the subordination of elementary

mathematics to higher mathematics, to overcome

its preliminary, preparatory character, and to overcome its

irrelevance to real life situations.

The findings of the Second International Mathematics

Study (SIMS) were used by Howard Russell, Ontario

Institute for Studies in Education, Canada, in his paper

*Mathematics for All: SIMS Data,* to argue that mathematics

is already taught to all pupils at the elementary level in

many countries. At the senior secondary level, however,

the prevailing pattern in most countries is for mathematics

to be taught only to an elite. At the lower level, the SIMS

data suggest that promotion by age, rather than by performance,

does not violate the concept of mathematics for all.

The SIMS data also appear to provide support for the

Cockcroft hypothesis that the pace of mathematics education

must be slowed if sufficient students are to be retained

in mathematics courses at the higher levels for it to be

accurately labelled mathematics for all. Alternatively, the

content of the curriculum could be trimmed down as suggested

by Damerow. Russell proposed a market-oriented

rationale to construct such a core of material, particularly to

meet the needs of the middle level students who will be

required to use mathematics in their chosen work in the

market place.

Afzal Ahmed was a member of the Committee of Inquiry

into the Teaching of Mathematics in Schools in England

and Wales (Cockcroft Committee), and is now the director

of the Curriculum Development Project for Low Attaining

Pupils in Secondary School Mathematics. In his paper, *The*

*Foundations of Mathematics Education for All,* he discussed

implications of the Cockcroft Report, published in January

1982, concerning the major issues of the theme group. He

pointed out that a suitable mathematics curriculum for the

majority assumes greater importance as societies in the

world become more technological and sophisticated. But at

the same time, the evidence of failure at learning and

applying mathematics by a large proportion of the population

is also growing. The Cockcroft Report proposes a

Foundation List of Mathematical Topics that should form

part of the mathematics syllabus for all pupils. In his discussion

of the Cockcroft Report, Ahmed focussed

particularly on the classroom conditions which facilitate, or

inhibit the mastery of these fundamental topics.

In a paper on *Universal Mathematics Education,*

Achmad Arifin from the Bandung Institute of Te c h n o l o g y

in Indonesia, described how community participation

should be raised in carrying out universal mathematics

education through looking at the aspect of interaction

within and between social and cultural institutions. He

asked the questions which parts of mathematics can

function as a developer of an individual’s intelligence and

how should those parts that have been chosen be presented?

Any program to answer these questions has to

take into account three components of interaction. Firstly,

depending on its quality, social structure through interaction

can contribute to the improvement of peoples’ a b i l ities,

especially by making them appreciate mathematics.

S e c o n d l y, a special form of social interaction, which he

called positive interaction, can motivate mathematics

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learning and create opportunities to learn. Thirdly, school

interaction itself can inspire, stimulate, and direct learning

activities. In developing countries, local mathematicians in

particular are able to understand their cultural conditions,

the needs, the challenges and the wishes of their developing

nation. Taking into account the three components of

interaction, they have the ability and the opportunity to

spread and share their knowledge and to translate and utilise

the development of mathematics in universal

mathematics education for their nation.

In many countries, there is one mathematics syllabus for

each year of the education system. Andrew J. C. Begg, in

his paper, *Alternative Mathematics Programs,* questioned

this practice and argued for the introduction of alternative

mathematics programs which will meet the varied needs of

all students in a range of circumstances and with a range

of individual aspirations. All such courses should contribute

towards general educational aims such as the development

of self-respect, concern for others, and the urge to

enquire. Thus, mathematics courses should provide an

opportunity to develop skills of communication, responsibility,

criticism, and cooperation. Such an approach has

implications for the way in which students are organised in

mathematics classes; for the scheduling of mathematics

classes; for the choice of teaching and learning methods;

for the extent to which emphasis is placed on cooperation

as against competition; for the use of group methods of

teaching; and for the provision that should be made for students

from diverse cultural groups. In this way, mathematics

programs for all students should assist not only the

achievement of mathematical objectives, but also the

attainment of personal, vocational and humanistic aims in

education. By matching mathematics programs to the

needs of students, the development of the self-esteem of

every student becomes central in the mathematics curriculum.

The second group of papers was concerned with particular

concerns related to Mathematics for All in industrialised

countries.

In their paper, *Arithmetic Pedagogy at the Beginning of the*

*School System in Japan,* Genichi Matsubara and

Zennosuke Kusumoto traced the introduction of the teaching

of western arithmetic to Japan in the late nineteenth

century. At a time when universal elementary education

was only just approaching reality in Japan, the government

declared a policy of adopting western-style arithmetic in

order to enable the country to compete more successfully

internationally. This move faced obstacles in its implementation

because of the traditional use of the abacus and the

widespread lack of familiarity with the Hindu-Arabic notation.

Further, in a developing national system of education,

teachers were in short supply and little attention could be

given to teaching methods in the training courses. The

paper emphasised the need to make such changes slowly

and to take into account the situation of those closely involved

with the changes if they are to be successful in modifying

the curriculum for mathematics for all.

The extent to which the mathematics learnt at school is

retained and used in later life is the subject of research

reported in a paper by Takashi Izushi and Akira Yamashita

of Fukuoka University, Japan, entitled *On the Value of*

*Mathematical Education Retained by the Social Members of*

*Japan in General.* A study in 1955 was concerned with

people who had learnt their mathematics before the period

in Japan in which mathematics teaching was focussed on

daily life experience and before compulsory education was

extended to secondary schools. Although it was found that

most people retained the mathematics skills and knowledge

well, rather fewer claimed that this material was useful

in their work. A second more limited study in 1982 confirmed

these general findings in relation to geometry. It showed,

broadly speaking, that younger people tended to use

their school mathematics more directly while older people

relied more on common sense. The study covered a further

aspect, the application of the attitudes of deductive thinking

derived from the learning of geometry. The thinking and

reasoning powers inculcated by this approach were not forgotten

and were claimed to be useful in daily life, but not in

work. Izushi and Yamashita conclude that the inclusion of

an element of formal mathematical discipline in the curriculum

is supported by Japanese society.

Another attempt to create a modern course in advanced

mathematics which is also worthwhile for those students

who don’t intend to proceed to university was reported by

Ulla Kürstein Jensen from Denmark in her paper titled

*Upper Secondary Mathematics for All? An Evolution and a*

*Draft.* The increase from about 5% in former years to about

40So in 1983 of an age cohort completing upper secondary

education with at least some mathematics brought about

an evolution toward a curriculum concentrating on useful

mathematics and applications in daily life and mathematical

modelling. This evolution led to the draft of a new curriculum

which will be tested under school conditions, beginning

in autumn 1984. The origin of this development is

based on new regulations for mathematical education for

the upper secondary school in the year 1961. It was

influenced by the new mathematics ideas and designed to

serve the needs of the small proportion of the students

passing through upper secondary education at that time,

but soon had to be modified for the rapidly increasing number

of students in the following years. So the mathematics

teaching, particularly for students in the language stream of

the school system, was more and more influenced by ideas

and teaching materials of a further education program

which was much more related to usefulness for a broad

part of the population than the usual upper secondary

mathematics courses. In 1981, this development was legitimated

by new regulations and, by that time, even mathematics

teaching in classes concentrating on mathematics

and physics became more and more influenced by the tendency

to put more emphasis on applications leading ultimately

to the draft of the new unified curriculum which is

now going to be put into practice.

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The central topic of a paper entitled *Fight against School*

*Failure in Mathematics,* presented by Josette Adda from the

Université Paris 7, was an analysis of social selective functions

of mathematical education. She reported statistical

data showing the successive elimination of pupils from the

«normal way at each decision stage of the school system

until only 16% of the 17 year age cohort remain whereas

all others have been put backward or relegated to special

types of classes. These eliminations hit selectively

socioculturally disadvantaged families. Research studies,

particularly at the Université Paris 7, have been undertaken

to find out why mathematics teaching as it is practised

today is not neutral but produces a correlation between

school failure in mathematics and the sociocultural environment.

They indicate the existence of parasitic sources

of misunderstanding increasing the difficulties inherent in

mathematics, e. g. embodiments of mathematics in pseudo-

concrete situations which are difficult to understand for

many pupils. On the other hand, it had been found that children

failing at school are nevertheless able to perform

authentic mathematical activities and to master logical operations

on abstract objects.

Two papers were based on the work of the EQUALS

program in the United States. This is an intervention program

developed in response to a concern about the high

dropout rate from mathematics courses, particularly in the

case of women and minority students. The program aims

to develop students’ awareness of the importance of

mathematics to their future work, to increase their confidence

and competence in doing mathematics, and to

encourage their persistence in mathematics.

In the first of these papers, *EQUALS: An Inservice*

*Program to Promote the Participation of Underrepresented*

*Students in Mathematics,* Sherry Fraser described the way

in which the program has assisted teachers to become

more aware of the problem and the likely consequences

for individual students of cutting themselves off from a

mathematical education. By working with teachers and

providing them with learning materials and methods, with

strategies for problem solving in a range of mathematical

topics, together with the competence and confidence to

use these, EQUALS has facilitated and encouraged a

transfer of concern to the classroom and attracted and

retained greater numbers of underrepresented students in

mathematics classes. Since 1977, 10,000 educators have

participated in the program.

Although the main focus of activity in the EQUALS program

has been on working with teachers and administrators,

needs expressed by these educators for

materials to involve parents in their children’s mathematical

education led to the establishment of *Family Math.* Virginia

Thompson described how this project has developed a curriculum

for short courses where parents and their children

can meet weekly to learn mathematical activities together

to do at home. This work reinforces and complements the

school mathematics program. Although the activities are

suitable for all students, a major focus has been to ensure

that underrepresented students, primarily females and

minorities, are helped to increase their enjoyment of

mathematics. The project serves to reinforce the aims of

the EQUALS program.

The move over the past ten years or so towards applicable,

real world and daily life mathematics in the

Netherlands, inspired by the work of Freudenthal, was described

by Jan de Lange Jzn. of OW and OC, Utrecht, in his

*paper Mathematics for All is No Mathematics at All.* Textbooks

have been published for primary and lower secondary

schools which reflect this view of mathematics, and research

shows that the reaction of teachers and students has

been very favourable. De Lange illustrated the vital role

played by applications and modelling in a newly-introduced

curriculum for pre-university students. Many teachers

apparently view the applications-oriented approach to

mathematics very differently from the traditional mathematics

content. The ultimate outcome, de Lange suggested,

may be that science and general subjects will absorb the

daily life use of mathematics and consequently this type of

mathematics might disappear from the mathematics curriculum.

That is, the ultimate for all students as far as mathematics

is concerned could in reality become no mathematics

as such.

Roland Stowasser from the Technical University of West

Berlin proposed in his paper, *Problem Oriented Mathematics*

*Can be Taught to All,* to use examples from the history of

mathematics to overcome certain difficulties arising from

courses based on a single closed system, which increase

mathematical complexity but do not equally increase the

applicability to open problems. He stated that mathematics

for all does not necessarily have to be directly useful, but it

has to meet two criteria: The mathematical ideas have to

be simple, and on the other hand, they have to be powerful.

He illustrated these criteria through a historical

example. Regiomantus formulated the problem to find the

point from which a walking person sees a given length high

up above him (e. g. the minute hand of a clock if the person

walks in the same plane as the face of the clock) subtending

the largest possible angle. The solution with ruler

and compasses in the framework of Euclidean geometry is

somewhat tricky. But according to Stowasser the teaching

of elementary geometry should not be restricted to Greek

tricks. For problem solving he advocated free use of possible

tools, and the solution of the problem is very simple if

trial and error methods are allowed. So the solution of the

historical problem represents the simple but powerful idea

of approximation.

What are the characteristics of a mathematics program

suitable for all students, and do any such programs exist?

These questions were addressed by Allan Podbelsek of

the United States in his paper, *Realization of a Mathematics*

*Program for All.* Podbelsek listed a number of criteria for

such a program covering not only content knowledge and

skills but also attitudes towards, and beliefs about, mathematics

and the process skills involved in its use. Mathematics

must be seen to be a unified, integrated subject, rather

than a set of individual, isolated topics. T h e

Comprehensive School Mathematics Program

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(CSMP) developed over several years in the United States

for elementary (K-6) level classes is found to meet these

criteria successfully in almost every respect. Practical problems

involved in the introduction of such a program as

CSMP to a school were discussed by Podbelsek. These

problems centred on the provision of adequate teacher

training for those concerned, meeting the cost of materials,

securing the support of parents and the local community,

and ensuring that administrative staff were aware of the

goals of the program.

Those translating mathematical, scientific or technical

material should have a basic knowledge of mathematics to

do their job satisfactorily, yet because of their language

background they are not likely to have studied mathematics

to any great extent at school. This is the experience

which led Manfred Klika, of the Hochschule Hildesheim in

West Germany, to a consideration of the nature and adequacy

of present school mathematics programs in his

paper *Mathematics for Translators Specialised in Scientific*

*Texts - A Case Study on Teaching Mathematics to*

*Non-Mathematicians.* Conventional school programs, he

claimed, do not prepare students to comprehend and make

sense of mathematical ideas and terminology. The solution

is to construct the mathematics curriculum around fundamental

ideas. Two perspectives on this notion are offeredmajor

anathematising models (e. g. mathematical

concepts, principles, techniques, etc.) and field-specified

strategies suitable for problem solving in mathematics (e.g.

approximate methods, simulation, transformation strategies,

etc.). A curriculum based on such fundamental ideas

would result in more meaningful learning and thus a more

positive attitude to the subject. A course based on this

approach has been established at the Hochschule

Hildesheim within the program for training specialist translators

for work in technical fields.

The major concern of the preceding contributions to the

topic “ Mathematics for All ” were problems of designing a

mathematics curriculum which is adequate to the needs

and the cognitive background of the majority in industrialised

countries. The organising committee of the theme

group was convinced that it is even more important to discuss

the corresponding problems in developing countries.

But it was much more difficult to get substantial contributions

in this domain. To stress the importance of the development

of mathematical education in developing countries,

the work of the theme group terminated with a panel discussion

on *Universal Mathematical Education in Developing*

*Countries,* with short statements of major arguments by

Bienvenido F. Nebres from the Philippines, Terezinha N.

Carraher from Brazil, and Achmad Arifin from Indonesia,

followed by the reactions of Peter Towns and Bill Barton,

both from New Zealand. The discussion concentrated on

the relation between micro-systems of mathematical education

like curricula, textbooks and teacher training and

macro-systems like economy, culture, language and general

educational systems which, particularly in the developing

countries, often determine what kind of developments

on the level of micro-systems are possible. Bienvenido F.

Nebres expressed the common conviction of the participants

when he argued that, in spite of the fact that often it

is impossible to get a substantial improvement of

mathematics education without fundamental changes in

the macro-systems of education, micro-changes are possible

and are indeed a necessary condition to make people

realise what has to be done to get a better fit between

mathematical education and the needs of the majority. This

result of the discussion highlights the importance of the

papers submitted to the theme group dealing with special

aspects of mathematical education in developing countries.

Three reports were given by David W. Carraher,

Terezinha N. Carraher and Analucia D. Schliemann about

research undertaken at the Universidade Federal de

Pernambuco in Recife, Brazil. David W. Carraher prepared

a paper titled *Having a Feel for Calculations* about a study

investigating the uses of mathematics by young, schooled

street vendors who belong to social classes characteristically

failing in grade school, often because of problems in

mathematics, but who often use mathematics in their jobs

in the informal sector of the economy. In this study, the

quality of mathematical performance was compared in the

natural setting of performing calculations in the market

place and in a formal setting similar to the situation in a

classroom. Similar or formally identical problems appeared

to be mastered significantly better in the natural setting.

The reasons were discussed and it was stated that the

results of the analysis strongly suggest that the errors

which the street vendors make in the formal setting do not

reflect a lack of understanding of arithmetical operations

but rather a failing of the educational system which is out

of touch with the cognitive background of its clientele.

There seems to be a gulf between the intuitive understanding

which the vendors display in the natural setting and

the understanding which educators try to impart or develop.

Terezinha N. Carraher reported in her paper *C a n*

*Mathematics Teachers Teach Proportions?* results of a

second research project. Problems involving proportionality

were presented to 300 pupils attending school in

Recife, in order to find out whether a child already understands

proportions if it only follows correctly the routines

being taught at school. The results indicate characteristic

types of difficulties appearing in certain problems, some of

which can be related to cognitive development. It is suggested

that teachers’ awareness of such difficulties may

help to improve their teaching of the subject. For if mathematics

is to be useful to everyone, mathematics teachers

must consider carefully issues related to the transfer of

knowledge acquired in the classroom to other problem solving

situations.

The third paper, presented by Analucia D. Schliemann,

*Mathematics Among Carpentry Apprentices: Implications for School*

*Teaching,* highlighted the discontinuity between formal school

methods of problem solving in mathematics and the informal

methods used in daily life. This research study contrasted the

approaches to a practical problem of quantity estima-

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tion and associated calculation taken by a group of experienced

professional carpenters without extensive schooling,

and a group of carpentry apprentices attending a formal

school system and with at least four years of mathematics

study. The results showed that apprentices approached

the task as a school assignment, that their strategies

were frequently meaningless and their answers absurd. On

the other hand, the professional carpenters took it as a

practical assignment and sought a feasible, realistic solution.

Very few computational mistakes were made by either

group but the apprentices appeared unable to use their formal

knowledge to solve a practical problem. Schliemann

concluded that problem solving should be taught in practical

contexts if it is to have transferability to daily life situations

out of school.

Pam Harris from the Warlpiri Bilingual School discussed

in her paper, *Is Primary Mathematics Relevant to Tribal*

*Aboriginal Communities?,* the problem that, in the remote

Aboriginal communities of Australia, teachers often get the

feeling that mathematics is not relevant. Several reasons

can be identified. Teachers often receive negative attitudes

from other people so that they go to an Aboriginal community

expecting that their pupils will not be able to do mathematics.

Furthermore, they observe a lack of reinforcement

of mathematics in the pupils’ home life. Teaching materials

mostly are culturally and linguistically biased. Teachers feel

discouraged because of the difficulties of teaching mathematics

under these conditions. Nevertheless, Pam Harris

stressed the importance of mathematics, because

Aboriginal children have to get an understanding of the

Second culture» of their country. They need mathematics

in their everyday life, in employment, and in the conduct of

community affairs. But to be successful, mathematics

teaching in Aboriginal communities has to allow for and

support local curriculum development. Individual schools

and language groups should make their own decisions on

the use of the children’s own language, the inclusion of

indigenous mathematical ideas, priorities of topics, and

sequencing the topics to be taught.

Kathryn Crawford, from the College of A d v a n c e d

Education in Canberra, presented a paper on *Bicultural*

*Teacher Training in Mathematics lEducation for Aboriginal*

*Trainees from Traditional Communities* in Central Australia.

She described a course which forms part of the Anagu

Teacher Education Program, an accredited teacher training

course intended for traditionally oriented Aboriginal

people currently residing in the Anagu communities who

wish to take on greater teaching responsibilities in South

Australian Anagu schools. The most important difference

between this teacher training course and many others is

that it will be carried out on site by a lecturer residing within

the communities and that, from the beginning, development

of the curriculum has been a cooperative venture between

lecturers and educators on the one hand, and community

leaders and prospective students on the other. The

first group of students will begin the course in August 1984.

The course is particularly designed to meet the fact that different

cultures emphasise different conceptual schemes.

Thus, temporal sequences and quantitative measurement

are dominant themes in industrialised Western cultures but

largely irrelevant in traditional Aboriginal cultures. To overcome

these difficulties, the focus of the problem is redirected

from the “failings” of Aboriginals and Aboriginal

culture to the inappropriateness of many teaching practices

for children from traditionally oriented communities.

The course has been developed based on a model designed

to maximise the possibility of interaction between the

world view expressed by Anagu culture and that of

Anglo-European culture as evidenced in school mathematics.

This is achieved by placing an emphasis on the student

expertise and contribution in providing information

about Anagu world views as a necessary part of the course.

In this community based teacher training course, it

seems that it is possible for the first time to develop procedures

for negotiating meanings between the two cultures.

**3. Conclusions**

The presentations given at the sessions of the theme

group summarised above can be considered as important

efforts to contribute to the great program of teaching

mathematics successfully not only to a minority of selected

students but teaching it successfully to all. But in spite of all

these efforts it has to be admitted that the answer to. the

question, What kind of mathematics curriculum is adequate

to the needs of the majority?», is still an essentially open

one. However, the great variety of the issues connected

with this problem which were raised in the presented

papers makes it at least clear that there will be no simple

answer. Thus the most important results of the work of this

theme group at ICME 5 may be that the problem was for

the first time a central topic of an international congress on

mathematical education, and that, as the contributions

undoubtedly made clear, this problem will be one of the

main problems of mathematical education in the next decade.

As far as the content of these contributions is concerned,

the conclusion can be drawn that there are at least three

very different dimensions to the problem which contribute

to and affect the complex difficulties of teaching mathematics

effectively to the majority:

- the influence of social and cultural conditions;

- the influence of the organisational structure of the school

system;

- the influence of classroom practice and classroom

interaction.

*Cultural Selectivity*

One of the major underlying causes of the above problem

is the fact that mathematical education in the traditional

sense had its origins in a specific western European cultural

tradition. The canonical curriculum of «Tr a d i t i o n a l

mathematics» was created in the 19th century as a study

for an elite group. It was

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created under the conditions of a system of universal basic

education which included the teaching of elementary computational

skills and the ability to use these skills in daily life

situations. There is a clear distinction between the aims

and objectives of this basic education and the curriculum of

traditional school mathematics which was aimed at formal

education not primarily directed at usefulness and relevance

for application and practice. This special character of

the canonical mathematics school curriculum is still essentially

the same today in many countries.

The transfer of the European mathematics curriculum to

developing countries was closely associated with the establishment

of schools for the elite by colonial administrations.

Under these circumstances it seemed natural to simply

copy European patterns. It is quite another problem to

build a system of mass education in the Third World and

embed mathematics education in both the school situation

and the specific social and cultural contexts of that world.

The papers summarised above point clearly to some of

the problems. Curricula exist which encourage students to

develop antipathies towards mathematics; this is commonly

the case in Europe. Further, such curricula have sometimes

been transferred to countries where the social

context lacks the culturally based consensus that is found

in Europe, namely, that abstract mathematical activity is

good in itself and must therefore be supported, even if it

seems on the surface to be useless. It has been proposed

on the one hand, that a sharp distinction should be made

between applicable arithmetic in basic education and essentially

pure mathematics in secondary education, and on

the other hand, that mathematics should be integrated into

basic technical education. This argument raises the question

of the relation between mathematics and culture which

may be the first problem to address when the idea of

mathematics for all is raised as a basis for a program of

action.

*Selectivity of the School System*

While the particular curricular patterns of different societies

vary, the subject is still constructed in most places so that

few of the students who begin the study of mathematics

continue taking the subject in their last secondary years.

The separation of students into groups who are tagged as

mathematically able and not able is endemic. Curricula are

constructed from above, starting with senior levels, and

adjusted downwards. The heart of mathematics teaching

is, moreover, widely seen as being centered on this curriculum

for the able, and this pattern is closely related to

the cultural contexts indicated above. However, we must

consider the problem of conceiving, even for industrialised

societies, a mathematics which is appropriate for those

who will not have contact with pure mathematics after their

school days. Up to now we have made most of our students

sit at a table without serving them dinner. Most

attempts to face the problem of a basic curriculum reduce

the traditional curriculum by watering down every

mathematical idea and every possible difficulty to make it

feasible to teach the remaining skeleton to the majority.

There is only a limited appeal to usefulness as an argument

or a rationale for curriculum building to avoid the

pitfalls of this situation. Students who will not have to deal

with an explicit area of pure mathematics in their adult lives

but will face instead only the exploitation of the developed

products of mathematical thinking (e. g. program packages),

will only be enabled by mathematics instruction if

they can translate the mathematical knowledge they have

acquired into the terms of real-life situations which are only

implicitly structured mathematically. Very little explicit

mathematics is required in such situations and it is possible

to survive in most situations without any substantial mathematical

attainments whatsoever.

Is the only alternative to offer mathematics to a few as a

subject of early specialisation and reject it as a substantial

part of the core curriculum of general education? This

approach would deny the significance of mathematics. To

draw this kind of conclusion we would be seen to be looking

backwards in order to determine educational aims for

the future. The ongoing relevance of mathematics suggests

that a program of mathematics for all implies the

need for a higher level of attainment than has been typically

produced under the conditions of traditional school

mathematics — and that this is especially true for mathematics

education at the level of general education. In other

words, we might claim that mathematics for all has to be

considered as a program to overcome the subordination of

elementary mathematics to higher mathematics, to overcome

its preliminary character, and to overcome its irrelevance

to life situations.

*Selectivity in Classroom Interaction*

Some of the papers presented in this theme group support

recent research studies which have suggested that it

is very likely that the structure of classroom interaction

creates ability differences among students which grow

during the years of schooling. In searching for causes of

increasing differences in mathematical aptitude, perhaps

the simplest explanation rests on the assumption that

such differences are due to predispositions to mathematical

thinking, with the further implication that nothing can

be done really to change the situation. But this explanation

is too simple to be the whole truth. The understanding

of elementary mathematics in the first years of primary

school is based on preconditions such as the acquisition

of notions of conversation of quantity which are, in their

turn, embedded in exploratory activity outside the school.

The genesis of general mathematical abilities is still little

understood. The possibility that extra-school experience

with mathematical or pre-mathematical ideas influences

school learning cannot be excluded. Furthermore, papers

presented to the theme group strongly suggest that the

d i fferences between intended mathematical understanding

and the understanding which is embedded in normal

classroom work is vast. We cannot exclude the possibility

that classroom interaction in fact produces growing differences

in mathematical aptitude and achievement by a

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system of positive feedback mechanisms which increase

high achievement and further decrease low achievement.

It is clear that to talk of mathematics for all entails an

intention to change general attitudes towards mathematics

as a subject, to eliminate divisions between those who are

motivated towards mathematics and those who are not,

and to diminish variance in the achievement outcomes of

mathematics teaching. This, in its turn, involves us in an

analysis of social contexts, curricula and teaching. It is

these forces together which create a web of pressures

which, in turn, create situations where mathematics

becomes one of the subjects in the secondary school in

which selection of students into aptitude and ability groups

is an omnipresent reality almost from the time of entry.

**Part I:**

**Mathematics for All –**

**General Perspectives**

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**A Necessary Renewal of Mathematics**

**Education**

Jean-Claude Martin

Mathematics for all must not only be accessible mathematics,

but interesting mathematics for all - or for the majority.

Such a theory leads one, in the case of the teaching of

mathematics in France, to raise problems of objectives and

curriculum organisation, but also of methods more than of

the content of the curriculum.

**1. The General Characteristics of Selective Education**

*(i) The Fundamental Teaching of Mathematics*

*for Mathematics Sake*

Mathematics as they are known today may be considered,

if not as a whole, as a system. The training of the highest

level of generalist mathematicians may a priori be defined

as leading to knowledge of this system.

Dividing the system of mathematics into parts going

from the simplest element to the most complicated may

represent, as a first approximation only, but quite logically,

a curriculum of study for the training of mathematicians.

That is what we shall call, to serve as a reference for our

later discussions, the teaching of mathematics for mathematics

sake. Its organisation in the form of a continuous

upward progression implies that each level reached will be

a prerequisite for the level immediately following.

Such a curriculum does not exist in the pure state but it

appears to be the foundation, the skeleton of most programs

of general mathematical training in many countries,

being a reflection of the European rationalist cultural tradition.

Adaptations of this consist essentially in heavier or lighter

pruning, stretching to varying degrees the progression,

or illustrating it to some extent by an appeal to real-life

experience (either in order to introduce a notion or to

demonstrate some application of it).

The first question raised then is whether such teaching

is a suitable basis for mathematics for all.

On the level of objectives, the reply is obviously negative:

The training of mathematicians can interest only a

minute portion of students.

*(ii) Selection by Means of Mathematics*

In France, statistics show that of any 1,000 students entering

secondary education, fewer than 100 will obtain seven

years later a scientific baccalaureat (including section D)

and a maximum of five will complete tertiary studies in

mathematics or related disciplines (computer science in

particular).

Referring again to statistics indicates that only about

one successful candidate at the baccalaureat in six holds

one of the types of baccalaureat (C or E) in which mathematics

are preponderant. That fact, together with other

indications concerning class counselling, brings out sufficiently

clearly the importance of selection — a well enough

known phenomenon anyway — by mathematics in the

secondary school. This selectivity appears moreover to be

relatively stronger than at university. This situation makes

mathematics a dominant subject. French, which formerly

shared the essential role in selection, is now relegated to a

secondary position.

This selection is manifested most often by a process of

orientation through failure for students at certain levels. But

in fact, this sanction is usually only the deferred result of an

ongoing selection process which takes effect cumulatively.

From the primary school, or as early as the first years of

secondary school, the classification between «Maths» and

«non-maths» students becomes inexorably stratified.

In recent years, the idea that selection through mathematics

is equivalent to a selection of intelligent students

has made some progress, even if it is only very rarely

expressed in such a clear way.

This function as the principal filter of the education system

has considerably harmed the prime constructive function

of mathematics as a means of training thought processes

by the practice of logical reasoning. Just as a filter

naturally catches waste, so mathematics produce academic

failures inherent in the system, in other words, not due

to intrinsically biological or psycho-effective causes but to

the teaching process itself.

The type of evaluation used is not unconnected. It has

the general fault of all standardised evaluation as is still too

widely practised.

It supposes a definition of the child’s normality that

pediatricians and psychologists contests1,2 : ranges of

development, differences in maturity are just as normal

and natural as differences in height and body weight. The

same applies to the formation and development of abstract

thought, which one must expect to be facilitated by the teaching

process and not measured and sanctioned by it.

Aptitude for abstraction seems to be generally considered,

with intelligence, as having an essentially innate character,

whereas it is admitted by researchers that the share

acquired in the social and family milieu and then at school

is probably preponderant.

The demands of «Levels of intelligence» are also judged

excessive for the teaching situation (first years of

secondary school). They would necessitate2 a clearly

above average IQ.

On this subject we may note a very important lack of

coordination between the quite reasonable programs and

instruction of the Inspectorate General of Teaching and the

contents of textbooks.

We shall see later some questions concerning the vocabulary

used, but where the program considers only arithmetic

or operations on whole numbers or rational numbers,

it can be seen that in fact a veritable introduction to algebra

is carried out.

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*(iii) Emotional Responses and Mathematics*

All teaching is obviously subject to emotional responses:

the student likes this, doesn’t like that, prefers this, and so

on. As far as mathematics is concerned, successful students

acquire an assessment based on a harmonious relationship,

but those who have difficulties feel strong emotions

that can induce suffering and anguish.4

**II. Problems of Language, Symbolic Writing**

A mathematical apprenticeship requires the acquisition of a

special language which is characterised by the interlocking

of a conventional language (with nevertheless its own

semantics and syntax) and a symbolic language.

If, beside their communicative aim, all languages serve

as a medium of thought — according to Sapir: The feeling

that one could think and indeed reason without language is

an illusions — the language of mathematics, more than any

other, is adapted to that very end. The sentence (containing

words) and the formula with its symbols are vehicles

of logical reasoning. In this area, symbolic writing is considerably

more powerful than conventional writing: one can

say that it is a motive force driving thought ahead more

rapidly.

*(i) The Power of Symbols*

On the occasion of the 4th International Congress on the

Teaching of Mathematics (ICME 4), Howsons5 clearly showed

the power of symbols, which one could have thought

in the first analysis to be only tricks of abbreviation, whereas

they do generate new meanings.

As essential elements of mathematics, they permit the

discipline to develop without its being necessary to burden

our thought processes with all the meanings with which

they are charged. A language open to independent development,

symbolic writing lends itself to operations the

automatic nature of which, once it is acquired, saves

conscious thought or at the very least permits considerable

economies in the process of reflexion.

An apprenticeship in symbolic writing and the attendant

operational procedures is therefore essential in the teaching

of mathematics.

*(ii) The Importance of Language Acquisition*

The nature of symbolic writing being a capacity for

self-development, if what has been learned in this area is

already considerable, the student will have no major difficulty

in acquiring the language necessary if he is to pass to

the next stage. Thus his difficulties will reside rather in the

structures of reasoning than in a knowledge of symbols. It

may be considered that this is the case of students in the

upper classes of secondary school.

On the other hand, at the beginning of this apprenticeship

(notably when algebra is introduced), the change

from the natural language to symbolic language, because

it is a prerequisite, no doubt has a special place in the hierarchy

of difficulties.

*(Iii) The Difficult Changeover to Symbolism*

The changeover from natural language to symbolic language,

as well as the problems caused by too rapid or too

early an introduction (poorly adapted to the development of

the thought processes of the student and his maturity) carries

with it some more technical difficulties, which in our

view have not been satisfactorily solved.

Symbolic formulation is more than mere translation. The

physicist is well aware of this, considering as he does

today this operation, called (mathematical) modelling, as

being of prime importance in the analysis of complex phenomena

or systems. In the same way, the return from the

formula to realist is an exercise that is not self-evident and

a table of correspondences and a dictionary will not suffice.

Symbolism introduces first of all a complication.

Afterward, naturally, when the obstacle is overcome, one

profits as a result of a simplification of procedures (automatic

responses in operations and their reproduction).

If one can solve a problem in ordinary language with a

level of difficulty N1, to use for its solution a poor knowledge

of symbolic language makes it more difficult (level N2).

On this subject the tests of C. Laborde7 seem significant.

Confronted with solving concrete problems or describing

mathematical objects, students do not use the codes they

have learned. But once the symbolism is better known, the

level of effort to attain the same goal is less. Level N3 is for

example the level of effort required of the master mathematician.

This summary demonstrates at the same time the

advantage of learning mathematics and the difficulty there

is, starting with the concrete description of a problem to formulate

it in mathematical terms. It also shows that the teacher

should give considerably more attention to lessening

the difficulty of acquiring the mathematical metalanguage

than accumulating purely mathematical knowledge.

*(iv) The Necessity of Introducing Stages Useful for*

*Conceptualisation*

G. Vergnaud8 has demonstrated that, when solving problems,

students used Faction theorems» or implicit theorems,

which were simply the products of their personal

conceptualisation revealing the workings of individual

thought processes. Several researchers have noted that

such processes did not follow the shortest path of the

mathematics taught nor the best method from the point of

view of logical rigour.

Because of this, it is often considered by the teacher to

be bad reasoning - to be done away with as quickly as possible

in favour of classical mathematical reasoning — whereas

it is rather logical reasoning in the process of developing.

The act of teaching, instead of ignoring or indeed rejecting

the representation constructed by the pupil, his own

personal mechanisms of thought, should consist, on the

contrary, in revealing these, understanding them and using

them.

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As long as educational research does not provide practical

ways of accomplishing this development, it is no doubt

right to give to the acquisition of symbolic logic a more

important share in the teaching process.

Inspiration may come from the evolution of symbolism in

mathematics through the centuries.

Howson alludes to this5 and the analogy of the evolution

of the individual’s knowledge according to Piaget’s theory.

Those noted in physics are arguments in the same direction;

if one begins with the hypothesis that human logic can

exist, it is likely that there are similarities.

But above all, so that the student finds his way naturally,

we should propose to him varied representations of the

same thing: “a supple and changing, suggestive and logical

formalism ” according to Lowenthal.9 We come back to

the recommendation of Howson and Brandsond: «no symbol

or contraction should be introduced if the student is not

ready fully and reasonably to appreciate the advantage it

offers».

We consider that the use of natural language along with

symbolic language can not only better guarantee the

acquisition of the symbolic language5,6 but above all serve

as a better basis or guide for the logical reasoning associated

with mathematical development.

*(v) Avoidable Difficulties*

As well as the intrinsic difficulties in the acquisition of the

symbolic language of mathematics, there exist difficulties

that one could avoid, growing out of the language used to

mediate between natural language and symbolic language.

This is the language used by teachers or school text to

give definitions, enunciate properties and theorems and to

provide the necessary explanations for beginners.

The language used by teachers is obviously very diverse

and varied, and there is no doubt that large numbers

of them know how to adapt as is necessary. In France the

General Inspectorate of Education encourages them to do

so. It recommends in particular that they avoid the introduction

of too many new words.

But if one considers school textbooks, one can ponder

whether these instructions have been taken into consideration.

The intellectual worth of the authors is not in question,

and one must seek the reason in an insufficient realisation

of the importance of the linguistic vehicle. We have

used a textbook for the level known as «5e» where, exceptionally,

a first chapter is devoted to helping in understanding

the terms used in the body of the text. So as to draw

a conclusion «a fortiori» we subjected this chapter to a test

for the «classification of texts according to the difficulty of

the approach required for understanding them» used in

technical education to select documents for students

according to their academic level.

This test has no pretentions to scientific perfection but

the results achieved demonstrate its pertinence.

The result is edifying: With respect to the French used,

this test should be given only to students three or four

years older. The analysis of difficulties shows essentially:

1. that the vocabulary used includes too many words

which are not part of the everyday language of the

student;

2. that certain known words are used in different

senses (paronyms);

3. that there is a supposition of certain references of

experience (not only mathematical but also of a

cultural nature);

4. that the structure of typical phrases aimed at

mathematical precision causes ambiguities on the

level of the French language.

As for the first two of these four observations, we carried

out a summary evaluation of the vocabulary requirements

of five of the most widely used textbooks. With respect to

the first level of the basic French vocabulary (representing

between 1,300 and 1,500 words) the comprehension of

the French used as a vehicle for mathematics teaching

(not including symbols) requires the knowledge of 100 to

150 new words or expressions.

In this body of material, the words that seem to be

known but which are used with a different meaning created

a doubly negative effect; they are not passive obstacles

to comprehension, but introduce confusion.

Several researchers1 0 have demonstrated this undesirable

effect of the most common of these: if (and only

if), then, and, or (exclusive), all ... These fundamental

words should be introduced with the same care as symbols

for they are not stepping stones to symbolic language,

they are merely its image.

Elements supporting reasoning, they need to be perfectly

assimilated so that the correct reasoning may be carried

out. But they are not the only ones that cause the specialised

language of mathematics to be in fact very different

from natural language. Other less frequent uses, as

well as syntax, increase the difficulty.

On this subject, one can raise questions (ii) concerning

the origin of language difficulties in mathematics. Do they

come simply from an insufficient mastery of the natural language?

Such a deficiency obviously introduces a handicap.

But the quite widespread existence of students classified

as «Literary» and «non-mathematical» shows clearly

enough that it is not sufficient to know French better in

order to understand mathematics.

Does not the difficulty of access to formal language also

reside in the incapacity of natural language to translate it

without weighing it down or even deforming it? This is

obvious for the initiate, to whom the formula offers a richer

meaning than the theorem that attempts to express it. The

connection, indeed the interdependence between the

mechanisms of formation of thought and of formal language,

still insufficiently known (cf. different hypotheses of

Piaget, Bruner, etc.) also lead to questions about an

influence of one upon the other (and vice versa).

But the fact that the question is so open to discussion

does not free the teacher from considering the more down

to earth problems of vocabulary and syntax. One should

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not write an introductory manual of algebra (or of other

areas in mathematics that make considerable use of symbolic

writing) without having the French corrected by a specialist

in reading.

Thus one would create the most direct contact possible

between natural expression and symbolic expression. And

if it were realised—which is likely— that short-cut explanations,

by means of Typical mathematical discoursed are not

practical, perhaps one would attain, at the cost of an apparent

waste of time, better comprehension.

**III. Proposals for the Teaching of Mathematics for A l l**

**Students**

From this analysis of faults and difficulties result some

paths that may lead to improvements in teaching.

*(i) Restore the Role of Mathematics as a Tool*

In our highly technological age, everybody no doubt needs

some background in mathematics.

That is why the teaching of mathematics as a tool ought to

be of interest to the majority on condition that, by taking

certain precautions, it is made sufficiently accessible. On

this condition it appears to us the only viable basis on

which to found the structure of mathematics for all.

But what form can such instruction take? It could not be

limited to a curriculum adapted to professional ends

constructed on a basis comparable with that of basic teaching.

For example, although the successive introduction

to algebra and then differential and integral calculus can

furnish a tool for the solution of problems of mechanical

physics, if it does not gradually reveal its concrete basis

and its applications we shall not consider it as instruction in

mathematics as a tool.

The pedagogical procedure too often used consists of

asking the student to acquire numerous prerequisites and

to await the whole construction piece by piece of the cognitive

edifice in order to perceive at last the end to which it

can be put means the teacher is avoiding his responsibilities

and it kills the student’s motivation .

Teaching mathematics as a tool means giving permanent

priority to the solving of problems and not to learning formal

aspects of the discipline. Pedagogically there are two

great advantages in this:

- we have seen that excessive formalism or too early an

introduction of symbolism was an obstacle in the early

stages;

- it is now allowed9 that the development of logical reasoning

is carried out essentially on the basis of experience

in problem solving.

The basic notion is to replace the upward progression in

mathematics isolated by its formalism by a spiral progression

dependent on other disciplines. This presupposes

undertaking at each stage of initiation an adaptation of teaching

methods inspired by research on language and

conceptualisation5,6,7,9,12.

- arouse interest in a problem ‘set the right type of problem);

- bring the student to pose it in logical terms, to translate

it into already familiar mathematical terms (modelling)

and thus bring home the practical application of mathematics;

- give practice in the corresponding operations;

- show the polyvalence and indeed the universality of

methods of logical reasoning, the utility of formalism;

- let the student measure from time to time the resultant

enrichment of his capacities in the area to which the

subject of the problem belongs;

- bring the student finally to a higher mathematical level.

*(ii) The Place of Mathematical Modelling*

One point that seems to us fundamental is the introduction

of «modelling». Here again one can see the fruit of the physicist’s

experience, but such a procedure is in our view

necessary more as a result of our earlier pedagogical

considerations concerning the difficulty of acquiring symbolic

language. «Modelling» or translating the concrete

problem into pertinent mathematical terms does not come

easily. The teacher must make a special study of the question:

- how does one, when faced with a more or less complex

system, observe it, identify it, express its workings in formal

relationships?

- how, when faced with a concrete problem, does one

describe it, translate it into equations?

- how, thanks to the tools of mathematics, does one progress

in one’s understanding, one’s solution of it?

- how finally, at the more sophisticated stages, does one

iterate identification and modelling to the limits of one’s

own knowledge?

Thanks to such a process, the teacher will be able to facilitate

the student’s conceptualisation. As a result of special

attention to the problem and the development of an open

educative process, the teacher will be able to follow the

student’s «natural» principles of reasoning, reveal the formation

of «theorems in action» mentioned above, and thus

facilitate by an appropriate pedagogical method the development

of these into true theorems.

Modelling thus leads into creativity and technological processes.

*(iii) The Necessity of an Interdisciplinary Approach*

An intensive use of modelling requires the mathematics

teacher to have a good knowledge of the applications of

mathematical tools in a variety of areas and makes it

necessary for the development of his teaching to be kept in

line with other disciplines using mathematics. This supposes

not only a basis in team work, but also in a national

curriculum and general interdisciplinary planning. A better

solution would no doubt be the creation of true multidisciplinary

subjects, an added advantage of which would be to

link up again areas of knowledge that the division into disciplines

has fragmented or simply overlooked.

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*(iv) Mathematics as a Tool and Mathematical Culture*

There is no inherent conflict between mathematics as a

tool and mathematical culture, one being able to lead to the

other and vice versa. Restoring the teaching of mathematics

as a tool will allow us to interest students and to offer

them greater possibilities of success and self-development

in the modern environment.

This is the path that seems the most certain to lead to

mathematics for the majority, through a process of success

and not of lowering standards.

*(v) Teaching Through Goals with a Differentiated*

*Progression*

The working out of such a system of teaching would imply

avoiding a drop in standards through an evaluative process

based on objectives that clearly marked out the development

of the curriculum, the chronology of which would be

subject to modification and would permit the most gifted

students to advance more quickly and those in difficulty to

follow at a different pace.

It is accepted that between the beginning of secondary

school and the baccalaureat the majority of students repeat

a year once or twice, and this gives room in the curriculum

and the means of attaining a differentiated progression.

The abrupt and penalising nature of repeating a year

when one begins everything over again, even the things in

which one has been successful, would be attenuated and

greater consideration would be given to the timing of initiation,

work and development.

It has always been accepted for a diploma like the baccalaureat

that differences of level should be tolerated in

various disciplines. Would it be fatal to experience failure in

mathematics between preschool kindergarten and the A or

B baccalaureat? Would it not be better to reach this stage

by a well organised progression and natural orientation

rather than in fits and starts with futile intermediate sanctions,

since in the end students will reject or avoid mathematics

if they cannot succeed.

Would it not be better to provide for success in slow

stages, or related to more limited objectives, rather than to

suffer failure so fully and so prematurely internalised that it

leads numerous students and then adults to a veritable

lack of mathematical culture?

**Conclusion**

Underlying the no doubt imperfect proposals presented

above is a deeper question of objectives.

Will mathematics, rather than being a filter of the elite,

recover their principal function of being the most wonderful

of tools (albeit an immaterial one), of being the way of teaching

logical reasoning?

By laying aside the attributes that make them forbidding

(language, abstraction), by capitalising on their interest and

power, mathematics could be accessible and interesting for

the majority of students, who would all reach their appropriate

level.

Finally, even in the event of only relative success, one

could restore the supply of scientists and mathematicians

that has dried up radically in recent years. Statistically one

would no doubt also achieve a better quality elite.

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17

**The Problem of Universal Mathematics**

**Education in Developing Countries**

Bienvenido F. Nebres, S. J.

In his paper «Mathematics for All— Ideas, Problems,

Implications,» Peter Damerow underlines two c e n t r a l

concerns. The first is that the canonical school curriculum

for mathematics was designed for an *elite* and so there are

serious adjustment problems when it is made universal.

The second is that it was designed for a *European* elite and

so the adjustment problems become even more serious

when it is introduced into the mass educational system of

a developing country.

«I think firstly we have to consider the fact that mathematical educa -

tion in the traditional sense has its origin in a specific Western

European cultural tradition, where the canonical curriculum of traditional

school mathematics was created in the course of the l9th century.

The transfer of this curriculum to developing countries in most

cases has been closely linked with the institutionalization of schools

by colonial administrations in these countries. It is well known that

these schools generally were attended only by an elite, which adopted

the Western European culture and often studied afterwards at

European universities. Under these conditions it seemed natural simply

to copy the curriculum of higher education. But it is quite another

problem to build up a system of mass education in the countries of

the Third World and to embed mathematical education into the specific

cultural contexts of these countries. «1

In a paper entitled *Problems of Mathematical Education in*

*and for Changing Societies: Problems in Southeast Asian*

*Countries,* which was presented at the October, 1983

Regional Conference in Tokyo, Japan, I tried to classify

mathematical education problems into two types,

micro-problems and macro-problems.

«*We can classify problems of mathematical education into two types:*

*the* first *we might can* micro-problems *or problems internal to mathe -*

*matical education. These would relate to questions of curriculum,*

*teacher-training, textbooks, use of calculators, problem-solving and*

*the like. The* second *we might can* macro-problems. *These are pro -*

*blems affecting mathematics education because of pressures from*

*other sectors of society: economy, politics, culture, language, etc.*

*One of the features of a* developed *society is a reasonable differen -*

*tiation of sectors and functions of society. While given sectors are, of*

*course, interdependent and affect one another, they also have some*

*reasonable autonomy. School budgets may increase or decrease,*

*but they have some stability and so it is possible to plan. Teachers*

*get a sufficient {though not high} salary so they can concentrate on*

*their teaching chores. But in contrast, structures in developing socie -*

*ties are not sufficiently developed to provide (for example) education*

*and culture with sufficient freedom from the pressures of politics and*

*economics Teachers may be called upon to perform many civic*

*duties - to the detriment of their classroom work. Their salaries may*

*nor be sufficient for them to be able to concentrate on their work.*

*Budgets may be unstable and information and opinion tightly control -*

*led ,* «*2*

In that paper I discussed the problem of universal mathematical

education for developing countries, mainly in terms

of economic constraints.

«The problem I would like to concentrate on here is that of the great

number of students who are in school only for four to six years. One

must, therefore, give them functional numeracy within severe

constraints. The time constraint is obvious. There are also problems

of scarcity of textbooks, not-so-well-trained teachers, language. We

might focus the question on only one aspect of the problem: curriculum.

In the Philippines, at least, the curriculum is the same whether

a student goes on for ten years through high school (or even beyond

to university) or whether the student stops after four to six years. I

propose the following questions. From a study of the curriculum and

from experience, at what point is functional numeracy realistically

achieved? After four years? After six years? After eight years? If one

were to look at the curriculum from the point of view of best helping

a student who will stay only for four to six years, would one redesign

the curriculum?»3

**However, on further reflection it seems clear that the** deeper

problem is, as is noted by Peter Damerow, cultural. «So

I think the relations between mathematics and culture is

the first and maybe the most general question which

arises when mathematics for all is taken as a program.

«4 For developing countries, the problem of mathematics

education and culture may best be understood by reflecting

on the history of the school system in these countries.

«All of the countries of Southeast Asia, with the exception of

Thailand, went through a prolonged colonial period. During the colonial

period, the school system was patterned exactly after that of the

colonising country. The norms of fit between school and society were

quite precise: the school system was to come as close as possible to

that of the mother country. It should produce graduates that would fit

into the civil service and who would do well in universities in the

mother country. With independence the above norms of fit between

school and society were seen with mixed feelings. Leaders became

conscious that a school system developed according to such norms

would, among other things, simply contribute to the brain drain. They

also became conscious that the school system had to respond to different

cultures and classes in the country: a westernized elite, a growing

lower middle class, urban workers, a traditional rural sector. The

aspirations for progress and equality led to new questions about the

role of the school system in society:

- Can the school system provide functional literary and

functional numeracy to the great number who attend

school only for four to six years?

- Can the school system provide the scientific and mathematical

skills for different levels in the agricultural, commercial,

and industrial work force?

- Can the school system train sufficiently well the small but

important number needed for leadership in the scientific

and economic sectors?

These are, of course, very difficult tasks. The specific problem faced

by the school system in many developing societies is that the society

at large expects it to fulfill the society’s dream of progress and

equality. These place unrealistic pressures on the school system.»5

1 Damerow, P. (1984): Mathematics for All - Ideas, Problem,

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2 Nebres, B. (1983): Problems of Mathematical Education in and for

Changing Societies - Problems in Southeast Asian Countries. In:

Proceedings of the lCMl-JSME Regional Conference on

Mathematical Education, Tokyo, p. 10.

3 lbid., p. 16.

4 Damerow, P., loc. cit.

5 Nebres, B., op. cit., p. 12.

18

**I. The Lack of Fit Between School Mathematics**

**and the Socio-Cultural Context of Developing**

**Countries**

There have been some very interesting examples in the

papers presented for the theme group «Mathematics for

All» at Adelaide regarding the lack of fit between school

mathematics and socio-cultural context.

In the paper «Having a Feel for Calculations,» several

examples are given of young street vendors using

non-school algorithms to do fast and accurate calculations.

«Customer: How much is one coconut?

Vendor (12 years old, 3rd grade): 35.

Customer: I’d like 10. How much is that?

Vendor: Three will be 105, with 3 more, that will be 210,

l need 4 more ... that is 315 ... l think it is 35O.»6

Yet these same young people did very poorly in the same calculations

in the school setting. The paper concludes:

«There appears to be a gulf between the rich intuitive understanding

which these vendors display and the understanding which educators,

with good reason, would like to impart or develop. While one could

argue that the youngsters are out of touch with the formal systems of

notation and numerical operations, it could be argued that the educational

system is out of touch with its clientele.»7

In another paper on «Mathematics Among Carpentry

Apprentices,» Analucia D. Schliemann compares the performance

and computational methods of professional carpenters

with apprentices. What was most striking was the

fact that the apprentices insisted on following «school» procedures

even when a little reflection would have shown

them that these were, in practice, absurd. fit seems that the

task was approached by the apprentices as a school assignment

and they did not try to judge the suitability of the

answers.»8

In an earlier discussion on universal primary education, I

had noted our failure to respond to an immediate need of

farming communities throughout the Philippines.9 T h e

introduction of high-yield varieties of rice brought in, of

course, greater productivity. However, it also demanded

higher inputs in terms of fertilisers, pesticides, labor or

machinery for weeding. Farmers had to take out loans to

avail of this new technological input. The farmers were lost

in the new economics of the system. As many of them put

it, «I know I am getting bigger harvests. But I also know I

am sinking deeper into debt.» Our school system in the

rural areas continued happily teaching sets and

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6 Carraher, D., Carraher, T., and Schliemann, A.: Having a Feel for

Calculations. lCME 5 Mathematics for All Collection, p. 2.

7 lbid., p. 6.

8 Schliemann, A.: Mathematics Among Carpentry A p p r e n t i c e s :

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9 Barcellos, A. (1981): Universal Primary Education. Te a c h i n g

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123.

commutativity, oblivious of the need for simple bookkeeping.

The plenary address of Ubiratan d’Ambrosio on

*Ethno-Mathematics* places this discussion in an even more

fundamental setting. Can we develop mathematics in

countries with different cultural traditions, which may be

quite different from the mathematics developed in Greece

and Western Europe? Would such a mathematics serve

the needs of other cultures better?

There are other things we could do to understand better

the gap between school mathematics and our socio-cultural

context. I have, for example, studied the textbooks

generally in use in the Philippines. They are either direct

copies or relatively mild modifications of textbooks in the

Western world. There is little awareness that there is a different

context outside. I have also analyzed test items

given in an assessment study of sixth graders throughout

the Philippines. There were 40 items, 10 on computational

skills, 12 on concepts such as place notation, 10 on routine-

type applications, 8 on analysis of data. They are the

usual types of exercises we put in textbooks to develop

manipulative skills. The problem is that most of the

concepts or skills developed would have no relevance for

the young person dropping out of school after six years.

**II. The Historical Development of Institutions in**

**Developing Countries**

The above analysis highlights the serious gap between

school mathematics and the socio-cultural context of developing

countries. I would like to locate the primary cause of

these problems in the history of our social and cultural institutions.

The first I would call *vertical,* that is, the relationship

between *similar* institutions, like schools, in *different*

*societies.* The second I would call *horizontal,* that is the relationship

between *different* institutions in the *same country.*

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I would also add a third relationship of *rootedness,* that is,

the insertion of these institutions in the socio-economic-cultural

matrix that underlies the given society.

To understand the situation, we should note that the history

of social, industrial, educational institutions in most

developing countries has been guided mainly by *vertical*

relationships. For example, to understand the school system

in the Philippines, it is less necessary to understand

the local social and cultural situation as it is to understand

the American school system and to note the adaptations

that have been made. If one were looking at Malaysia, one

would turn to the British school system. The same can be

said regarding the system of hospitals and health care,

financial institutions, etc.

If we were to picture the development of institutions like

that of a tree, where the institutions represent the leaves,

branches, fruits (the visible developments in society), then

we would picture our institutions like upside-down trees.

They are rooted not so much in the socio-cultural matrix of

the country, as in the socio-cultural matrix of model countries

abroad. This whole pattern of development of institutions

according to *vertical* relationships has produced what

is usually called the modernized sector, which includes the

best of the educational system. This modemized sector is

not a product of the socio-cultural matrix of the country, but

is very much a foreign transplant. Much of the air it

breathes is imported air, whether this be imported curricula,

imported talent, or imported management techniques.

What are the consequences of this development guided

mainly by vertical relationships?

(1) The development of our institutions is directed by their

sources abroad, not by complementary institutions or

needs in our society. Thus a leading educator is invited to

an international conference and is introduced to the new

mathematics or to computer assisted learning or distance

educational techniques and he comes back and wishes

immediately to implement what he has learned. Because it

is the latest and the best, whether or not it has serious relevance

to the country. The norms for judging the value of our

schools are often in terms of how well our students do in

graduate school in England or the United States, rather

than in how they fit and contribute to the larger society.

(2) The analysis in the earlier part of the paper shows that

*horizontal* relationships and *insertion* into the local culture

are weak. The mathematics classroom is totally unaware of

the «street» mathematics of the young pupils inside. There

is no linkage between the needs of a rural community for

better bookkeeping and the new mathematics being taught

to its children.

These results define for me a crucial task for the future:

how to develop better horizontal fit and better rootedness

in the socio-cultural context.

**III. Tasks that We Might Attempt to Improve the**

**Situation of Universal Mathematical Education**

**in Developing Countries**

I would like to propose *two tasks.* One is in the area of bringing

about a cultural shift in our countries. The second is a

more specific task of working towards a better integration

between universal mathematical education and the outside

world to which our students will go.

*(1) Bringing about a cultural shift in our countries*

I would propose that mathematics educators, together with

other educators and other leaders of society, take up this

need of having the social and cultural institutions of the

country be better integrated with one another and be better

inserted into the culture. This is not to deny the importance

of linkage with other institutions in the Western world or in

the other developing countries. It is simply to accentuate

the need to have these imported developments be integrated

into the social and cultural milieu of the country. It

is important for us to accentuate the high cost to development

of this lack of integration. Whether the cost be in

terms of brain-drain or in terns of graduates who cannot

find jobs in our society. Or a young population without the

skills for a productive life.

*(2) How to proceed concretely to bring about a better*

*integration between the universal mathematics*

*curriculum and the world into which our students*

*will be going*

In the proceedings of the Fourth Intentional Congress on

Mathematical Education, Shirley Frye has a suggested

mathematics curriculum for students who leave school at

an early age.

«The particular goals of a minimal mathematics education include

having:

1. a sense of number;

2. the ability to quantify and estimate;

3. skills in measuring

4. usable knowledge of the basic facts

5. the ability to select the appropriate operation to find a solution;

6. the ability to use a calculator to perform operations

7 . a ‘money-wise’ sense.

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The last skill relating to being ‘money-wise’ is most important since

an individual should have the ability to decide whether wages are

being paid correctly and if purchasing transactions are fair. «10

The proposal is that mathematics educators in our country

attempt the following tasks:

(1) Study the actual articulation in the curriculum of

the seven goals stated by Shirley Frye, that is,

how are they translated into mathematics con-

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10 Frye, S. (1983): Suggested Mathematics Curricula for Students

Who Leave School at Early Ages. In Praceedings of the Fouth

International Congress on Mathematical Education. Zweng et al.,

eds., Birkhäuser, p.32.

cepts, into mathematical skills, into problems in the textbooks.

(2) Look for how these goals appear in the social environment

of the student. That is, what tasks or events

bring about a sense of number or require the ability to

quantify and estimate and so forth (cf. Carraher and

Schliemann papers).

(3) We should then compare the two, that is, the curriculum

inside the school and the appearance of these goals

and concepts in the outside world and see how we can

bring about a better integration between the two.

(4) Share results with one another and detemmine to show

progress in ICME 6.

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**Conclusions Drawn from the Experiences of the**

**New Mathematics Movement\***

Peter Damerow and Ian Westbury

**Introduction**

«Mathematics for All» is the title of our theme group. The

group’s implicit goal is the consideration of an issue which

is fundamental to the idea of general education - and to be

«successful» the theme has to be engaged in a way that

goes beyond the limitations which appear to be endemic to

contemporary curriculum studies: preaching the necessity

of a new program and then arguing in hindsight that widespread

adoption was, from the beginning, an unrealistic expectation.

Needless to say, no such goal can ever be

achieved in one discussion, but the issue which the group

canvasses does not go away because it is a difficult one to

address.

It is easy to make pointed comments on the emptiness of

the kind of conclusion from curriculum research we parodied

above. But at the same time, there is a sense in which

such a conclusion is an inevitable result of a web of problems

that all educational reformers, and particularly subject-

based reformers, have faced as they think about the

scope of their work and their agendas. *There is a need to*

*find ways of changing the contents and conditions of general*

*education as part of a larger concern for changing the fit bet -*

*ween the work of the schools and the rapidly changing scien -*

*tific and social demand for qualifications,* but how this is to be

done is clearly totally elusive.

Within mathematics education the history of the so-called

Knew» mathematics is one instance of this larger issue.

There was a world-wide movement to introduce a Knew»

mathematics—but we know that the effect of these efforts

was negligible: Little has changed in classrooms and the

change that has occurred bears little relationship to the

goals of the original reform movement. This fact defines the

parameters of our problem. Changing the fundamentals of

general education in a goal-oriented, systematic, time-limited

way poses innumerable unsolved issues. It may be true

that the needs of rapidly-changing societies do not allow us

to base the contents and the practices of general education

on tradition and the inner experience of the school. It may

be true that it is the task of our times to move the school

system from being a relatively autonomous, developing

social system into a guided institution which is continually

adapted to changing needs by planning decisions and

administrative action. But the fate of the new math

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\* An enlarged version of this paper is published in the Jour nal of

Curriculum Studies, 17, 1985, pp. 175 -184.

ematics movement shows that so far we only know what

the task is. We have not created the means needed to

address it.

*Mathematics for AID - As a Program of Reform*

The reason for the presence at ICME 5 of a group discussing

mathematics for all and the absence of a group

addressing how new mathematics might be introduced into

the school is intimately related to the experience of the new

mathematics movement. At the height of that movement it

was commonly assumed that the new mathematics *was* a

mathematics for all. It was, for instance, the first addition to

traditional arithmetic and so the first alternative to the Folds

curriculum of the universal primary school for about 200

years. But as Damerow et al. have shown, the claims that

were sometimes made for the real-world relevance of the

new mathematics, the claims that would have been needed

to establish a belief that the new mathematics had a

real-world relevance, were often even more difficult to sustain

than those associated with traditional mathematics.1

More important, the «new mathematics» showed decisively

how problematic major change in a subject can be:

While there were some admirable experiments which showed

what might have been done in elementary schools

under the name of new mathematics, «experimental» outcomes

were not generalized to school systems as wholes.

Perhaps the best that can be said about the widespread

introduction of the new mathematics was that its teaching

did not inhibit the traditional teaching of arithmetic too

much.

And what was the result of the implicit, though often tacit

assumption of the period in which new mathematics was

the vogue that the yield of a a “subject” reform could be

secured equally in every culture independent of the degree

to which formal education was institutionalised? There can

be no doubt that Dienes’ efforts to introduce a modern

mathematics project in Papua-New Guinea in the mid-

1960s was as successful as he claimed to be. But 20 or so

years later, after intensive attempts to adapt Dienes’ curriculum

to local conditions, Souviney comments, in his discussion

of mathematics education in Papua New Guinea,

that it is not enough for the educational establishment

«simply to institute a selection procedure which identifies

and promotes children who exhibit ‘high’ e d u c a t i o n a l

potential while failing to address adequately the needs of

the vast majority.» «Increased attention must be paid to the

needs of [the group of children who will return to their villages

after completing community school] who presently

constitute 70% of the community school graduates.»2 By

implication, something much larger than the new

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1 Damerow, P. et al. (1974): Elementarmathematik: Lernen fur die

Praxis? Ein exemplarischer Versuch zur Bestimmung fachuberschreitender

Curriculumziele. Stuttgart: Klett.

2 Souviney, R. J. (1983): Mathematics Achievement Language and

Cognitive Development: Classroom Practices in Papua-N e w

Guinea. In: Educational Studies in Mathematics 14.

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mathematics seems to have surfaced in the context which

rendered the Dienes’ «modern» program moot.

This instance is from the developing world but parallel instances

can be found in the industrialized world. It is cultural

and contextual factors which, as they interact with

mathematics itself (or any subject area), pose the most

serious problem which slogans like «new mathematics and

«mathematics (or science) for all» must face. Do we keep,

for example, the highly selective frameworks and methods

of traditional mathematics education but give up the privileged

position of the subject as part of the core of general

education? Or do we seek to keep mathematics at the core

of the curriculum but find a way of teaching the subject to

all students?

*Two Alternatives*

What might these alternatives mean — for ideology and for

reality? The first possibility would make « mathematics» a

subject of early specialization, with the present role of the

subject being taken over by physics, technical education,

economics, etc. In this way most students would experience

mathematics as a useful tool and concentrate on creative

mastery of and application of the problem-solving techniques

which result from mathematical thinking. The core

of mathematics, its ideas, conceptual structures, methods

of proof and the like, would only be taught to those who

specialize in some way or other in the subject.

This suggestion comes close to the actual situation of

mathematics education in many nations. In the Federal

Republic of Germany, for example, only 11% entering university

students enroll in the subject areas of mathematics

and natural sciences; 21% enter programs in engineering

and the remaining 68% range over all other fields. For most

students, therefore, mathematics is but a potential tool, and

all we are saying is that mathematics programs in school

might reflect this situation. But mathematics in Germany is

not taught in this way and most mathematics teachers

would probably deny the possibility and would instead

emphasize the specialist, pure mathematical aspects of

their work. It is worth, however, mentioning that the alternative

possibility we sketched above once played a substantial

role in curricular thinking. When the influential

«Verein zur Förderung des mathematisch-naturwissenschaftlichen

Unterrichts» (Association for the Support of

Mathematics and Science Education) was mooted in 1890,

the majority of its potential members argued *against* a

continuing role for pure mathematics as a core subject in

the high school curriculum. At the founding of the association

in 1891 a motion was passed against the teaching

of pure mathematics. It was only as a result of the later

influence of the Gottingen Professor of Mathematics, Felix

Klein, that this policy was changed. But Klein promised

that, in the near future, there would be reintegration of

mathematics with its applications in other sciences and

practices and therefore the continuation of the traditional

kind and place of mathematics education was justified as

an interim measure. The possibility of such a new mathematics

became the goal of the association - and this

t u r n-o f-t h e-century anew mathematic» was profoundly

influential both in Germany and internationally. Klein, of

course, did all that he could to promote the development of

such a mathematics with its implied integration with the

domains of practice but he failed and given this it can be

argued that the case for the abandonment of a «pure»

mathematics for all is still as relevant today as it was in

Gemmany in the 1890s.3

The second alternative we mentioned above is keeping

mathematics as a fundamental part of the school curriculum

but finding a way of teaching it effectively to the majority.

What problems must be faced as we contemplate this?

We have first to consider the fact that mathematical education

in the traditional sense had its origins in a specific

cultural tradition. The canonical curriculum of Traditional

mathematics» was created in the l9th century as a study

for an elite and this pattern persists. In Germany, for

example, «advanced» school mathematics (i.e. analytical

geometry and calculus) are only offered in Gymnasium

which in 1981 enrolled only 10.9% of the 18-year-old age

cohort. And as enrolments in Gymnasium have increased it

has seemed necessary to relax the once-fixed expectation

that all students in the Gymnasium would complete a full

program in school mathematics in order to maintain «standards.

» In 1977—78 only about 70% of German students in

grade 12 (the second last year in the Gymnasium) were

taking the traditional sequence in mathematics, i.e. about

10% of the relevant age cohort.4 While the particular curricular

patterns of different societies vary, mathematics is

constructed in most places in ways that lead to few of the

students who begin mathematics in the early years of the

secondary school continuing to take it in their last secondary

years. The separation of students into groups who

are tagged as «able-mathematically» and, «less abler is

endemic. The heart of mathematics teaching is, moreover,

widely seen as being centered on this curriculum for the

able - although all students *begin* the study of mathematics.

There are some important differences between countries in

their retention rates but in the main we see the patterns

which were created in the l9th century still holding; advanced

mathematics is a study for a few.

The transfer of the European mathematics curriculum to

developing countries was, of course, closely associated

with the creation of schools for elites by colonial administrations.

Under these circumstances it seemed natural to

simply copy

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3 Lorey, W. (1938): Der Deutsche Verein zur Förderung mathematischen

und naturwissenschaftlichen Unterrichts, E. V. 1891-1938.

Ein Rückblick zugleich auch auf die mathematische und natur vissenschaftliche

Erziehung und Bildung in den letzten fünfzig

Jahren. Frankfurt a. M.: Salle.

4 S t e i n e r, H.-G. (1983): Mathematical and Experimental Sciences in the

FRG - Upper Secondary Schools. Arbeiten aus dem Institut fur

Didaktik der Mathematik. Universität Bielefeld, Occasional Paper 40.

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European patterns — but, as Souviney makes clear, it is

quite another problem to build a system of mass education

in the Third World and embed mathematics education in

the specific cultural contexts of that world.5 How is this to

be done? Is a mathematics curriculum desirable if it causes

students in these countries to develop the antipathies

against mathematics which are commonly found in

Europe—but in social contexts which lack the

culturally-based consensus found in Europe that abstract

mathematical activity is as good as such and must therefore

be supported even if it seems on its surface to be useless.

Even if this argument is inappropriate, it does raise

the question of the relation between mathematics and culture

which may be the first problem which arises when the

idea of mathematics for all is raised as a platform for a program

of action.

We must consider, second, the problem of conceiving,

even for industrialized societies, a mathematics which is

appropriate for those who will not have contact with pure

mathematics after their school days. Most current attempts

to face the problem of a basic, «minimal-competency» curriculum

reduce the traditional curriculum by pushing out

every mathematical idea and every possible difficulty to

make it feasible to teach the remaining skeleton to the

majority. But there is only a limited basis for an appeal to

«utility» as an argument or a rationale for curriculum building

to support this approach. *Students who* win *not have to*

*deal with an explicit pure mathematics in their adult lives but*

*will face instead only the exploitation of the developed pro -*

*ducts of mathematical thinking {e. g program packages) will*

*only be enabled by mathematics instruction in school if they*

*can translate the mathematical knowledge they have acquired*

*into the terms of real-life situations which are only implicitly*

*structured mathematically.* Very little explicit mathematics is

required in such situations and it is possible to survive

without any substantial mathematical attainments whatsoever.

6

But is this kind of argument a way of making the case for

the first alternative we considered earlier? And is that

alternative the only one we might be left with? It may be,

but if this is true it would seem to deny the significance of

the topic we are concerned with. Thus we might observe

that to draw this kind of conclusion is to look backwards

in order to determine educational aims for a future. T h e

facts we have cited suggest that a program of mathematics

for all implies the need for a *higher* level of attainment

that has been typically produced under the conditions of

traditional school mathematics—and this is especially

true for mathematics education at the level of general

education. To put this another ways we might claim

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5 Souviney, R. J. (1983): See Footnote 2.

6 See, for example, Bailey, D. (1981): Mathematics in Employment

(16-18) (University of Bath); Sewell, B.: Use of Mathematics by

Adults in Daily Life: Enquiry Officer’s Report (Advisory Council for

Adult and Continuing Education, London); ADVISORY COUNCIL

FOR ADULT AND CONTINUING EDUCATION (1982): Adult’s

Mathematical Ability and Performance. London: ACACE.

that *mathematics for all has to be considered as a program to*

*overcome the subordination of elementary mathematics to*

*higher mathematics, to overcome its preliminary character,*

*and to overcome its irrelevance to real-life situations.*

*The Mathematics Classroom*

A number of recent research studies have suggested that

it is very likely that the structure of classroom interaction

itself creates ability differences among students which

grow during the school years. What might cause these growing

differences in mathematical aptitude? The simplest

explanation rests on the assumption that these differences

are due to predispositions for mathematical thinking - with

the implication that nothing can really be done to change

the situation. But this explanation is too simple to be the

whole truth.

The understanding of elementary mathematics in the first

classes of primary school is, we know, based on preconditions

like the acquisition of notions like conservation of

quantity which are, in their turn, embedded in exploratory

activity outside the school. As long as the genesis of general

mathematical abilities is as little understood as it is, the

possibility that extra-school experience with mathematical

or premathematical ideas influences school learning cannot

be excluded. Furthermore, we know from classroom

interaction studies that the differences between intended

mathematical understandings and the understanding which

is embedded in normal classroom work is vast. We cannot

exclude the possibility (aggressively suggested by

Lundgren) that classroom interaction itself in fact produces

growing differences in mathematical aptitude and achievement

by a system of positive feedback mechanisms which

increase high achievement and decrease further low achievement.

7

Such classroom level phenomena also interact in profound

ways with curricular factors. The English Cockcroft

Committee on teaching of mathematics pointed to the

significance of the notion of *curricular pace* as a critical

variable affecting school achievement. If a pace necessary

to cover an overall curriculum (i. e. to reach the levels of

understanding necessary for, say, English sixth-form work)

is to be sustained, a given rate of coverage is required of

teachers. The Cockoroft Report claims that in England

there has been little change in this implicit rate since the

pre-war years despite the fact that the cohorts of children

ostensibly learning mathematics are now drawn from the

second and third quartiles of the general ability distributions

(as a result of increased access to secondary schooling).

The result, the Cockcroft Committee has suggested,

is an overall rate of coverage and pace of instruction which

is far too fast for many if not most pupils. For such pupils

math-

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7 Lundgren, U. P. (1977): Model Analysis of Pedagogical Processes.

Stockholm Institute of Education, Studies in Curriculum Theory

and Cultural Reproduction 2. Lund: CWK Gleerup.

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matics as a subject is abstract, mechanical, and procedure-

based, and success is hard to come by.8

*Implications*

Our discussions to this point make it clear that to talk of

mathematics for all entails an intention to change general

attitudes towards mathematics as a subject, to slow the

pace of teaching, to eliminate divisions between those who

are friends of mathematics and those who are not, to diminish

variance in the achievement outcomes of mathematics

teaching. This, in its turn, involves us in an analysis of

the forces found in social contexts, curricula, and teaching

inasmuch as it is these forces together which create a set

of frames which *create* situations in which mathematics becomes

one of the subjects in the secondary school in which

*selection* of students into aptitude and ability groups is an

omnipresent reality from almost the earliest days of secondary

schooling.9

As we ponder what such notions might mean we have to

address three very different levels of analysis of the mathematics

curriculum.

*1. The distribution of knowledge.* With the implication that we

reject assumptions that mathematical knowledge is the

prerogative of some cultural communities and not others

and instead see mathematics as something potentially

appropriate to all people. At this level the idea of mathematics

for all involves

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8 Cockcroft Committee (1982): Mathematics Counts. Report of the

Committee on Inquiry into the Teaching of Mathematics in Schools

under the Chairmanship of Dr. W. H. Cockcroft. London: HMSO.

9 For an English study see Ball, S. J. (1981): Beachside

Comprehensive: A Case Study of Secondary Schooling.

Cambridge: Cambridge University Press.

issues of cultural exchange and intercultural understanding

- within and between social groups and geopolitical communities.

*2. The school system and its integration into the society.* The

idea of mathematics for all poses an issue of general education

rather than elite education. At this level the idea of

mathematics for all involves us in a rethinking of the traditional

concerns of mathematics education—away from the

«needs» of elites and towards the needs of both elites and

average students; our sense of crowning achievements

would come not from the achievements of the few but from

the achievements of the many. Our index of accomplishment

would be the overall *yield* of the school system (i. e.

the percentage of a cohort mastering given bodies of

content and skill) rather than content and skill achievement

of the most able.10

*3. Classroom interaction.* Mathematics for all is a problem of

opportunities to learn and their relationship to the dynamics

of the learning process. This level of concern must include

an analysis of the assumptions, patterns, and practices of

within-school division of students into ability groups, sets

and streams — for setting/streaming is ubiquitous in

mathematics education from the early secondary years.

It is quite clear, of course, that these levels are very closely

linked together and that they serve to do no more than

*define* the dimensions of the complex but coherent problem

labelled by the slogan «mathematics for all.»

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10 For a conceptualization of «yield» see Postlethwaite T. N. (1967):

School Organization and Student Achievement. Stockholm

Studies in Educational Psychology 15. Stockholm: Almquist &

Wiksell.

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**Implications of Some Results of the**

**Second International Mathematics Study**

Howard Russell

Peter Damerow has introduced some important suggestions

which must be considered carefully if there is to be

any progress made towards the goal of «mathematics for

all» in the classrooms of our various countries.l His suggestions

regarding pace and coverage are grounded in discussions

and presentations which have been summarized

in Mathematics Counts2 and they have emerged as debatable

issues in the period of the mid-eighties. The present

paper is offered as a contribution to the debate and the

data presented constitute an important part of a recently

established data set.

The Second International Mathematics Study (SIMS3)

data constitute this data set and these data show substantial

variations among countries in the extent to which

mathematics education is provided for all students. The

data which reveal these variations are coverage data and

retention data and the product of these two, i. e., coverage

x retention, are presented as «yield.» While it is true that

there is not much new in the concepts involved, the data

are sometimes revealing when presented in the new form

used in SIMS. These data may prove to be helpful as we

proceed to consider a shift in mathematics education from

content for the elite, to a more marketable,

mathematics-for-all. The SIMS data may be helpful because

they foreshadow relationships among key variables

which need to be manipulated if the suggested shift in

mathematics education is to take place in the mathematics

classrooms of the world, as opposed to taking place only in

the minds and the writings of educational leaders.

In this paper I propose to consider the SIMS pop A

data first, i. e., from the 13-y e a r-olds, and to use

these data to suggest that already we have mathem

a t i c s-f o r-all at the elementary level in many countries.

There are variations in what mathematics is, at

this level, but whatever it is, all youngsters get it. T h e

SIMS pop B data, i.e., from 18-y e a r-olds more or less,

show signs that mathematics for an elite is still the

prevailing plan in most countries. It is at this age

1 Damerow, P. (1984): Mathematics for All - Ideas, Problems,

Implications. Paper presented to the ICMI Symposium at the

International Congress of Mathematics. Warsaw, August, 1983.

Zentralblatt fur Didaktik der Mathematik, 16, pp. 81-85.

2 Mathematics Counts (1982): Report of the Committee on Inquiry

into the Teaching of Mathematics in Schools under the Chairmanship

of Dr. W. H. Cockcroft. London.

3 Travers, K. et al.: The Second International Mathematics Study,

Vol. 1, (for publication 1985) Pergamon Press.

level that some countries have tried to move towards

mathematics-for-all and comparisons of these countries

with the others which have not moved so far gives us good

information with which to plan future strategies for implementing

a mathematics-for-all curriculum throughout the

secondary school years. But first let us look at pop A

where, although it is true that virtually all youngsters are

retained in school, and in mathematics, it is nevertheless

clearly evident that different amounts of mathematics and

different parts of mathematics are taught.

The pop A coverage data are shown in Table 1. These are

topic by topic coverage data which are presented in country

rows (see Figure 1). Thus, the coverage index for arithmetic

for Country D is .74 and the standard deviation is .12.

This means that the average teacher with a pop A class in

Country D claims his/her students have been taught the

material involved in 74% of the items under consideration.

The standard deviation is the measure of variation in coverage

C among teachers in Country D on that particular set

of arithmetic items.

Since it is true that virtually all youngsters in a nation

eventually make it through the pop A level, the variation in

coverage is the main source of vacation in the mathematics

which is offered to pop A students. The data in Table 1

show substantial variation from country to country, and as

well the standard deviations show sizeable variations

among classes within countries. It is true then that there

are many youngsters who miss out on instruction in a substantial

amount of the content defined by the SIMS Pool,

even in such popular topics as arithmetic. Since this is a

phenomenon which affects all countries, it may be one of

the features of the present mathematics curricula that is difficult

to change. This would be especially interesting if it

can be shown that the youngsters who miss out on coverage

are the ones who cluster in classes which spend time

on unlearned content of earlier grades and/or require more

than the usual amount of time to cover most topics. If this

is true then it would appear that the teachers of slow

classes have adjusted the pace to the needs of their students.

Although retention is uniformly high through the pop A

years, it is nevertheless true that there is considerable

variation in the amount of time taken by students to get

through to the pop A level. This suggests variation in pace.

Table 2 shows the mean age in months for both the pop B

and the pop A students. The first message which emerges

from this table is that some countries appear to take much

less time to get through to the end of the pop A year than

others. Country A requires only 162 months. Country K

requires 166 months, and the other countries spread themselves

over a range of many months. What interpretation

such data have for deliberations about mathematics-for-all

may be clearer when the associated p-values for student

performance are presented at some future point in time. In

their absence it appears that mathematics-for-all can be

pursued as effectively by following the lead of countries

with a low mean age as those with high mean age.

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The above argument seems to be in conflict with the

hypothesis proposed by Cockoroft4 that the pace of mathematics

education must be slowed down if we are to keep

enough students in the mathematics courses to accurately

label it mathematics-for-all. Table 2 showed us there is

wide variation in pace because there is wide variation in

age among counties. If pace were related to retention at

pop A or retention later then the age variation should be

somehow related to retention. We have made the supposition

that pace, for our purposes, is really coverage divided

by time (age in months) and we have found that pace at

pop A is unrelated to retention or any other variable of central

interest. What seems to have happened then is that

most contries have maintained a «promotion-

by-performance» standard policy and this, in turn,

has lead to retardation or failing of significant numbers of

students. The failing of students, or the forced repetition of

grades by students thus shows up a slowing of pace, but

this type of slowing the pace seems not to have provided

any positive outcomes. Another way of stating it is that no

violence is done to the concept of mathematics-for-all

when age promotion rather than the slower paced, promotion-

byperformance standards is adopted at the elementary

grade levels. Indeed, it can be postulated that violence

may be done to the concept of mathematics-for-all if annual

p r o m o t i o n-b y-performance standards has the effect of

retarding student progress to the point that many students

give up schooling and mathematics the minute that they

attain the legal leaving age. Clearly, more data and more

study are required before such generalisations can be

widely accepted.

The story is different at the pop B level. Table 3 shows the

coverage data means and standard deviations. Again,

there is wide variation both among contries and within. But

these coverage data cannot speak to the issue of mathematics-

for-all until they are augmented by the retention

data. Table 4 shows coverage, retention and yield data for

each country which possessed the necessary data. Now

the variation among countries is even more evident, and

the possibility of finding relationships seems promising.

In order to get one central relationship quickly we

can look back at Table 2 which shows the mean age

for the students in pop B. Along with these mean

ages I have introduced the difference in ages between

pop A and pop B. Now it seems evident that

the countries which are attaining the highest yields

are those which provided their students with the longest

time interval between pop A and pop B. namely

Countries E, O. A and K. What emerges then is a

tentative conclusion which supports the Cockcroft

hypothesis and as well reinforces the Damerow position

elaborated in his paper on mathematics-f o r-a l l .

It must be conceded that Countries C and F possess

high yield, but a more thorough analysis of these

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4 Cf. Footnote 2.

countries in the Volume I of SIMS indicates that Country C

has the lowest level of attainment by far and hence what is

taught is no indication at all of what is learned. Country F

also pays a price of low student attainment, and for purposes

of the present argument that country as well as

Country C is discounted. The SIMS data not only offer

assistance in our debate by providing empirical support for

the Cockcroft and the Damerow positions but also they

help us to identify at least tentatively some of the boundaries

beyond which this central pace hypothesis may be falsified.

Perhaps the first boundary worthy of attention is the one

which separates pop A from pop B. On the surface it

appears that the «slower pace produced higher yield»

hypothesis breaks down between pop A and pop B. An

alternative explanation of the data may be that the type of

slowing down which is produced by annual performance

promotions, i.e., failing the lowest students, has its own

negative effects which in turn counteracts any benefits of

the apparent slowdown. Another point arises here which

requires attention. It is the possibility, indeed probability,

that pace is a variable which has an optimum level such

that either an increase or a decrease from it is likely to produce

a reduction in learning. It is also likely true that optimum

pace is unique to individuals, and hence a general

increase in pace will be beneficial to some students and

detrimental to others. If we return to the data now, and

focus on the fact that the two countries with the lowest

mean age at pop A are also high in yield at pop B. we may

suggest that the pace in the elementary grades could be

too slow in many countries. If this were not true then why

would the apparently fast-paced nations do so well? When

we move to the pace issue at the secondary level the situation

is quite clearly reversed and the nations with the slowest

pace through the pop B years are the ones which

seem to be benefitting the most. Thus, the

Cockcroft-Damerow hypothesis is most likely to be helpful

in our analysis of the secondary school program.

The SIMS data provide another way of looking at the

pace issue. The «opportunity-t o-learn» data have been

aggregated in a matrix form which reveals item-by-i t e m

and class-b y-class coverage. In the case of Country E,

for instance, there are five matrix displays shown in

Figure 4, one for each of the item clusters arithmetic,

algebra, geometry, probability and statistics and measurement.

Each row corresponds to a test item in the

SIMS Pool and each column corresponds to a class

(Country E had a sample of 85 classes), and hence

there are 85 columns in each of the matrix displays.

The reason that this form of display takes on meaning

in the pace debate is that the ordered columns clearly

distinguish between classes with higher and lower

coverage. This same distinguishing among classes on

coverage is, in fact, a distinguishing among classes on

the variable we have called pace. The fast-p a c e d

classes are on the left; the slow-paced on the right. My

discussions with classroom teachers about these

matrices has reinforced my own view that we have at

present wide variations in pace among our

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classes and that these variations are likely the result of the

classroom teachers’ attempt to accommodate the pace to

the capabilities of the students. The fact that the matrix displays

for virtually all countries reveal similar variations in

pace among classes suggests that there is some common

reality tied to such data. What that reality is, and how it can

be utilized in moving towards a viable mathematics-for-all

is the present problem. My feeling is that the reality here is

precisely the one we are looking for, namely that pace can

be slowed, and in fact is already slowed, for the advantage

of the below average student.

The potential for benefits from manipulating pace is suggested

by a consideration of some matrix displays at pop

B. Figure 5 shows Country E and Country F. and the feature

of these displays which is most evident at first glance

is the low coverage in «analysis» for Country F compared

to high coverage for Country E. If Country F should prefer

to cover the same amount of analysis as Country E it could

consider the possibility that the extra time in secondary

school (55 months versus 47 months) could do it. Also, the

possibility that lower retention could be a factor must be

considered. My own speculation is that the slower pace in

secondary school in Country E accounts for both higher

coverage (than Country F) and for higher retention (than

other countries except Country F).

There are insufficient data at the present time to warrant

definitive claims, but the debate is just beginning now, and

I believe the SIMS data and their various new forms of

aggregation will eventually lead us to general relationships

among truly fundamental educational variables. If we can

indeed manipulate learning through the manipulation of

pace then we may begin the process of moving towards

mathematics-for-all even before we know what the nature

of the content should be.

Table 1: Implemented Coverage Indices C\* (see Figure I

below)

I wish to close my contribution to the debate on mathematics-

for-all with a brief and simple analysis of the issue

of content selection. I wish to suggest that a rationale for

mathematics which appeals to the «new» clients of mathematics,

i.e., the middle level students who constitute the

backbone of society, must be carefully constructed. l believe

that a market oriented rationale is quite appropriate.

Such an orientation is likely to be widely accepted if it is

true, and if it can be shown to be true, that the students in

the middle and below the middle on our mathematics competence

scale will be required to use mathematics, or

«mathematics-for-all» in their chosen work in the marketplace.

Some observers suggest that

mathematical-skill-based sophistication in the marketplace

will increase dramatically as hi tech moves into a dominant

market position. Under such circumstances it is true that

mathematical-skill-based sophistication should be introduced

into the core of the mathematics curriculum which

Damerow is proposing. How to identify the precise ingredients

in this new core is not known, but there should be

mechanisms available for arriving at a best guess.

While it is true that many observers see an increasing

need for mathematical sophistication on the part of the average

worker, there are other observers who suggest that

the «user friendliness» of computers yet to be introduced in

the marketplace will place fewer demands of a mathematical

nature on the typical citizen in the workplace. I have not

yet seen a clear answer for this issue. My intuition suggests

dramatic increases in sophistication but I have no

data to support my intuition. Perhaps mathematics educators

should be prepared to collaborate with representatives

of government and business in an effort to identify the

generic skills needed in our new core mathematics-for-all.

Then we can hope to make significant progress in the

quest for mathematics-for-all.

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Table 2: Mean Age in Months

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Country Pop A Pop B Diff.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

A 162 217 55

B 171 218 47

C 171 217 46

D 170 217 47

E 167 222 55

F 168 215 47

G 170 214 44

K 166 223 57

L 170 217 47

N 168 214 46

O 167 228 61

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Table 3: Implemented Coverage Indices C

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Population B

POP B ALG ANAL NUM SETS FIN P&S GEO WGTD

(25) (45) (19) (7) (4) (7) (29) MEAN (SD)

MN (SD) MN (SD) MN (SD) MN (SD) MN (SD) MN (SD) MN (SD)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

CountryA 100 (1) 94 (7) 82 (32) 95 (9) 99 (0) 83 (26) 85 (28) 91

E 91 (10) 92 (10) 81 (15) 83 (19) 93 (6) 72 (16) 74 (23) 85

J 92 (10) 94 (11) 88 (12) 85 (22) 52 (10) 86 (11) 68 (28) 85

K 92 (19) 88 (13) 87 (13) 87 (16) 83 (6) 85 (17) 70(37) 84

L 91 (10) 87 (13) 76 (21) 89 (11) 63 (3) 44 (16) 76 (24) 82

D 85 (12) 86 (13) 73 (18) 55 (19) 60 (6) 67 (20) 62 (30) 75

O 84 (18) 83 (18) 80 (21) 56 (28) 83 (4) 73 (16) 55 (37) 75

C 62 (12) 73 (16) 68 (14) 75 (8) 70(22) 77 (20) 62 (19) 68

H 87 (16) 57 (24) 80 (19) 81 (19) 55 (17) 46 (30) 52 (37) 65

B 70 (14) 60 (27) 69 (13) 75 (12) 85 (6) 81 (7) 53 (28) 64

N 56 (20) 73 (20) 50 (21) 28 (17) 43 (4) 17 (6) 35 (35) 52

F 81 (22) 35 (33) 74 (28) 66 (27) 9 (12) 28 (34) 43 (38) 51

MEAN 82 (13) 77 (18) 76 (10) 73 (19) 66 (25) 63 (24) 61 (14) 73 (13)

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\* (Not collecting OTL data were: Belgium (FR), Hong Kong, Nigeria and Scotland.)

Table 4: Coverage Retention and Yield

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Population B

Country Ct R Y

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

A 89 12 107

B 59

C 65 50 325

D 71 6 43

E 84 19 160

F 49 30 147

H 65 12 78

J 84 11 72

K 84 15 126

L 84 10 84

N 46 6 28

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Figure 1 : Item Classification Figure 5 : Country E

Figure 4 : Country E Figure 6 : Country F

**Implications of the Cockcroft Report**

Afzal Ahmed

1. Mathematics permeates the whole society and its use

seems to assume ever increasing importance as our societies

become more technological and complex.

Mathematical methods and thinking are not the prerogative

of scientists, engineers and technologists only, they are

used by people in making everyday decisions. Their use in

analysing individual behaviour, to study opinions and attitudes

is also increasing. The place of mathematics in both

primary and secondary school curricula for all pupils is also

evidence of general agreement that the study of mathematics,

along with language is regarded by most as essential.

Is it not ironic that the subject which has assumed such

prominence in society is also that which is most closely

related to failure? Low attainment in mathematics has certainly

been at the centre of the education debate in Britain

throughout this century.

2. The Committee of Inquiry into the Teaching of

Mathematics in Schools in England and Wales, of which I

was a member, was set up in 1978 as a result of concern

about the mathematical attainment of pupils.

I must be careful not even to attempt to summarise the

report of this committee «Mathematics Counts», which was

published in January 1982.1 I would, however, use relevant

evidence from this report to discuss the aims and focus of

the Curriculum Development Project for Low Attaining

Pupils in Secondary Schools Mathematics which I am

directing. This is a three-year project which commenced in

September 1983 and is one of three projects commissioned

by the Department of Education and Science as

a result of the concern raised in the Cockcroft Report (The

other two projects concern the assessment procedures for

low attaining pupils.) I shall not confine this paper to the

work of this project nor does the paper contain official

views of the project. I merely use the findings of the

Cockcroft Report and my own project to support the views

expressed in this paper.

Paragraph 334 of the Cockcroft Report begins with the following

sentence:

Low attainment in mathematics can occur in children whose general

ability is not low. «

This, of course, is true of adults too, and this fact is illustrated

quite vividly in Brigid Sewell’s report on

\_\_\_\_\_\_\_\_\_\_\_\_\_

1 Great Britain Department of Education and Science (1982):

Mathematics Counts Report of the committee of Enquiry into the

Teaching of Mathematics in Schools the Cockcroft Reporter

HMSO.

the use of mathematics by adults in daily life.2 This enquiry

was undertaken in association with the Cockcroft inquiry,

Section 6(ii) of this report states:

«Many of the people interviewed during this enquiry were inhibited

about using mathematics, this led them to avoid it as much as possible

and in some instances it has affected their careers. The inhibition

was most marked among women who had specialised in arts

subjects. The more educated were affected to a much greater extent

than the less educated.»

3. The solution to the problems of low attainers in mathematics

is not simply dividing pupils into the following three

groups and then providing separate curricula for each:

a. those who are good at mathematics;

b. those whose general ability is not low but are failing at

mathematics;

c. those whose general ability is low and are failing at

mathematics.

The problem is much more complex since we do not know

enough about the way children learn, we are not agreed on

the nature of mathematics and there is little known about

effective teaching methods. Moreover, the differentiates at

which pupils learn and a wide variation in attainment at a

particular age, make it impossible to categorise pupils in

the above groupings with any degree of permanence.

Another main factor is that teachers want to keep all

options open to enable pupils to enter public examinations

at the highest level possible. There are further factors such

as previous school experience, environment, attitude and

motivation which influence the attainment of pupils in

mathematics, so the idea of separate maths curricula for

separate groups does not offer much promise.

4. Past attempts at making suitable curriculum provision for

mathematics for all pupils have focused on change of

content, groupings of pupils and management of

resources. The impact of these changes on mathematics

education has not been significant Mathematics for the

M a j o r i t y.3 The Schools Council Project in Secondary

School Mathematics was set up in 1967 to help teachers

construct for pupils of average and above ability. The

courses relate mathematics to pupils’ experience and provide

them with some insight into the processes that lie

behind the use of mathematics as the language of science

and a source of interest in everyday life.

This was an admirable aim and the project was inspired

by the «Newsom Report» published in 1963 under the title

Half our Future.4

The following two quotes would, I hope, indicate the inspiring

nature of this report:

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2 Sewell, B. (1982): Use of Mathematics by Adults in Daily Life.

Advisory Council for Adult and Continuing Education.

3 Mathematics for the Majority (1970): Chatto & Windus for the

Schools Council.

4 Ministry of Education (1963): Half our Future. HMSO.

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«Our aim in the teaching of mathematics to all pupils, to those with

average and below average ability no less than to those with marked

academic talent, should be to bring them to an interest in the content

of mathematics itself at however modest a level.» (Paragraph 459)

«Few, if any, of our pupils are ever likely to become mathematicians,

but some may well come to find satisfaction in mathematical work if

its purpose has first been clearly seen and confidence established

through the successful use of mathematics as a tool.» (Paragraph

422)

Note: *Our pupils* refers to half the pupils of secondary schools, i. e

those for whom public examinations were not originally designed.

This project contained some very exciting, attractive and

relevant material for teachers and pupils but the evidence

of its lack of impact on schools is provided by the following

comments from recent reports on mathematics teaching in

secondary schools.

«The work was predominantly teacher controlled: teachers explained,

illustrated, demonstrated, and perhaps gave notes on procedures

and examples (...) A common pattern particularly with lower

ability pupils was to show a few examples on the board at the start

of the lesson and then set similar exercises for the pupils to work on

their own (. ) At the worst it became direct ‘telling how’by the teacher,

followed by incomprehension on the part of the pupils (...)» (H M I

Secondary Survey, Chapter 7, Section 6.35)

«(. . .) and in the majority of the classrooms the teaching did not aspire

to do more than prepare the pupils for examination (...)» (H. M. 1.

Secondary Survey, Chapter 7, Section 6.25)

From the Cockcroft Report6

«(...) Mathematics lessons in secondary schools are very often not

about anything. You collect like terms, or learn the law of indices, with

no perception of why anyone needs to do such things. There is

excessive preoccupation with a sequence of skills and quite inadequate

opportunity to see the skills emerging from the solution of problems.

As a consequence of this approach, school mathematics

contains very little incidental information. A French lesson might well

contain incidental information about France - so on across the

curriculum; but in mathematics the incidental information which one

might expect (current exchange and interest rates, general knowledge

of climate, communications and geography, the rules and scoring

systems of games; social statistics) is rarely there, because most

teachers in no way see this as part of their responsibility when teaching

mathematics We believe that this points out in a very succinct

way the need -which is by no means confined only to courses for

low-attaining pupils - to relate the content of the mathematics course

to pupils experience of everyday life.» (Paragraph 462)

5. It is interesting to note that although recent reports point

out that teachers seem to concentrate on teaching rigidly to

examination syllabuses and spend a vast amount of time

on teaching routine skills they are not successful in increasing

the proportions of pupils who Can perform these skills.

Interesting evidence of this is provided by Dr. Margaret

Brown in her article Rules Without Reasons? «7 According

to her, in some cases the proportion actually decreases!

5 Great Britain, Department of Education and Science (1979):

Aspects of Secondary Education in England. HMSO.

6 See Footnote 1.

7 Brown, M. (1982): Rules without Reasons? In: International

Journal of Mathematics Education, Science and Technology. Vol.

13, No. 4, pp. 449-461.

Further serious and disturbing evidence has been quoted

in Paragraph 444 of the Cockcroft Report. It points out that

according to examination board regulations, the

16-year-old pupil of average ability who has applied himself

to a course of study regarded by teachers of the subject as

appropriate to his age, ability and aptitude may reasonably

expect to secure grade 4 in the certificate of secondary

education. The mark required to achieve grade 4 in mathematics

is often little more than 30% ! This implies that

pupils of average ability can only obtain one-third of the

possible marks, and it can only damage pupils’ confidence

since the examinations are normally set on the syllabuses

which teachers say they need to spend most of their time

on.

6. So what are the reasons for teachers continuing to teach

in largely ineffective methods? The reasons are complex

but not unsusceptible to some analysis. In many cases teachers

are not unaware of the failure of the system they are

operating but their perception of the constraints which force

them to operate in a restricted way is often misleading and

mixed up. If one were to ask them the reasons for not changing

their teaching approaches, as I often do, one is likely

to receive a fairly standard catalogue of reasons such as

resources, time, class size, disruptive pupils, rigid examination

system, lack of pupil motivation, demands from

employers and universities, pressure from parents, political

pressure, lack of suitably qualified mathematics teachers,

lack of technical support in mathematics departments and

so on. Some of these pressures are real and others’ perceived

ones only but they do keep teachers locked up in

operating an ineffective system.

7. The Cockcroft Report has pointed out that the mathematical

education which many pupils are receiving is not

satisfactory and that major changes are essential .

The major changes as far as the teaching approach is

concerned are outlined in Paragraph 243, the most quoted

paragraph of the report—

" Mathematics teaching at all levels should include

opportunities for

- exposition by the teacher;

- discussion between teacher and pupils and between

pupils themselves;

- appropriate practical work;

- consolidation and practice of fundamental skills and routines;

- problem solving, including the application of mathematics

to everyday situations;

- investigational work "

Unlike previous reports which were mainly aimed at teachers,

the Cockcroft Report in Chapter 17 has recommended

active co-operation of the six main agencies for effective

change. Teachers at the front local education authorities

providing support, examining boards, central government,

teachers’ training institutions and those funding curriculum

development and educational research. Co-operation

is also sought from employers and the public at large.

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8. In this climate of support from the above agencies, with

a nation-wide program of «post Cockcroft» activities supporting

us, we have found that the following aims of our

project for low attaining pupils in secondary schools can be

realistic and help focus attention on aspects crucial in bringing

about change in the teaching of mathematics:

- to encourage teachers to change their attitudes about

ways in which low attainers learn mathematics;

- to help teachers to interpret the Cockcroft Committee’s

«Foundation Lists» (Paragraph 458) in the spirit of

Paragraph 455 to 457 and 460 to 466, i.e., suggest activities

which should involve low attaining pupils in a wider

range of mathematics than the usual restrictive diet of

«basics»;

- to provide teachers with ideas and strategies which

should enable pupils to change their perceptions of

mathematics, encourage them not to view the subject

just as a body of knowledge which has to be «passed

on» fact by fact;

- to suggest ways in which teachers can continually gain

insight into pupils’ mathematics without having to rely on

formal tests;

- to suggest ways in which pupils can arrive at conventional

methods and terminology through their participation

in problem-solving activities and investigatory

mathematics;

- to suggest ways of working which should enable pupils

to see links between mathematics and other subject

areas;

- to suggest ways of working which should help teachers

to develop pupils’ confidence and independence in

handling mathematics;

- to suggest approaches which should help teachers cope

with different rates of learning amongst low attainers.

9. We have begun with a viewpoint that there is already a

large amount of material available for pupils, and the problem

lies in its incorporation into the classroom. Transfer of

material from one teacher to another, or one classroom to

another is not straightforward and often causes sufficient

difficulties for the second teacher to reject the material as

unsuitable or justify its failure by cataloguing external pressures.

We have concentrated in the first instance on developing

«good practice» in twelve chosen schools from six local

counties. One teacher/researcher has been released from

timetable commitment from each case study school on a

fixed day every week. We find it essential that

teacher/researchers are not released full-time and for the

rest of their working week they are in the real environment

of schools where changes are intended to take place. The

value of this work will lie in the opportunity to find common

and distinctive features, to follow these up and probe further

in order to provide a basis for making decisions about

any contributory reasons for success and failure of material

and methods.

These case studies should enable other teachers to identify

with situations and experience and to predict the likely

measures of success in their own classrooms. We are also

convinced that the growth in implementation of changes

will not come about by written publication only. We are

using a «cellular-growth» model for development and dissemination

of ideas. In all the six counties we have developed

a growing network of teachers, through full and

part-time inservice courses, who are participating in trials

and feedback. Naturally, the teacher/researchers are

increasingly involving all the other teachers of mathematics

in their own school, and we have considered it most important

that teachers of other subjects appreciate the changes

taking place and support them.

We are hoping that inservice packages (including video

tapes) related to case studies will be produced for general

dissemination to advisory staff, heads of departments in

schools and teacher-training institutions. It is even now

apparent that this will serve to outline the development process

outlined above. We think it very unlikely that there is

any short cut.

10. It is not my intention to discuss in detail all the areas of

exploration we have undertaken so far, but only to provide

a glimpse of some significant issues relevant to the theme

of this paper.

In considering the courses for 11- to 16-year-old pupils the

Cockcroft Report in Paragraph 451 states:

«we believe it should be a fundamental principle that no topic should

be included unless it can be developed sufficiently for it to be applied

in ways which the pupils can understand (...)»

The chief reason offered by teachers for not using methods

which enable pupils to apply their mathematics is the lack

of time. I believe that the issues are more complex than this

and are associated with confidence and the scale of perceptual

leap required in changing their beliefs about how

children learn mathematics.

The Cockcroft Report points out:

«In order to present mathematics to pupils in the ways we

have described it will be necessary for many teachers to

make very great changes in the ways in which they work at

present (...) (Paragraph 465)

Enabling these changes to take place so that teachers

implement them from the position of conviction and confidence

is at the centre of our project.

11. One major obstacle for teachers in changing their

methods of teaching is their anxiety about «covering» the

syllabus. This is further complicated by the fact that very little

effort has been made to disentangle the teaching of those

aspects which almost all pupils need to come across such as

reading charts, diagrams and tables, interpreting simple statistical

data, simple ideas of probability, ideas of inference

and logical deductions, developing a feel for simple measurements,

visualising simple mechanical movements and

many other areas outlined in the Cockcroft Foundation List

(Paragraph 458) from the teaching of those more sophisticated

parts of mathematics which the able and interested

pupils might study, e. g. deductive geometry, calculus, algebraic

manipulation etc. The able minority often misses out

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on the aspects of mathematics for all outlined earlier since

these are glossed over by their teachers who regard these

topics not as an area of experience but as «bits» of knowledge

pupils need to possess.

Teachers of mathematics, who themselves were mainly

an able minority, as well as pupils tend to resort to the way

they were taught mathematics. There is little change in the

methods used to teach the aspects of mathematics which

the able minority will learn compared with those aspects

covered by most pupils (including the able). The main

change consists in diluting, or what is referred to as «watering

down» the content and presenting topics in small easy

steps with plenty of practice examples. The result is trivialising

and makes mathematics meaningless for most

pupils. A large number of pupils, even those are able, tend

to lose interest and find little meaning in this activity and

hence lose confidence in the subject. There is also a tendency

for teachers to teach topics on a syllabus, systematically,

item by item and believe that if all the topics have

not been taught their pupils will suffer.

In order to ensure that topics are covered, it is possible to

rush these through in a narrow, restricted manner rather

than embedding them in a wider context allowing pupils to

reflect upon the use of the mathematics presented to them

and discussing the appropriateness of methods used by

them. Algorithms are often taught to pupils too soon, and

there seems to be an assumption that once taught they are

remembered. The Chelsea Report, Understanding Mathematics

11 - 168 points out:

«The teaching of algorithms when the child does not understand may

be positively harmful in that what the child sees the teacher doing is

‘magic’ and entirely ‘divorced’ from problem solving.» (Chapter 14)

Mathematics, in daily life, is not encountered in small packages

as taught by fragmenting a syllabus or in the form of

the straight-forward command of the textbooks. It appears

in context in a variety of spoken or written language and

social situations. The Cockcroft Report proposes that its

Foundation List of Mathematical Topics (Paragraph 455)

should form a part of the mathematics syllabus for all

pupils. This list reflects situations in which people meet

mathematics in life, and the emphasis of the report is on

presenting mathematics in a context in which it will be

applied to solving problems.

12. «At all stages pupils should be encouraged to discuss

and justify the methods which they use.» (Cockcroft

Report, Paragraph 458)

The implications of the suggested changes in teaching

styles for teachers are great for those who have not worked

in this manner. It would certainly call in question the

conventional method of designing curriculum in terms of

topics, concepts and skills. For example, it could mean that

a teacher would need to have a collection of «situations»,

problems and inves-

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8 Hart, K. M. et al. (1981): Children’s Understanding of Mathematics

11-16. John Murray.

tigations to offer to pupils and the consideration of mathematical

outcome of these in terms of content and processes

for individual pupils would be a retrospective activity

by the teacher. For example, consider a situation

where a pupil is presented with 5 pence and 7 pence

stamps and asked:

a. What totals can he make if he has as many of each as

he wishes?

b. Are there more?

c. Can he convince others?

d. What happens with other stamps,

3 pence and 5 pence?

5 pence and 9 pence?

Some of the outcomes of this activity, depending on the

levels at which individuals tackle this problem, could be

listed as follows:

Generalisation and Factors

Organisation of work Multiples

Pattern spotting Addition and

multiplication

Hypothesising and Algebra

testing Permutation

Formalising Combinations

Exclusion Primes and composites

Checking etc. Triangles

Modular arithmetic etc.

For a confident and experienced teacher, this way of working

could enable him to overcome the major problem of

not limiting a child’s attainment since the choice of rich starters

would enable all pupils to become engaged in the activity

and offer an opportunity for follow-up work in depth for

those who were interested and able to do so: This would

certainly help to overcome the problem of offering motivating,

relevant and challenging mathematics without restricting

pupils’ opportunities for taking any external examinations.

Although the change in emphasis may appear

small, the solution is not as simple as it appears. Teachers

who are used to assuming the major control of their pupils’

learning find it extremely difficult to change their focus.

Pupils also become used to the idea that teachers will

always have the right answers and the right method for all

problems, and if they wait long enough these answers

would be provided by the teacher directly or through the

«bright» pupils in their group. Under these circumstances,

pupils do not find it easy to change their role and assume

responsibility for their own learning.

These changed perceptions of mathematics learning and

teaching need to develop in a climate of mutual trust and

confidence.

13. In her publication «Generating Mathematical Activity in

the Classroom»9 Marion Bird has used written records of a

class of 11-year-old pupils to demonstrate that it is possible

to teach in a way which encourages pupils to begin to ask

their own

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9 Bird, M. (February 1983): Generating Mathematical Activity in the

classroom. west Sussex Institute of Higher Education.

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questions, to control the direction of their investigations, to

make conjectures and think how to test them to make and

agree on definitions and equivalences, to search for patterns,

to make generalisations and to seek for reasons for

what seems to be happening.

One of our project activities has been to examine a large

number of case studies developed by teachers who have

been trying out these teaching approaches in the classroom

and identifying general features which facilitate the

work and the mathematical activities and those features

which inhibit them. These «facilitators» and «inhibitors»

have been found very helpful by teachers attempting such

an approach in the classroom. We have been exploring

methods of involving teachers of other subjects in supporting

these developments since their attitude can have a significant

influence on pupils. We have also been exploring

simple but effective methods of sustaining networks of teachers

who can offer each other support, encouragement

and stimulus - all important ingredients of bringing about

effective change. A collection of ideas and strategies which

would enable teachers to initiate a greater change of focus

in the control of learning in the classroom has been compiled.

This has also provided some initial starting strategies

which have been tried out in several classrooms.

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10»The Mathematical Association Diploma in the Teaching of

Mathematics to Low Attaining Pupils in Secondary Schools» was

piloted at the West Sussex Institute of Higher Education, the

Mathematics Education Centre and at Bishop Grosseteste

College Lincoln.

We are also observing and developing case studies on

effectiveness of various strategies for inservice support,

e.g. teachers visiting each others schools, teachers released

to work with other teachers in their own schools etc.

Finally, it seems clear to us that the most effective change

can come about by starting from the teachers’ o w n

strengths and building from there. This, in the time of great

pressure on inservice finance, can be an extremely slow

process. It would be counterproductive and a retrograde

step if we allowed ourselves to be seduced by short cuts

which entail offering crutches to support teachers.

The development of the Mathematical Association Diploma

in the Teaching of Mathematics to Low Attaining Pupils 10,

for which some teachers can be released for one day a

week over a year under a central government scheme is

already proving an effective agent for change. We need to

think seriously about the continuation of these developments

undertaken by teachers who have completed these

courses and widening the means of supporting those who

are not able to have such an opportunity. The extreme importance

and enormous benefits of teachers released from

schools to work with other teachers and the effective programs

for such activities can be a theme for another paper!

Details available from:

The Secretary of the Diploma

Board

The Mathematical Association

259 London Road

Leicester LE2 3BE

England

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**Universal Mathematics Education and its**

**Conditions in Social Interaction**

Achmad Arifin

As far as I understand, universal mathematics education is

mathematics education having everyone in a society as its

target. Its objective is to educate the society to be more

intelligent in utilising the available resources or opportunities

to improve their welfare and prosperity. What we need

is a mechanism to bring mathematics education to everyone

in the society. This mechanism should have the ability

to disseminate the intended changes or improvements at

any time. School system should be in the mechanism. But

the school system alone cannot be made responsible for

bringing mathematics education to everyone or to carry out

improvements in mathematics education.

Improvement followed by another improvement, or change

followed by another change, will be the feature of mathematics

education in particular as a consequence of the

rapid development of mathematics as well as that of science

and technology.

The mechanism should be able to channel the needed

changes and improvements to the school system without

any disturbances, that is to say, as smooth as possible. For

a country with a large population the need in such mechanism

can be very urgent.

In the implementation of universal mathematics education

the active participation of the community can contribute a

great deal in reaching the target.

In this paper I will try to describe how we should raise

community participation in carrying out universal mathematics

education through looking at the aspect of interaction.

In developing the mechanism the mathematics itself

and its process of development should be brought as close

as possible to the place where mathematics education is

going to be developed. Because the role of the mathematicians

and their activities is also significant in initiating the

development of universal mathematics education.

*1. Social Structure*

Setting the ultimate objective of education as to develop

the society toward attaining a better standard of living

means that education should be the concern of everyone in

the society. To enable the society or the community to participate

in the process of education a certain interrelationship

has to be developed between the society and the

school system. Since interaction is a component which

basically supports the educational process, we may examine

the interrelationship mentioned above from the

aspect of interaction. To pave our way to this purpose, we

generalise our examination by looking at the interactions

that happen in a society.

Social structure in a society is understood to be the totality

of interactions among people or groups of people.

Depending on its quality, social structure can influence the

survival and the development of the society. Through interactions

the society improves its ability for continuing its survival

by utilising the resources and opportunities that are

available in its disposal. These interactions are social interactions.

Let us direct our attention to the social interactions which

contribute to the improvement of people’s ability and refer

to this particular interaction as a positive interaction. Since

interaction can happen between people and their environment,

social as well as natural one, we generalise the meaning

of positive interaction as to include not just the social

one.

*2. Positive Interaction*

Looking at a particular society we may always ask whether

interactions happen among individuals in that society.

Assuming that interactions happen among them we may

further ask, what are the kinds of interactions and how

intensive they are. What we should identify are those interactions

which have the effect on increasing individual’s

knowledge or skill, or individual’s ability in general.

On the other hand, someone may gain additional knowledge

or skill through reading books or observing natural

phenomena. Can we say that someone gains additional

knowledge and skill through interaction between him or her

and the books he or she is reading or between him or her

and the natural phenomena he or she is observing? The

answer is yes, but this will depend on the way he or she

reads the books and observes the phenomena. Certainly

someone needs a certain ability in order to undergo these

kinds of interactions.

We generalise the meaning of positive interaction to include

any interactions which contribute to the development of

abilities of individuals or groups of individuals. It can happen

in various kinds and patterns between individuals and

their surroundings. This positive interaction is actually an

important aspect in the educational process. It provides

opportunities to individuals to improve their ability. With

their continually improving ability the people will have

opportunities to contribute to the development of their

social structure.

Positive interaction is expected to happen life-long for

each individual. Someone needs some knowledge and skill

to initiate positive interaction with his or her surroundings.

In particular, someone should be able to utilise information

to get some additional knowledge and skill, or to improve

his or her ability in general. Facilities like libraries or

museums which exhibit new developments, for instance in

science and technology, form resources of information.

These can provide stimuli to motivate positive interaction to

happen continuously with increasing intensity and quality

as to fulfill the needed ability.

Someone’s continual efforts for improving abilities, or

developing new abilities can be considered as the consequences

of the rapid development in various sectors. The

developments in science and technology in particular crea-

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te challenges in their utilisation. To cope with these challenges,

particular abilities have to be made to exist.

Leaving the development of such needed abilities to individuals’

motivation and initiation, through their involvement in

positive interaction, might take a long time. Therefore, an

organised effort is needed to design positive interaction to

happen in a certain time period and space.

*3. School Interaction*

Someone needs to have certain abilities in order to be able

to get involved in a positive interaction. Someone needs to

know the language of the book he is reading, or perhaps he

needs some knowledge about the topics discussed in the

book in order to get some additional knowledge about the

topics. In general, someone needs some basic knowledge

and skill to draw additional knowledge from the book he is

reading, from the discussion with someone else, from the

observation of a natural phenomenon, etc.

To organise positive interaction to happen in a certain time

period and space is particularly aimed at providing individuals

with some basic knowledge and skill. Positive interaction

which is designed to happen in a certain time period

and space is an important aspect pertinent to what we

understand as school. I refer to this positive interaction as

school interaction or classroom interaction. The ability of

individuals in carrying out further development of their own

ability as well as that of their society independently is usually

set as the main objective of school interaction. This further

development of ability can be carried out through positive

interaction.

A school is a place where people who have different backgrounds

and come from different environments come together

with the intention to learn. The people who come to

learn at a certain school constitute a society of students

with a certain characteristic, that is a certain level of readiness

to get involved in positive interactions.

On the other hand, the school is equipped with a curriculum

containing a series of programs of teaching and learning

process as a means to achieve the objective as set in

the curriculum. In this junction where the interface between

student background and school curriculum take place we

expect the teachers to play their role; that is to manage the

school interaction to happen as to give an optimal result.

In managing school interaction we should pay attention to

the individual’s background as differences related to value

systems or behaviour patterns might occur. Referring to

those differences the teachers or someone else who acts

as a facilitator should manage the interaction so as to happen

without disturbances.

Each of the three components: social structure, positive

interaction and school interaction should have the ability to

absorb some developments in mathematics that are relevant

to the society development and to utilise them in educating

the people in accordance with their respective role.

The parts of each component include:

- Social structure: To appreciate and support necessary

changes in school mathematics and create conducive

environment outside school for learning mathematics.

- Positive interaction: To set up facilities other than

schools that motivate and create opportunities for

mathematics learning.

- School interaction: To develop the environment in

school and the teaching methodology in mathematics

class as well as that for individual approach so to enable

them to inspire, to stimulate and to direct learning activities.

Referring to the roles of the components as described

above, we are not looking at them with their passive meaning;

but with their active participation in providing opportunities

and stimulation for mathematics learning. This justifies

the purpose of the three-component functional relationship:

social structure - positive interaction - school interaction,

that is to provide opportunities and stimulation for

mathematics learning through interaction, aimed at everyone

in the society.

*4. MathematicsforALL*

Once again, universal mathematics education is mathematics

education for everyone in a society, so it is mathematics

education for all. The school system is one of the

places where universal mathematics education can be

channeled to reach a part of the members of a society. In

other words, school interaction is one of the components

for reaching individuals in a society and to manage them to

get involved in learning mathematics. Since learning

mathematics at school is related to the development of a

society as a whole we also have to examine the components

which can contribute in paving the way to make the

relationship really happen. These components are social

structure and positive interaction.

What parts of mathematics that a person learns at school

should be accepted and appreciated by the society as a

part of the society development process. On the other

hand, the school should be knowledgeable and be aware

of the needs of the society, particularly those concerning

the society development and those related to increasing

the individual’s ability.

From now on, we need to attach more operational meaning

to positive interaction. That is, some facilities need to

be created or to be established in order to stimulate and

provide opportunities and means for positive interaction to

happen. In this connection, the targets are the society and

the school system.

From what has been explained above, we can see that

the interrelation among social structure, positive interaction

and school interaction enable the development of the

school system as well as the quality and the intensity of

social interaction continually. This will result in increasing

the ability of individuals. Furthermore, referring to the role

of individuals in increasing the quality and the intensity of

interaction the continual development of school mathematics

teaching will, after all, result in increasing the ability of

the society itself. Therefore, taking also into consideration

the individual’s background during teaching-learning processes

we can describe a functional relationship among

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social structure, positive interaction and school interaction as

follows.

This three-component functional relationship forms a

mechanism for developing the needed ability of a society in

coping with challenges. Therefore, it can also be utilised as

a means of transferring mathematics and its development

to everyone in the society, through various kinds and patterns

of interaction.

This is aimed at achieving certain mathematical abilities.

This gives an idea as to how to carry out universal mathematics

education. To initiate universal mathematics education

it is therefore desirable that in any society the

three-component functional relationship can be identified.

In this era of science and technology development, the

intelligence of the people will contribute a great deal and

meaningfully to the society’s ability. Mathematics education

can play a role in developing the intelligence of the people.

But this will depend on the persons who have to make

opportunities to enable mathematics to play its role fully;

that is to create a conducive environment where mathematics

learning can happen through interaction. Referring to

this, we are dealing with the following questions.

1. Which part of mathematics that can function as a developer

of an individual’s intelligence?

2. Those parts of mathematics that have been chosen,

how should they be presented?

To learn mathematics through interaction can take place

everywhere and any time, not necessarily in a classroom

during a mathematics class. Therefore, those two questions

are relevant to the school interaction as well as to the

other two components: social structure and positive interaction.

Referring to the purpose of developing a society,

we should apply the two questions to the three components:

social structure, positive interaction and

school interaction equally balanced. The answer will

depend much on the existing cultural condition, for

example, the average mathematical knowledge possessed

by the people in the society, the intensity and the quality of

interaction that constitute the social structure. Furthermore,

the existence of mathematicians and their activities related

to mathematics should provide some assets in seeking

answers to the questions.

In developing countries mathematics for all can be earmarked

as universal mathematics education, that is mathematics

education for everybody. The mechanism for carrying

out universal mathematics education is the

three-component functional relationship. In developing universal

mathematics education it is always necessary that

we examine the mechanism whether it is an appropriate

condition to carry out the new changes. Otherwise, we

have to develop the mechanism itself, that is to develop the

components or the interrelation among the components.

In developing countries the problems related to the development

of universal mathematics education always are

concerned with the development of the mechanism

besides those of mathematics and its teaching. In developing

countries with a large population, the problem will

mainly concern the development of the mechanism, to

enable it to function accordingly. This means that in carrying

out universal mathematics education we have to pay

attention to the three-component functional relationship:

social structure — positive interaction —school interaction.

Its development includes the development of the components

and their interrelationship.

The role of the mathematicians from the country concerned

is to form a pool of expertise for the development of the

mathematical content and the way it should be presented.

This includes the development of the teaching methodologies

as well as that of the mechanism, that is the

three-component functional relationship. Therefore, the

development of universal mathematics education will never

be free from the need to develop mathematical activities in

the developing country concerned and the willingness of

the mathematicians to take part in the efforts to improve the

intelligence of their people.

*5. The Role of Mathematicians*

The two questions related to what and how as mentioned

previously are: Which part of mathematics that can function

as a developer of an individual’s intelligence and how

should they be presented, always occur during the development

of universal mathematics education. The development

of mathematics provides us with many choices in fulfilling

the mathematical content, while the development in

technology provides us with many alternatives for presenting

certain mathematical topics.

The questions related to what and how will always

be relevant from time to time. Therefore, what is more

important here is the availability of a mechanism

which is able to provide answers to the questions of

what and how in accordance with the needs and

conditions of the developing society or nation. T h e

mechanism should include the three-component functional

relationship which carries out the universal

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mathematics education and the local mathematicians who

provide information and expertise for the development of

the mathematical content.

On the other hand, the local mathematicians should be

actively involved in stimulating and upgrading the three

components that are aimed at developing the three-component

functional relationship.

To fulfil this task the mathematicians need to keep themselves

informed on the development of mathematics and

actively maintain their communication with mathematical

activities as wide as possible.

*6. Conclusion*

Referring to the role of mathematics education in developing

individuals’ intelligence that is aimed at developing

the ability of a society or nation, the role of mathematicians

in providing information and expertise, the

expected active involvement of mathematicians in the

development of mathematics education, the fact that

mathematics and also science and technology are developing

rapidly, the fact that the growth of education depends

on the cultural condition of the society, we may close this

paper with the following remarks:

1. Universal mathematics education can function as a

means for increasing the ability of a society or a nation

through developing individuals’ intelligence.

2. Universal mathematics education needs to undergo

continual development to maintain its function in a developing

society or nation.

3. The three-component functional relationship can function

as a mechanism for universal mathematics education

to reach everyone in a society through the school

system as well as outside the school system.

4. The local mathematicians and their mathematical activities

could be supportive to the continual development of

universal mathematics education including the

three-component functional relationship as its mechanism.

5. The development of the three-component functional

relationship as a mechanism and the supportive attitude

and activities of the local mathematician reflect the

effectiveness of the community participation.

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**Alternative Mathematics Programs**

Andrew J. C. Begg

**Introduction**

Whenever a group of high school teachers discuss their

mathematics programs they make certain assumptions.

They usually take for granted the way the programs have

developed over recent years, the available resources, and

the purpose of the programs. In this paper I want to suggest

a number of questions that may cause us to question

our syllabi and teaching methods and consider alternative

programs. Before looking at these questions we need to

consider some aims, assumptions, and constraints that we

probably all share.

*Aims*

In mathematics education the three most common aims of

our programs are summed up as:

Personal — to help students solve the everyday problems

of adult life;

Vocational — to give a foundation upon which a range of

specialised skills can be built;

Humanistic — to show mathematics as part of our cultural

heritage.

These three aims imply that basics are necessary but not

sufficient, that we must present a broad-based course as

the future needs of our students will vary tremendously,

that historical topics and career information should be

included, and mathematics should be given a warm and

human flavour rather than a formal or logical one.

*Assumptions*

In my country we assume that universal secondary education,

including mathematics for at least two years, and

usually for three or four years, is the right of all students.

This assumption varies in other countries and may make

some considerations less relevant.

The other assumption I would make is that for many students

mathematics is not inherently interesting — indeed

they may have been turned off the subject. Usefulness is

not enough, we want motivation and fun, and our mathematics

programs must build in this motivation so that students

look forward to our classes.

*Other Constraints*

A huge variety of schools exist: large/small, urban/ rural,

traditional/alternative, wealthy/poor, academic/ technical

etc. In spite of this variety many places, including New

Zealand, have a national system of education that has one

syllabus for each year of schooling.

Our yearly programs are based on 3 or 4 hours week of

mathematics. In New Zealand this used to be taught in 5 x

40 minute periods but is now usually 3 x 60 minute periods.

This change should cause significant movements in the

teaching of our mathematics program.

The third constraint is caused by the pressures on our

school syllabi. We are asked to cover numerous new items.

Computer education, careers education, outdoor education,

health education, and multicultural education are

examples of areas that either require time from the total

school program or need to be integrated across the curricula

and this includes the mathematics curricula.

*General or Special Purpose Mathematics*

It is usually possible to look at a group of students and see

some aim they have in mind, e. g. to graduate from high

school, to pass an external exam, or to prepare for employment

or unemployment. When we look more closely we

can usually isolate a number of subsets of students with

differing needs. In the same class we find students who are

terminating their mathematics education, others who hope

to pass an exam and others who expect to take the subject

on to a higher stage.

In large schools it may be possible to separate these students

into different classes which cope with their specific

needs, but in small schools this is not possible. Further, the

students may not be certain about their future plans and

needs.

In New Zealand I find that many classes contain students

requiring enrichment while others need remedial assistance,

some are sitting one exam while others are sitting another,

some expect to leave school for a job, others for

unemployment; yet generally students are not given alternative

programs to cater for these needs. At the most

senior level in New Zealand we are looking at alternative

mathematics programs with a statistics or calculus bias

according to the student’s needs, but for the majority of

school leavers no basic course exists with significant elements

of budgeting, tax, insurance, rents and the other

skills needed by school leavers. I believe alternative programs

are needed where content decisions are based on

the needs of the participating students.

*Teaching: Mathematics or Students*

A mathematics program is part of a total educational package.

As teachers and program designers we must consider

the aims of this whole package and then adjust our programs

to suit. These general aims would include the development

of

- self-respect,

- concern for others,

- urge to enquire

and we would want to develop the skills of - communication,

- responsibility,

- criticism,

- cooperation.

If we wish to achieve some of these aims we must stress

cooperative learning, encourage project work

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and displays of work by individuals and groups. We must

use discovery approaches and not try to shortcut this

time-consuming process by presenting results too quickly.

We must give all students success in their eyes, in their

peers’ eyes and in their teachers’ eyes. Here alternative

programs are needed as our present ones are not achieving

these goals, and we must consider other teaching

methods as an important part of these programs.

*Monocultural or Multicultural*

Traditionally, our programs, both contents and methods,

reflect the traditions of European education. Many of us

have assumed this European background and the associated

attitudes to learning. Now we find that mathematics

programs are required for monocultural groups that are not

European and for multicultural groups that reflect a broad

range of cultures.

In New Zealand the two main non-European groups are

Maoris and Pacific Islanders. Practically no research has

been done on their different views and attitudes to mathematics,

nor to the way their language reflects different

views of the subject. What we are aware of is that many

Polynesians prefer group work and do not enjoy being ranked

apart from their peers. This fact has obvious implications

in designing a program that stresses group rather

than individual success.

Other problems experienced by groups from other cultures

and in particular by new immigrants are the problems

associated with language. If teachers are aware of these

difficulties and modify their programs these difficulties can

be reduced, but it is difficult for a teacher to cope when a

very broad range of backgrounds occur at the same time.

In building self-esteem we must at least build respect and

understanding for the differences that our students reflect.

We can at least use names and subjects from other cultures

in our examples, but we must be careful not to offend.

Assuming this cultural sensitivity, opportunities exist in

numerous areas to incorporate ideas from other cultures

and help students of all races appreciate the differences

between members of their communities. This two-way

appreciation helps our students «stand tall».

*Streamed or Mixed Ability*

Disregarding all other differences and factors affecting the

achievement of our students we all accept that students of

a particular age group vary from very talented to those of

low ability. One way of handling this variation is streaming.

The difficulty with streaming (or even broad-banding) is

that it is practically impossible to stream exactly, and when

one considers the factors affecting achievement (illness,

schoolchange, bad teaching etc.) one realises that streaming

can never be perfect.

Many teachers believe that because of social factors it is

desirable to keep mixed-ability forms. Certainly mixed-ability

forms mean we must offer alternative programs within

our classroom where with streamed groups it is easy to

think one program is suitable for all students when in fact it

is only suitable for the majority of them.

In smaller schools and in schools where option structures

affect mathematics we are forced to cope with mixed-ability

classes and to design alternatives within the program.

These alternatives include enrichment and extension for

the more able students and more practice in basic skills for

the less able. These alternative programs need to consider

the appropriateness of topics according to whether or not

students are ready for the subject and whether or not they

expect to continue their mathematics education into the

future.

*Class, Individual, or Group Programs*

Traditionally, most of us taught our classes as one group.

More recently, with the advent of mixedability classes,

some of us have tried individual programs. Having tried

both methods, I would believe that neither a class approach

nor an individual program is satisfactory if used all the

time, so again I look for alternatives.

Analysing the possibilities we have six main teaching

modes:

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Teaching Mode Work Rate

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Class teaching Same Same

Individual program Same Different

Individual program Same Different

Group teaching Same Different (between groups)

Group teaching Different Different (between groups)

Group teaching Different Different (within groups)

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No approach is necessarily correct and variation of style

throughout the program is probably the most desirable answer

but within this varied program I think we need to use

groups to a much greater extent than at present.

Sometimes groups will be doing the same work at different

speeds, sometimes the same work at different speeds but

achieving the work at different levels, and sometimes different

work. All these modes need consideration in our programs

and obviously our decisions will affect the resources

we need for these modes of teaching.

*Cooperative or Competitive Learning*

I have mentioned the way some cultural groups prefer working

in groups and how using groups may help overcome

some problems associated with aspects of whole-class

and individual programs. I want to suggest that a group

approach is the real-life approach to many problem-solving

situations and that our programs should reflect this cooperative

and realistic approach.

I know that we all value excellence and feel that the student

who ranks first in our class deserves praise and

reward, but I believe that this should not be achieved using

a competitive strategy which recognises one at the expense

of ranking others in lower positions. It can be done with

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comments rather than with numbers. At the same time,

other students also do well in presentation or in attitude

while others improve in some facet of their work, and these

aspects need positive encouragement and praise too.

When a cooperative group is working together (whether it

is mixed ability, streamed, special needs or whatever) then

the whole group should share the praise and encouragement,

and there is no need to separate out individual skills

to achieve our objectives.

*Traditional or Alternative Teaching Styles*

The teaching style of many of us still reflects the style of

many years ago, the way we were taught and even the lecture/

tutorial system of traditional universities. Meanwhile,

our clients have changed, they represent a broader range

of backgrounds and abilities, their interest in mathematics

is different, and they are more sophisticated. Some teachers

have altered their teaching styles and some have

tried discovery approaches. The new maths revolution had

some teachers trying more formal approaches and others

more intuitive approaches.

I believe that, in terms of our general aims, we need to

encourage open-ended approaches whenever appropriate,

through discovery learning and through project activities.

The important aspect here is that a variety of approaches

is needed, and it is often useful to have one group doing

projects while another group requires more teacher attention.

We sometimes talk of integrating mathematics with other

subjects in the curriculum. Science, economics, geography,

technical drawing and home economics are examples

of subjects where obvious overlaps exist. From our knowledge

of transfer of training we must assume that subject

overlap must be made explicit and that when we talk of

applications in mathematics, surely the most relevant applications

are those ones which the students are at present

involved in with their other class work. This change must be

reflected in our programs and will vary according to the

other option choices of our students.

A final aspect of teaching style that worries me in New

Zealand is a result of the change in school structures from

7 x 40 minute periods each day to 5 x 60 minute periods.

In mathematics this has usually meant 3 x 60 minute periods

replacing 5 x 40 minute periods. At the same time,

many teachers have not significantly adjusted their teaching

style. I believe this 60 minutes should be split into at

least 4 shorter periods with a greater variety of activities

occurring during the hour. Maintenance should be built in,

the range of activity should include not only oral and written

work, but also physical activity and a greater use of

visuals both in teaching and summarising. I believe this

variation should be built into our programs in recognition of

the short attention span of many young students. The need

to provide varied and interesting stimulation over an extended

period is the only way we can avoid having our students

bored.

*Text-oriented or Multi-media*

Textbooks are the most commonly used resource in our

classrooms. They have usually been written for a particular

course and are the cheapest available resource. With the

advent of alternative programs we see the need to build up

numerous supplementary resources. With groups working

on different topics in the same classroom and with students

with reading difficulties, we see other reasons for more resources.

Mathcards and worksheets are needed to direct

students into alternative activities such as games for maintenance

work, project starters, and enrichment. Much of

the work on these cards could be self- m o t i v a t i n g .

Computer-assisted learning has obvious applications to

remedial, revision and enrichment work. Films, slides and

videos all have their place in providing variation. Our

mathematics programs must «build in» these resources so

that within a school all the students are getting these advantages

and not merely those in the classes of one or two

keen teachers.

Imaginative texts are still necessary, and we must remember

that every student should learn to learn from a book

without assistance.

*Logical or Humanistic Approach*

Mathematics was taught very formally, then with the «new

math» we saw logical approaches, mathematical

approaches and psychological approaches. Some people

have tried more intuitive approaches, and I understand one

or two have used an historical approach.

What I would prefer is a humanistic approach, I mean an

approach that is student-centred and develops from the

students’ particular interests and needs. An approach that

links their work to real life and to applications that are relevant

to them.

I want to see a warm approach that treats every student

as someone special, that works positively to avoid sex or

race stereotyping, and that builds self-esteem in our students.

I am sure that once this self-esteem is present, teachers

will be amazed at the progress students can make.

**Conclusion**

I know that schools have limited resources, that teachers

have limited time, and that numerous other constraints are

put on us by our schools, but I believe we can all introduce

more alternative elements into our programs. I know most

of us like to have a class start together, but we can still produce

various endpoints, we can use group work, and we

can encourage more cooperative problem solving.

I am sure we must give students the opportunity to make

decisions that are relevant to their education and each of

us should be «the guide on the side not the sage on the

stage».

Program development will keep happening, it is our responsibility

to make sure it helps our students achieve the

aims of education in general as well as the aims of mathematics

education.

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**Part II:**

**Problems and Developments**

**in Industrialised Countries**

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**Arithmetic Pedagogy at the Beginning of the**

**School System of Japan**

Genichi Matsubara and Zennosuke Kusumoto

In Japan the Emperor had held the reins of government,

although the Samurai held it for some time in Japanese

history. In the Meiji revolution in 1868, the Emperor took the

reins of government again and the new government in the

Meiji Era was born. To make the nation modernise, it made

great efforts to learn a great deal from foreign countries

and to adopt new educational policies.

About 110 years have passed since the elementary school

system was established in Japan. Here I’d like to tell something

about arithmetic pedagogy at the beginning of the

Meiji Era in Japan.

Today everyone thinks it is the turning point in education in

Japan, so that to reflect on arithmetic pedagogy in the Meiji

Era will provide some suggestions on how to introduce a

new arithmetic pedagogy.

**I. Arithmetic Pedagogy at the Beginning of the**

**Meiji Era**

First, I’d like to discuss the «Terakoya» (private schools) in

the age of the Samurai, since they played an important role

in forming the basis of arithmetic pedagogy at the beginning

of the Meiji Era.

*A. Arithmetic in «Terakoya»*

At the end of the Edo Era, the Samurai took the reins of

government in Japan. In order to educate the Samurai’s

children, each feudal clan had a school of its own. But they

did not take into consideration the education for ordinary

people. So they established their own schools to get a

minimum knowledge to go into the world—these were called

«Terakoya». There were many «Terakoyas» but there

was no communication in terms of teaching methodologies

among them. Arithmetic in «Terakoya» was mostly concerned

with how to handle abacuses. Many kinds of textbooks

used at «Terakoya» were published in the Edo Era.

Among these publications, about 1,126 books were mathematics

books such as UJinkoki-, «Sanpo zukai». These

were used as textbooks for children and teachers at

«Terakoya» or other private schools. Next I’ll introduce to

you something about «Jinkoki» to let you know the

contents of arithmetic pedagogy in those days.

*B. «Jinkoki»*

This is the book on abacuses written by the mathematician

Mitsuyoshi Yoshida in 1627. This was modelled after

«Sampotoshu» which was a manual for abacuses in China.

But the contents mainly consisted of how to use abacuses

for business transactions in daily life. It is noticeable that

what was dealt with in it was just the same as what we

dealt with in problem solving, which was discussed all over

the world in the early 20th century. But teaching methods

were not dealt with in it. After many teachers of «Terakoya»

used it, the contents were revised and published several

times.

*C. «Terakoyosho «*

The content of arithmetic at that time was to learn the four

operations by using the abacus. The teaching materials

connected with daily life transactions were arranged in the

same way as in problem solving. There were many children

who were able to do division of two-digit numbers.

Graduating from «Terakoya», they went to another private

school where they studied the latter half of «Jinkoki».

Teachers of «Terakoya» taught them according to their own

philosophy. So, when the government introduced school

systems in education and build elementary schools,

«Terakoya» was one of the models in mathematical education.

In December 1873, Director David Murray reported

to the Ministry of Education that the average standard of

education was very high.

II. **Arithmetic Pedagogy After the Proclamation of**

**School Systems**

After the revolution of the Meiji Era, the Samurai reign was

replaced by the Tenno (Emperor) reign. Then the unified

Meiji government was born. One of the policies of the

government was to build elementary schools all over Japan

and to put an emphasis on the 3R’s. Prior to this, there

were already a few private elementary schools established.

But feudal clan «Terakoya» and other private schools exerted

a great influence to make the new educational policies

possible.

In March 1869, the government ordered the building of elementary

schools in every prefecture. The government promoted

the new educational policy, so people were eager to

equalise ordinary education. After the Samurai Era passed

on to the Tenno reign, each local government took over the

policies. The curriculum of each school, similar to that of

«Terakoya», was as follows: The official age of enrollment

was five, but it was usually six. Almost all children could

understand basic addition and subtraction. In To k y o

«Terakoya» was not admitted as a school, but later was

admitted as a private elementary school. In those days the

government authorised two kinds of schools.

(1) One was a school for people going into business after

graduation, which was established in each prefecture. By

establishing these schools, the government aimed at the

decentralisation of education.

(2) The other was a school for people going on to college and

the university after graduation. Then there were three kinds of

elementary schools considered from other perspectives.

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a. the national school — established by the Ministry of

Education; the attached school - established as the

preparatory course for colleges and universities; b. b.

elementary schools of each prefecture — established by

private funds;

c. the elementary school — established by feudal clans.

**III. Proclamation of School Systems**

*A. Establishment of the Ministry of Education and*

*Proclamation of School Systems*

When the Meiji government started, much thought was

given to educational policies. It established a kind of school

administration section in the government of Kyoto. Then

the capital was moved to Tokyo and the same section was

established in Tokyo. So, it was thought necessary to establish

the Ministry of Education, which deals with educational

matters such as making rules, building schools and

assisting financing. In September 1871, the government

decided to abolish schools for the Samurai and to establish

the Ministry of Education.

The Ministry of Education had the task of making rules

about elementary schools and junior high schools and did

everything for schools. It made efforts to equalise the standard

of education of every school in Japan. This made it

reinforce school systems in Japan.

In Japan the law on school systems was the first one that

was established for education. It controlled the systems

and the curricula of schools from elementary level to university

level in Japan. To make the law, the government not

only gathered the materials from other countries, but also

established pilot schools to get data from them.

In August 1872, the outline of school systems was

decreed as follows:

a. the necessity of schools in terms of man’s development;

b. to study regardless of positions and sex;

c. to explain the mistakes of traditional learning;

d. to give everyone opportunities of learning;

e. to make parents responsible for children’s education;

f. to make parents pay money for children’s education;

The contents which children had to study were arithmetic

or abacuses. We can find in «Gakumon no susume»

*(Encouragement of Study* written by Yukichi Fukuzawa),

how to learn the angular style of the Japanese, how to write

Chinese characters and to drill using abacuses, to deal

with the balance etc. were indicated.

Now I stall summarise the school systems.

*1) The large, middle and small districts*

The Ministry of Education, which was responsible for

controlling and managing schools in Japan, divided all

areas as follows:

*The large districts —* the country was divided into eight. A

university was established in each large district.

*The middle districts —* each large district was divided into

32. A junior high school was established in each middle district.

*The small districts —* each middle district was divided into

210. A elementary school was established in each small

district.

So, there were 6,720 elementary schools in one large district

and 53,760 elementary schools in all.

*2) Schools*

There were three kinds of schools - universities, junior high

schools and elementary schools.

The curricula were prepared in a special book.

*3) Elementary schools*

Elementary schools were considered to be the primary

stage of school education, so all the people had to go to

school by all means. Schools were generally called elementary

schools, but there were several kinds.

a. Infant schools - preschools for children under six years

of age.

b. Private elementary schools - the man who had a licence

taught in his private home.

c. Schools for the poor— schools for the children of poor

people who could not support themselves. The rich

contributed money to these schools.

d. Village elementary schools — schools in the remote

areas. The teacher omitted a part of curriculum or attended

evening school.

e. Girls’ elementary schools — schools for girls in which

they were taught handicrafts and ordinary subjects.

f. Ordinary elementary schools — It was divided into lower

and upper schools.

*4) The subjects of elementary schools*

At first there were about 20 subjects. They were revised

soon and only a few of them remained. I’ll introduce something

about arithmetic here.

*Arithmetic —* order and the four operations; they were

explained in the western style.

*Geometry—*the subject for upper schools;

*Arithmetic* was taught using a western-style method and

avoided teaching how to use the abacus.

In the lower elementary school systems, it was prescribed

that elementary education was compulsory. Since all the

laws had not yet been put in order, I think that the school

systems remained just a model which could not be followed

as an ideal at that time. Not all the children of school age

could go to school.

In 1883 the percentage of school attendance was about

50%. It was still a very low rate at that time.

In 1870 compulsory education started in England for the

first time in the world. Two years later the school systems

started also in Japan, so it may be interesting to note that

compulsory education started in Japan next to England,

but elementary school education was already spreading in

advanced countries.

*B. The Elementary School Syllabus*

Following the proclamation of the school system laws, the

elementary school syllabus was proclaimed.

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It was a little similar to the course of study which was introduced

from the USA after World War II. But it was just the

syllabus before World War II, or the one which was used in

countries except the USA.

The syllabus is as follows:

The elementary school was divided into two stages — an

upper and a lower one. The lower one was from the age of

6 to 9. The upper one was from the age of 10 to 12. So the

children studied for eight years. The course of the lower

elementary school was divided into eight grades. The term

of each grade was six months. The contents of each grade

were given in this period. But as it was only a model, it was

recom mended to revise and to use it in each prefecture.

Next, I shall introduce arithmetic only. I can only find in the

eighth grade the same course of study of arithmetic as

today’s. But I cannot find it in the first grade to the seventh

grade.

- the 8th grade: 6 months, 5 hours a day, 30hours a week,

except Sunday; Western-style arithmetic: 6 hours a

week; Using textbooks, the teacher wrote the Arabic

numbers, order and the fundamental calculations of

addition and subtraction on the blackboard, and the children

wrote them on their paper. Children practised the

calculation of figures and mental arithmetic every other

day. When a teacher made the children do mental arithmetic,

children could not use paper, and only answered

questions on the blackboard. The questions which were

answered in exercises the day before remained unerased

on the blackboard, and the next day, all children

were made to do the exercise again.

- the 7th grade: 6 months; arithmetic: 6 hours a week; to

teach multiplication and division as in the 8th grade, and

to do exercises on calculation of figures and mental

arithmetic every other day;

- the 6th grade: 6 months; arithmetic: 6 hours a week; to

teach multiplication and division;

- the 5th grade: 6 months; arithmetic: 6 hours a week; to

study the application of the four operations, and to do

the exercises on the calculation of figures and mental

arithmetic every day;

- the 4th grade: 6 months; arithmetic: 6 hours a week; to

teach the four operations with compound numbers;

- the 3rd and 2nd grades: 6 months; arithmetic: 6 hours

a week; to teach fractions;

- the 1st grade: 6 months; arithmetic: 6 hours a week; to

teach fractions and proportions; review of each subject:

2 hours a week; to review all subjects studied before.

After passing the test, children went to upper ele- v o l u -

me mentary schools. Children who could not pass the test

studied for six months more in the 1st grade.

The lower elementary school was more modern ised compared

with the schools of the clans and «Terakoya». But

the Ministry of Education did not consider the unification of

teaching methods all over Japan soon.

In the syllabus of the lower elementary school, arithmetic

consisted of teaching western-style arithmetic and not teaching

how to use the abacus. But at that time there were

few who could teach westernstyle arithmetic, so the

government took the measure of making plans to train teachers.

There was a big objection against only teaching

western-style arithmetic and not teaching to use the abacus.

There were few books about western-style arithmetic till

«Shogaku sanzitsuyo» was published by the Ministry of

Education. So it introduced «Hitssan kunmo», «Yozan

sogaku» as textbooks, and explained a little about the teaching

methods. I do not think all the teachers could teach

the children. In the Ministry of Education there occurred

discussions about it. In 1874 it gave a notice, «We do not

intend to use only western-style arithmetic, but we will let

the children study the abacus, too.»

Arithmetic in the syllabus of elementary schools did not

limit the size of numbers for each grade. Today it is said

that each teacher determines the limit of numbers according

to the children’s ability. It is not necessary to show the

details. At that time, not everyone knew about the modern

school, so it could not be helped. It might be unavoidable

when the development of children was unknown.

Geometry was the subject for upper elementary schools.

Measurement was only thought as the adaptation of calculation.

Now I shall introduce to you «Hitssan kunmo» to

show how it was taught at that time.

It was published in September 1869. The writer was Meiki

Tsukamoto — a geographer. He was a talented man who

played an active part in the navy and bureaucracy. He was

the first writer who offered arithmetic in a systematic and

modernised textbook,» said Kinnosuke Ogura. It was

because he studied western-style arithmetic from the

beginning — he was not a man of the abacus. This book

was published as a primer, but it was of high level.

This book was the first that showed the style of the subjects

as it is common now.

- The first stage is a general explanation.

- The second stage is a detailed explanation of the

methods with examples.

- The third stage are exercises of calculations.

- The fourth stage are exercises of applications and problems.

The book contained four volumes. For each volume

existed another book which contained formulas and answers.

- the first volume: number, four operations; the second

volume: fractions;

- the third volume: proportions;

Now I shall introduce to you a part of the first

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*1) Number*

The number in Chinese character

The large number (larger than 10)

The decimal number (smaller than 1)

The cardinal number

The notation

the number placing and notation of the decimal

system

*2) The four operations*

Addition: the method of calculation of figures, the

addition of numbers of 4 or 5 figures; the

number divides 4 figures; applied problems

Subtraction: the same way as addition

Multiplication: explained as addition; the calculation

Division: with 10 to 12 is added to the fundamental

multiplication table; to explain multiplication

of the number of I or 2 figures to explain the

method of division when the divisor

consists of I or 2 figures

*C. Upper Elementary School Syllabus*

- the 8th grade: *arithmetic:* 6 hours a week

- the 7th grade: *arithmetic:* 6 hours a week

- the 6th grade: *arithmetic:* 6 hours a week

to teach proportionate distribution

*drawing:* 2 hours a week

to draw point, line and regular

polygon

- the 5th grade: *arithmetic:* 6 hours a week

to teach proportionate distribution

*geometry:* 4 hours a week

to teach regular polygon

*drawing:* 2 hours a week

the same as the 6th grade

- the 4th grade: *arithmetic: 6* hours a week

to teach proportionate distribution

*geometry:* 2 hours a week

to teach line, angle and triangle

*drawing:* 2 hours a week

to draw plain, straight line and

volume

- the 3rd grade: *arithmetic: 6* hours a week

to teach square and square root

*geometry:* 4 hours a week

to teach circle and polygon

*drawing:* 2 hours a week

to draw plain, straight line and

volume without shadow

- the 2nd grade: *arithmetic: 6* hours a week

to teach the calculation of interest

*geometry:* 4 hours a week

to teach comparison with each figure

*drawing:* 3 hours a week

to draw arc, line and volume

- the 1st grade: *arithmetic: 6* hours a week

to teach series and logarithm

*geometry:* 4 hours a week

to teach practical use

*drawing:* 4 hours a week

to draw a map and others

«Sokuchi ryaku» was the appointed textbook for geometry.

It was written by Tora Uryu in 1872. It was for the measurement

of land. So part of the book is used for the text. The

contents of it were almost definitions and their explanation.

**IV. TeacherTraining**

Positive educational policies of the Ministry of Education

were to build schools, to arrange syllabuses and to train

teachers.

To get rid of defects of traditional education, it was necessary

to adopt western-style training and to train teachers by

foreign teachers.

In September 1872, the normal school was built in Tokyo

and lectures began. The following April, the attached elementary

school was established. This was the first attached

school for the pupils of the normal school. It was used

not only to practice teaching but also to investigate teaching

methods. Then it developed into the research center

of the normal school and the model school for all prefectures.

At that time, the contents of teaching in normal

schools were mainly concerning teaching methods. In June

1873, it adopted academic subjects of study. Until then the

term of the school year was not determined, so pupils

could graduate according to the results at any time. They

were dispatched to each prefecture as teachers. The term

of the school year for upper and lower schools was determined

to be two years and each grade was divided into two

stages.

As schools were built in each prefecture, there arose a

problem of shortage of teachers.

In general, people thought that the contents of elementary

schools were three subjects: reading, writing and abacus

just like in «Terakoya» and other private schools. So teachers

of punctuation, writing and abacus were employed

for special subjects.

At that time it was easy to get teachers from «Terakoya»

and other private schools. The Ministry of Education made

efforts to train prospective teachers. In 1874 each prefecture

established teachers’ training schools without permission

of the Ministry of Education. About 46 of this kind were

built all over the country.

Many books on education were published by the Ministry

of Education. It thought that it was of no use only to teach

teaching methods, but it was neces-

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sary to explain to newly appointed teachers in detail the

book of «The Primer of Elementary Schools».

Then the principal of Normal School, Nobuzumi Morokuzu

published «The Required Manual for Elementary School

Teachers», and many other books of this kind were published.

On the other hand, M. M. Scott taught carefully the teaching

method of modernised schools, not through the

books. The students who graduated from the schools went

to provincial normal schools to teach the teaching

methods. But as the percentage of school attendance

increased, the number of schools increased. So there was

a shortage of teachers for a long time.

It was necessary not only for teachers but also for the

people to have knowledge about modernised schools —

«What is education? Explanation of the mission of the

school, the system and management of modernised school

and teaching method in details.» I introduce one of the

popular editions of the book, «The Required Manual for

Elementary Teachers» written by Nobuzumi Morokuzu in

1873.

In this book he explained the important duties of teachers

in three items - «The teachers arrange the seats of pupils

according to their results. They move to upper classes after

the test. Where does the teacher stand in the classroom?

How are the desks arranged? etc.»

This description was easy to understand for teachers. It

may have been difficult to teach the style of modernised

school lessons and to get rid of the «Terakoya’s» teaching

concept.

Now I shall introduce a part of arithmetic education.

*The 8th grade: arithmetic*

To show the picture of numbers; it was necessary first to

know how to read the numbers and to teach Arabic numbers.

Teachers wrote both numbers on the blackboard and

asked the pupils to read them or teachers read them themselves.

Then children wrote on their slate board.

Afterwards, some of them wrote on the blackboard.

Teachers checked them and said to the children, «Raise

your right hand if it is right.»

*The 6th grade: question and answer*

Using pictures about shape, volume, line and angle, teachers

taught shapes and surfaces of material or the kinds

and names of lines etc.

**V. «The Book of Elementary Arithmetic»**

The Book of Elementary Arithmetic was chosen as the textbook

for the elementary school syllabus of the normal

school and the elementary school syllabus of many prefectures.

It was written by the Tokyo Normal School and published

by the Ministry of Education. It was a progressive syllabus

compared with the textbooks of other subjects at that

time. It was considered to be one of the best books on

arithmetic education in the country.

The textbook was much influenced by W. Colburn — his

thought was based on the «intuition» of J. H. Pestalozzi.

In this edition the main editors were M. M. Scott and C.

Davis. It was used in many prefectures. It was excellent,

but it was doubted whether it was actually used by teachers.

In March 1873 the first volume was published; in April

1873 the second volume was published;in May 1873 the

third and fourth volumes were published; in September

1876 the fifth volume was published.

I introduce a part of it.

*The first volume*

«The Elementary School Syllabus» of the normal school is

taught in the 6th grade. In the 7th and 8th grades, the primer

of elementary school — numbers, the memorisation of

the calculation tables of addition, subtraction and multiplication

were already taught.

Japanese numbered

Arabic number

Explained in two pages, the first edition of the book written

by W. Colburn was published in 1821; it was used for 60

years with several revised editions. The last version of ‘84

was published after his death. The Book of Elementary

Arithmetic was, although it was published in 1873, similar

to the hook of ‘84. Material selection and explanation of the

problems were like that of ‘84. Considering the fact of M. M.

Scott’s coming to Japan, it may have been modelled after

the book of ‘63. In the first volume, children study addition

- example: add 1 to 1-10 and 2 to 1-10. Question and answer

were just as in the book of ‘63.

So, how did M. M. Scott see the Japanese? Recently, the

following letter of M. M. Scott was found at the Griffis collection

in Rutgers University. A part of his letter to Griffis is

cited below.

«You ask me what I think of the Japanese after thirty-five years’

experience of them. I may say that I always had a very high opinion

of their intellectual qualities but had some misgiving as to the practical

application of what they could so easily learn.

Those misgivings have been completely brushed away from my

mind. They have proven themselves great in nearly every department

of human effort and I predict for them even in the near future

greater achievement still. When I left Japan it was with much regret

at what I thought then to be disserverance of a ten years’ acquaintance

much appreciated by me, but I have had now twenty years’

experience with a large number of Japanese in Hawaii, with a different

class indeed, but still with their able and amiable official here.

You are quite right in thinking this country a very interesting one. I call

it ‘a museum of ethnology’. It would pay you to take a vacation and

come here in the near future, and with your powers of observation

and your slashing pen you could show us to ourselves as others see

us. Pray do come. l will give you a welcome.»

**Vl. The Situation of Arithmetic Education at the**

**Beginning of the School Systems**

A superintendent visited schools throughout Japan.

How did he feel?

*A. The report of the superintendent D. Murrey*

He said, «There remains traditional style of education.» But

he thought that it was better to change

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only slowly. I think he understood that making gradual progress

was better for Japan. He held the same opinion

concerning the abacus.

*B. The situation in the country*

Over a short period of time, each prefecture established

training centers and normal schools to lessen the shortage

of teachers. Each prefecture devised the method of a circuit

teaching independent of the school administration.

Developing school systems in the Meiji Era meant fighting

against poverty. They were hard pressed to pay money for

education, but realised the value of education gradually

and became interested in it.

All the heads of the Ministry of Education made a tour of

inspection. It was because people complained of the

school systems, and also because there arose a tendency

of revising the syllabus of schools.

The school buildings were temples or people’s houses.

The shortage of teachers was a crucial problem. So the

excellent children who left elementary schools with an

excellent score were adopted as teachers. The children

aged more than 15 years were adopted as assistants. The

children aged more than 13 years were adopted «leaders

in the lesson». Their ability was just the same as the 6th

graders of today.

As the subsidy of government and prefecture was very

small, the money for school was expanded for the elementary

school district. Farmers and fishermen in those days

were so poor that they could not afford it.

There was a time when there were no blackboards and

notebooks in the classrooms. Children wrote the numbers

and words on trays which were filled with sand or rice bran.

Some of the teachers were not competent to handle the

four operations with calculation figures and did not understand

why the product of the decimal was less than the multiplicand.

When a child learned the Arabic numbers, he

asked, «Is number 6 just like the shape of the nose?»

There were a few regular teachers who began to study

themselves in each prefecture.

*C. Promotion and examination*

In elementary schools, pupils had to pass the examination

to go up to an upper grade. They took the examination for

each grade, and one who succeeded in the examination

could go up to the higher grade.

When the pupils left the elementary school, the middle

school and the university, they had to take the examination

to leave school.

Each prefecture established «the test of the elementary

school». There were many books on teaching methods

which were published by a private company. I found the

background and the method of pedagogy in them.

*D. Conclusion*

The leaders of the Meiji government were overwhelmed

with civilisations of foreign countries. They thought that

there was not a moment to lose to equalise the standard of

education to catch up with the advanced countries.

The government made efforts especially to establish elementary

schools compulsorily, and also to let children go to

school, but they did not know people’s opinion on education.

The government intended to change the existing system

to develop modernised schools in Japan.

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**On the Value of Mathematical Education**

**Retained by Japanese Society as a Whole**

Takashi Izushi and Akira Yamashita

We would like to talk about «On the Value of Mathematical

Education Retained by Japanese Society as a Whole».

This is a research on how school mathematics, which students

learned at junior and senior high schools, has been

retained by them when they grew up into society. The purpose

of the study is to try to get a fundamental viewpoint

for the organisation of curriculum for the department of

mathematics in the future.

**1. The History of Mathematical Education in Japan**

We think that the history of mathematical education in

Japan may be divided into the following six stages:

*The first stage*

This stage was the period of about 40 years of the Meiji

Period (1868 — 1911) through the Taisho (1912—1925) to

the beginning of the Showa Period (from 1926). In this

stage, the main objectives in mathematical education were

to get skills in calculating, to train thinking power and to

gain practical knowledge.

*The second stage*

This stage was the period of about ten years in which

mathematical education was seriously affected by the

Perry Movement. The main objectives were to develop the

concept of function and to foster one’s power of space

observation. World War II took place during this stage.

*The third stage*

This stage was the period of about ten years after World

War II and was influenced by the USA. The method in

mathematics teaching was focused on daily life experience.

In this stage, the six-year term of compulsory education

was extended to nine years.

*The fourth stage*

This stage was the period of about ten years which was a

period of review of the former stage. Mathematics teaching

was taken as the matter of mathematics systems seriously.

And Japan’s economic growth began in this stage.

*The fifth stage*

This stage was the period of about ten years in which

mathematical education was seriously affected by modernisation.

The percentage of the number of students going

on to high school was over 90%.

*The sixth stage*

We are now in this stage, the period of about ten years

which is a period of review of the modernisation efforts.

And efforts are made in mathematics curricula to meet the

differences among the students.

**2. Purposes**

The purposes are classified into the following three.

The *first purpose is* to examine the following: Do they

remember or understand mathematics contents which

were learned at junior and senior high school?

The *second purpose is* to examine the following: What

kind of contents of mathematics are useful to their work?

The *third purpose* is to examine what may be called formal

discipline. In the first stage of the history of mathematical

education in Japan, formal discipline was emphasised

and Euclidean geometry took a great part as formal discipline

for a great guiding principle.

So, we examined what impression was made on them.

From the examinations mentioned above, we try to get the

mathematics contents which are retained by Japanese

society, and we would like to use their knowledge to organise

the curriculum for the department of mathematics.

**3. The Way of Examination**

The examinations were carried out twice.

*(1) The first examination*

This examination was carried out in 1955. The time was

during the fourth stage of the history of mathematical education

in Japan.

*a) Participants*

The participants are members of society who learned

mathematics before the third stage. They have contributed

to economic growth in Japan, more or less. The number of

participants was 976 and they were sampled from the

whole society according to their occupations: technologist,

teacher, specialist, administrator, office worker, farmer,

fisher, seller, etc.

*(b) by Content*

They were asked to solve problems connected to the following

contents:

1 Calculation (including positive and negative

number, literal expression)

2 Round number, percentage

3 Proportion, reciprocal proportion

4 Fundamental figure

5 Solid of revolution

6 Scale

7 Projective figure

8 Word problem (equation of the first degree)

9 Congruence and similarity of triangles

10 Statistics (graph)

11 Pythagoras’ theorem

12 Trigonometric function

13 Coordinates

14 Word problem (simultaneous equation)

Furthermore, they were given the following questionnaires

corresponding to the problem: «If you understand the

content related to this problem, is it useful to your work?»

*(2) The second examination*

This examination was carried out in 1982. A part of the

result of this examination was presented at the

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I C M I-JSME Regional Conference in Mathematical

Education (1983, Japan).

*a) Participants*

The participants were graduated from a senior high school

belonging to a national university. This senior high school

is an eminent school for the graduates going on to college.

The 62 persons of the object were chosen corresponding

to the years they were in senior high school, 19 persons

from the years of the third stage, 18 persons from the years

of the fourth stage and 25 persons from the years of the

fifth stage. Their occupations were technologist, scholar,

doctor, etc. and they contributed to the improvement of

technology and science in Japan, more or less.

*b) Content*

They were asked to solve problems which were mainly

related to elementary geometry, because we wanted to

make a study of formal discipline.

**4. The Outline of the Examinations**

*(I ) The result of the first examination*

The problems were connected to the following

contents:

I questionnaires of elementary geometry

II 1 perpendicular of line and plane

2 parallel

3 sum of internal angles of triangle

4 projective figure

5 equivalent transformation

6 condition of triangle construction

7 congruence and isosceles triangle

8 measurement

III logic

IV axiomatic method

V non-geometric model of the axioms of the affine

geometry

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Table 1 : Correct Answer (%)

a specialist and administrator (218 persons)

b office worker (219 persons)

Table 2 : Answer That is Useful to Your Work (%)

a specialist and administrator (218 persons)

b office worker (219 persons)

*(I ) The result of the second examination*

Table 3 : Correct Answer (%)

a specialist and administrator (218 persons)

*(3) The outline of consideration*

*a) Knowledge*

Simple mathematics knowledge which was learned in

junior and senior high school is well remembered by every

person, though many years have passed since they learned

it.

In these tables (Table 1, Table 3) only a, b are displaced,

but the persons of other occupations answered also more

than 40% of the problems correctly.

It is not suprising that a gets higher percentage of correct

answers than b. But the difference is not so much.

The problems in which a relatively differs from b in percentage,

are connected to proportion and reciprocal proportion.

As for the problems of calculation, b is better than

a.

The response about the problems of calculation, round

number, fundamental figure, and congruence and similarity

of triangles, shows high percentages for each person.

On the other hand, the problems of projective figures and

trigonometric functions, show low percentages. This is a

consequence of the times in which they learned. We cannot

find any differences between the results of the first

examination and that of the second examination.

In the second examination, the objects were chosen

concerning the years of graduation from senior high

school. We find that knowledge, such as of perpendicular

line and plane, which may be observed in daily life are forgotten

as the time passes after they learned them.

*b) Usefulness*

The number of persons who agreed to the questionnaire «If

you understand the content related to this problem, is it

useful to your work?» is smaller than the number of those

who answered the problem correctly .

The contents judged relatively useful for the persons of

both a and b are scales and statistics. The contents of proportion,

reciprocal proportion and fundamental figure are

thought to be more useful by the persons of a.

Many persons answered correctly the problems of calculation

and solids of revolution, but few persons thought that

these contents were useful. And many persons judged that

content of coordinates not useful to their work.

The persons of a more than b think that the contents of

proportion, reciprocal proportion and fundamental figure

are useful.

*c) The way of thinking*

It is examined here whether the attitude of deductive thinking

which was obtained by learning elementary geometry

is still in their mind or not.

IV and V of the second examination are the problems

which need deductive thought. V is a problem of a

non-geometric model of the axioms of affine geometry.

The result of examinations is satisfactory. It has been a

long time since they learned, but they have not forgotten

the attitude of thinking deductively.

The result of examinations about equivalent transformation

is satisfactory too. The result of examinations about

logic is satisfactory.

Y ounger persons replied that they used mathematics

knowledge.

But older persons replied that they judged by common

sense, though they might use mathematics knowledge

unconsciously.

Many members of Japanese society judge that thinking

and reasoning powers are developed by learning elementary

geometry. And they judge that knowledge of elementary

geometry is useful to their daily life, but not to their

work.

From these findings, we may conclude that formal discipline

is supported by Japanese society as a whole.

What was mentioned above is the results of our examinations,

but it can show a viewpoint on what school

mathematics should be.

**5. Examples of Problems**

*(I ) Problems of First Examination*

1. Proportion and reciprocal proportion

Mark an x if the following statement is false: A train can

travel 100 km/h. The distance traveled

is proportional to the time traveled. ( )

2. Calculation

a) (+12) - (-7) + (-15)

b) (0.15 - 3 : 4) -0.3

c) 4a2b x 2a2b4

3. Fundamental figures

Mark an x if the following statement is false: For three

lines 1, m, n in a plane, if 1^m and 1^n, then m//n. ( )

4. Congruence and similarity

Which of the following are congruent or similar triangles?

a) ( ) and ( ) are congruent triangles.

b) ( ) and ( ) are similar triangles.

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5. Projective figures

Mark an x on a projective figure which expresses

true length.

(a) (b) (c)

( ) ( ) ( )

6. Scale

Find the actual distance using the scale:1 :50000

a) 3 cm b) 4 cm c) 14 cm

7. Solids of revolution

What are the following solids of revolution?

a) b)

8. Coordinates

a) Give the coordinate of point P.

b) Find the equation of the line PQ.

*(2) Problems of Second Examination*

Problems of perpendicular line and plane

Describe the definition of the following word.

a) A line is perpendicular to a plane (in a space).

b) A plane is perpendicular to another plane (in a space).

2. Problems which need deductive thought IV Assume the

following premise about the organisation of roads and bus

stops.

- There are at least two bus stops.

- For every two bus stops, there must be one and only

one road which is through them.

- There is one and only one bus stop on the intersection

where two roads cross.

- All bus stops don’t go along one road.

- For each road and a bus stop (not along this road), there

is one and only one other road which doesn’t intersect

the first road and goes through the bus stop.

According to these assumptions, show in the following

order that there are at least four bus stops.

Ql. There are at least two bus stops.

Why? Let these bus stops be named A and B.

Q2. There is only one road which goes through A and B.

Why?

Q3. There is at least one bus stop (not along the road in

Q2). Why? Let it be named C.

Q4. There is only one road which goes through A and C.

Why? Let the road be named b.

There is only one road which goes through B and C.

Why? Let the road be named a.

Q5. There is only one other road which goes through A

and does not intersect the road a in Q4. Why?

There is only one other road which goes through B and

does not intersect the road b in Q4. Why?

Q6. Two roads in Q5 certainly intersect. Why?

Q7. There is only one bus stop at the intersection in Q6.

Why?

VA’s family go on a journey. Assume the premise about the

organisation of cars and passengers.

- There are at least two cars.

- For every car, there are at least two passengers.

- For any two passengers there is only one car containing

both passengers.

For every car show in the following order that

there is a passenger who is not in the car.

1) There is another car which differs from car or. Why?

Let the car be named b.

2) A is in car b. Then there is a member other than A who

is in b. Let the passenger be named B.

3) Both A and B are not in car a. Why? So there is a passenger

B who is not in car a.

In addition to these, the following assumption holds good.

- For each car and a passenger (not in the car), there is

one and only one other car containing this passenger

but not containing any passenger in the first car.

For each car, there are at least two passengers who are

not in this car. Why?

For each passenger, there are at least two cars which do

not contain this passenger. Why?

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**An Evolution Towards Mathematics for All in Upper**

**Secondary Education in Denmark**

Ulla Kürstein Jensen

**1. Upper Secondary Education in Denmark**

A description of the evolution in the teaching of upper

secondary mathematics in Denmark would be built on sand

without a few introductory remarks about upper secondary

education in Denmark.

The traditional Danish upper secondary school called

gymnasium is a three-year education based on nine years

of compulsory education that contains mathematics throughout.

Nevertheless, many students choose to postpone

their start in the gymnasium until after the optional tenth

year offered by the schools where they receive their compulsory

education and originally meant for those who want

to increase their qualifications without going to the gymnasium.

The gymnasium consists of two lines, the languages line

and the mathematics line, each of which is divided into

several branches. Mathematics is compulsory on all

branches and is taught at three levels in the gymnasium.

The lowest is the one for the languages branches, and

there are two for the mathematics branches.

The branching doesn’t take place until after the first year,

and the maximum number of mathematics lessons per

week on a branch is six whereas the minimum is three. The

standard timetable on each branch comprises about ten

subjects all of which are compulsory.

In 1983 about 20,000 students finished the gymnasium

and about 12,000 of these came from the mathematics

line. Nearly 4,000 students finished another academically

oriented education, the so-called HF-education. Under certain

conditions the HF-examination offers the same opportunities

for advanced studies as does the «studentereksamen

» the final examination of the gymnasium.

Mathematics is a compulsory subject in the first year of the

HF-education.

In 1983 a total of about 24,000 students, which is approximately

40% of a year of Danish students, completed an

upper secondary education with some mathematics.

**2. The Evolution that Started in 1961 and a Step**

**Towards an** «**Upper Secondary Mathematics for**

**All» Course**

When describing the evolution in the teaching of upper

secondary mathematics it is natural to start in 1961 when

new curriculum regulations for the upper secondary school,

the gymnasium, were signed. The regulation for mathematics

was penetrated by the new-maths-wave and intended

for a small proportion of the students, but it was to be

applied by a rapidly increasing number of students during

the next 20 years. It is important to notice that it restored

mathematics as a compulsory subject for the languages

line students after a pause of a decade. This turned out to

be a small step towards mathematics for all.

The purpose of mathematics for the language students

was to give the pupils an impression of mathematical way

of thinking and method and to provide them with mathematical

tools that could be useful in other subjects at

school and during their future activities, so it wasn’t only

aiming at university studies.

The topics to be taught were: the concept of a function,

elementary functions, infinitesimal calculus, computation of

compound interest, combinatorics and probability theory.

The first textbooks were very theoretic and the whole

course nearly failed completely, but it was rescued by a

new textbook at a suitable level.

When the HF-education, that is a type of further education

meant as an offer to everybody who wishes to qualify

for more advanced theoretic education, came into existence,

it was but natural that the purpose of the mathematics

regulation 1967 was nearly the same as the one for the languages

line students. The course was to be more elementary

though, for instance, it shouldn’t comprise infinitesimal

calculus, but the textbooks included chapters from the textbooks

for language students. After a short time, it became

evident that fundamental changes were necessary in order

to change this compulsory one-year, five lessons per week

course to a success.

In the revised mathematics regulation for HF it was mentioned

as the first goal to provide the students with mathematical

knowledge that could be useful in other subjects

and in their daily life, and as the second, to give them an

impression of mathematical method and way of thinking.

Theoretical algebra and group theory were removed from

the curriculum and statistics, probability theory, and binomial

test were entered. The so-called free lessons meant

for cooperation with other subjects or for elaboration of previously

treated material appeared in the curriculum. A textbook

for the course was written on the basis of an experiment.

This textbook comprised many examples connected

to everyday life and it encouraged the pupils to find others.

The style of it made it easy to understand also for many

who had earlier given up mathematics as too difficult or as

uninteresting. It contained few proofs, but it was very good

at helping the students to create relevant concept images.

The text encouraged the teacher to let the students spend

much of the lessons working in groups of three to five persons

solving and discussing problems. In this course,

mathematics was no longer a terrifying subject.

In retrospect, the new HF-curriculum together with the new textbook

and the applied methods of teaching seem to have served as a catalyst

for the succeeding evolution. The course was the first course

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that might be called an upper secondary mathematics

course for all.

The HF-mathematics that at the beginning had been heavily

influenced by the mathematics curriculum of the languages

line now in turn influenced this curriculum little by

little. Among the reasons for this evolution should probably

be mentioned that the increasing number of students not

aiming at university studies or similar advanced studies

created difficulties for the mathematics teachers, especially

when they were teaching the languages line students,

and the fact that teachers often feel more free to experiment

when teaching these students than when teaching

those on the mathematics line because the former have

only an oral exam to pass whereas the latter have to pass

an oral as well as a written exam, the latter being centrally

set by the Ministry of Education. So the teachers tried

consciously or subconsciously to remove some of their difficulties

in the mathematics lessons for the languages line

students by using ideas or methods from the teaching of

the HF-students, and during the last half of the seventies

an increasing number of teachers chose an optional

mathematics syllabus for their languages line students.

The above mentioned optional curriculum had for some

years been used by 90% of the classes when in 1981 it

became regulation. Let me state a few remarks in order to

characterise it. The objectives are:

The students should acquire:

- some mathematical knowledge which can be of use in

other subjects and in their daily life,

- a knowledge of the framing and application of some

mathematical models,

- an impression of mathematical methodology and reasoning.

It is noteworthy that it is now explicitly mentioned in the

objectives that the students should acquire some mathematical

knowledge that can be of use in their daily life and

a knowledge of the framing and application of some mathematical

models. It is also remarkable that differential calculus

in this syllabus may be substituted by another coherent

material of the same extent and value if the teacher and the

students so wish. Another interesting feature is the so-called

free lessons. These approximately 25 lessons can be

used for going deeper into the compulsory topics, for working

with new topics, for instance some that are connected

to other subjects or for providing an introduction to electronic

data processing and its role in society. The teacher and

the students choose how to use these lessons.

**3**. **Drafts for Courses in Advanced Mathematics that**

**Are at the Same Time Worthwhile Courses for**

**Students Not Intending to Go to University**

The evolution on the mathematics line accelerated later.

During the last ten years, the number of students choosing

this line has increased from about 7,000 to about 12,000,

but at the same time the percentage going to university or

similar advanced education has dropped considerably. As

a result several booklets written by teachers as most textbooks

are and offering alternative and often more intuitive

and less formal approaches to many topics have appeared

during the last five years. The majority is intended to be

used either the first year in the gymnasium or at the lower

of the two levels. The booklets represent a new interpretation

of the regulations. The evolution in HF-mathematics

and mathematics on the languages line have inspired the

authors of the booklets.

A new interpretation of the old regulation isn’t enough to

take into account the change in the students’ qualifications

from their preceding schooling as well as the growing

influence of computers and the fact that for the majority the

mathematics course in the gymnasium isn’t just a kind of

an introduction course but the students’ final mathematics

education. Therefore, the Ministry has just issued a draft

for a new curriculum for mathematics for each of the

branches of the mathematics line.

The intentions leading to the construction of the draft for

the curriculum common to all branches but the one on

which mathematics is taught at the highest level, the

mathematics-physics branch, were to create a curriculum

that:

- within some central mathematical fields shows mathematics

as a subject with its own essential values,

- permits an all-round elucidation of the interaction between

mathematics and other subjects,

- allows time for absorption in major concepts and correlations,

- allows time to meet special wishes from the class or the

school,

- encourages that the content of the lessons should be

influenced by the teacher and the class to a greater

extent than before,

- guarantees that the student has received an all-round

mathematical education with perspective and width,

- prepares the students for a wide range of types of further

training for which a foundation in mathematics is

required.

Both drafts live up to these intentions by comprising not

only a list of topics to be taught but also a list of aspects

that are to be brought into the teaching, and various comments

on ways and means of teaching. The following

objectives, topics, aspects and comments are included in

both drafts.

Objectives:

The students should acquire insight into mathematics as a

form of cognition and as a means of description.

Topics:

1. Integers, rational and real numbers

2. Plane geometry

3. Functions

4. Infinitesimal calculus

5. Statistics and probability theory.

Aspects:

i) Through suitable examples the students should experience

how an algorithmic approach throws new light on

the mathematics they work with, and they

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should acquire a knowledge of the practical application of

electronic data processing.

ii) The students should acquire knowledge of parts of the

history of mathematics and of mathematics in cultural,

philosophical, and social context.

iii) The students should obtain knowledge of formulation

of mathematical models as idealised representations of

reality and get an impression of the possible applications

of mathematical models and of the limitations in

the applications.

iv) The students should learn about mathematics, they

should be aware of mathematics as a form of cognition

and as a language.

Comments:

As to the comments, I have chosen to include just the following:

- The choice of methods of work is to be adapted to the

students as well as the mathematical content, and the

students should be acquainted with several methods of

work so that they can take part in the choice.

- As to the use of textbooks and texts, it is desirable that

the students, apart from reading ordinary textbooks,

become acquainted with texts about mathematics. It is

also desirable that the students try to read a mathematical

text in a foreign language.

- Topics should be approached from different angles.

There has got to be deductive sequences as well as

intuitive ones. Also, the students should become increasingly

familiar with the language of mathematics including

symbols and concepts from set theory and logic.

- When planning the lessons, respect should be paid to

subjects where mathematics is applied.

The draft for the curriculum for the branch with the highest

level of mathematics, the mathematics-physics branch, differs

from the previously mentioned by including, for instance,

some numerical analysis, induction, mathematics from

an algorithmic standpoint and recursion that should provide

the students with a general theoretical background for

future work demanding and involving the use of computers.

It also comprises a considerable amount of free lessons,

the content of which has to be chosen by the teacher and

the students, and it requires a more elaborate treatment of

some topics. Among the additional demands on this branch

I should also like to draw attention to the fact that the

students should learn to express themselves precisely and

clearly orally as well as in writing, and to the fact that the

students should acquire an understanding of the deductive

nature of the subject by working with proofs in connection

with which they should also get the opportunity to construct

proofs of their own.

Teachers who want to cooperate and increase experience

by testing the drafts, thereby helping to formulate other

concepts of them, are going to use them now. Drafts for

textbooks are also being published. On the basis of these

efforts, curriculum regulations beneficial to many, not only

to future mathematicians, should appear in a couple of

years.

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**Fight Against Academic Failure in Mathematics**

Josette Adda

In France in recent years, studies on the rate of academic

failure have revealed the fact that the education system

functions by a process of successive elimination of pupils

from the normal streams at each level of orientation.

In the appended diagram, an extract from (12), it can be

seen that among children born in 1962 in France there

remained at the «theoretically normal level» only 72.2% at

age 7, 59.5% at 9, 44.1% at 11, 34% at 13, 21.9% at 15

and 16.1% at 17 (the remainder having repeated years or

having been put into marginal-type classes).

Moreover, it appears that, on the one hand, these eliminations

concern more selectively children of socio-culturally

deprived families and, on the other hand, that the responsibility

of «the teaching of mathematics» (and not

mathematics themselves) is essential in these orientations,

which have the effect of confirming social inequalities.

In order to evaluate this «inequality of opportunity» as far

as mathematics are concerned, it has been noted that, for

1976—77 (cf. (2)), 52% of the children of upper executives

in the corresponding age group were following the

C-stream (i. e. a course with a predominance of mathematics),

the rate being 6% for the children of workers; their

chances of reaching that particular class being respectively

91% and 23%. Thus, entry to these classes was far from

being equal for all and the «socio-cultural handicap» was

2.2 times more disastrous for the C-section than for all the

classes.

An examination of the socio-professional category of the

family head for students at the Ecole Polytechnique in

1978—79 (cf. (2)) reveals that, out of the 602 students, 422

come from the category of «liberal professions and upper

executives», (or 70%), whereas this category represented

8.3% of the French population for the age group under

consideration.

As for the final year mathematics specialists of the Ecole

Normale Superieure of the same year, one notes that, out

of the total of 21 students, 12 had at least one parent who

was a teacher.

What a lot of «wasted intelligence» (to use the expression

of M. Schiff)!

We shall sum up briefly some research carried out at the

University of Paris 7 which aims to analyse *why* certain

children fail and others succeed and *how* the process of

failure works, so as to find what changes should be made

in teaching to remedy the situation.

In order to analyse the phenomenon, it is first of all necessary

to be aware that children are not normally in direct

contact with mathematics, that be coming familiar with

mathematics is achieved by the intermediary of «mathematics

teaching», which in fact plays the role of a simple

intermediary for only a minority but is rather a filter for the

majority. A study of its workings is thus indispensable.

Classes are essentially carried out, of course, not in

«mathematical language» but in natural language, and so

create numerous external difficulties of linguistic origin (on

the semantic level rather than the lexical or syntactic levels

as is often thought). These difficulties have been analysed

at length by D. Lacombe (cf. his lectures at Paris 7).

However, the purely linguistic explanation is still insufficient,

and it is also in the pragmatic and the rhetoric of the

discourse of the mathematics teacher that we must seek

the sources of faulty comprehension and misunderstandings.

First of all, being abstract, the objects of mathematics that

are treated, the properties and the relations that are studied

can never be seen (in contrast, for example, with the

objects studied by the physical and natural sciences) and

so the distance between the *signified* and the *signifiers*

plays here a role that is more crucial than for any other type

of discourse. Signifiers such as mathematical symbols, diagrams,

graphic representations are necessary and yet are

the source of poor comprehension of the mathematical

objects signified (cf. (1), (4), (6), (8)). By studying the

«misunderstandings» brought about by this confusion between

signifier and signified we have observed the responsibility

they bear not only in a very great number of errors

but also in the impossibility of acquiring the concepts themselves.

For example, for many children (and teachers!)

there are no sets without a string and numerous adolescents

affirm that 2 is a whole number but neither decimal

nor rational, 2.00 is decimal but neither whole nor rational,

and 4/2 is rational but neither decimal nor whole!

Another way of representing mathematical concepts is the

use by teachers and textbooks of metaphors (cf. (14)) that

are supposed to refer to the experience of the student.

Almost always far too simplistic really to conform to any

recognisable reality, they are nevertheless still too complex

not to be burdened likewise with numerous extraneous

meanings that block access to mathematics.

Instead of seeking to show through appropriate exercises

the abstract and generical nature of mathematical

concepts, one type of pedagogy has sought for some years

to «make mathematics concrete» through the method of

teaching: an absurd enterprise and as such inevitably

bound for failure. The initial idea that starting with real-life

situations and anathematising them can be a form of motivation

in early stages was in itself quite reasonable. The

constraints of the school system, however, led to presenting

mathematical questions rigged out in «disguises» that

were extremely artificial, pseudo-concrete and the source

of misunderstandings, becoming thus extramathematical

causes of errors in problems claimed to be «mathematical

». Even problems of the type «Mummy goes shopping,

she buys (...)» are not really natural and the expenditure

calculated is often very different from that of actual

purchases (unrealistic prices, proportions out of line with

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commercial usage .. .). Moreover, this variable «Mummy»

(each pupil supposedly feeling involved) introduces an

emotional factor that is not necessarily positive: for

example, when the mother has financial difficulties, has little

time to do the shopping, is sick, far away or deceased .

. . (cf. (3)). Reactions in the face of these *academic*

«situationproblems» are very different from those of students

to whom one can give the opportunity to «mathematise

» a *real-life* «situation» problem: doing the shopping

themselves, for example.

F. Cerquetti has shown that when pupils in an apprentice

class for baker-pastrycook have to do all the calculations

for purchases necessary for making croissants and for selling

them, considerable success may be noted, whereas

the same students react against all the artificial «word»

problems put to them in textbooks and prefer and succeed

better in purely abstract games with numbers (cf. (9)).

Young children have a potential for abstraction which is

not exploited. The fact that primary school teachers are

often recruited from the students who have the least positive

feelings towards mathematics is very worrying in

France. It sets up an interlocking process of failure (cf. (8))

and declining performance is one of the most distressing

phenomena of our educational system (cf. work in progress

by F. Carayol and M. Olvera in particular). For example, the

use of clear symbolism is perfectly well allowed as a simplification

by young children (cf. the well-known experiments

of Davydov in the USSR), but certain ways of introducing

badly understood formalism are rejected by students in the

secondary system: in fact, when one seeks the causes of

rejection in the teaching of mathematics, one almost

always finds that it is a question of notions, the presentation

of which has been carried out in an inconsistent way,

with inner contradictions .

It is important to stress also that class use of questions

«disguising» mathematics, a method fraught with errors

because of the outside influences that are introduced, is

not «socially neutral» and this constitutes a further factor of

selection (cf. (5) and (10)). At the beginning of this century,

exercises referred above all to a rural universe of landowner

adults who exploited their holdings, transmitted inheritances,

invested their savings, and so on. Today, there is

an attempt to involve the child more and so school exercises

refer often to children but these are children living

in towns or cities (often the capital), receiving lots of presents,

making journeys, and so on. Thus, not only can certain

children not be familiar with certain of the elements

necessary to understand the questions but, above all,

these «disguises» contribute to giving many of them the

idea (immigrants or not — some speak of «home-grown

foreigners»!) that they are foreigners in this world, this universe

of schoolroom problems that they believe to be (the

ultimate mistake!) the universe of mathematics. It is striking

to see the archetypes that certain pupils propose when

they are asked to invent the text of a problem (cf. (3) for

example).

Questionings used in *evaluation* do not so much reveal

inequality but create it (cf. (7)). The formulation and the

presentation of mathematical problems present the same

sort of bias as those (denounced for years now) of IQ tests.

Moreover, poor results have an all the more disastrous

influence in that the present school system sets up a *loc -*

*ked-in process of failure* through the «Pygmalion effect»

(self-fulfilling prophecy) and above all by the irreversible

streaming off towards poorly thought of types of classes.

The struggle against academic failure in mathematics is

not a question of change in curriculum. It requires a

concerted attack on the true basic problems, for otherwise

all the sources of difficulty can recur, in a more or less

serious form, on any chapter of mathematics. T h e

«reform» of recent years has been a good example of this,

with the result that all the criticisms expressed at the time

of the survey on teaching carried out by the review *L*

*‘Enseignement scientifique* in 1932 are easily transposed

into the present situation.

Teachers must become aware of those aspects of their

teaching practice which create misunderstandings and lack

of comprehension; they must not only have a solid knowledge

of the mathematics that they have to teach but also

be capable of understanding how these are to be transmitted

in the mathematics class and study the ethnological

features of that universe where, in the interrelationship between

teacher, pupils and mathematics, communication is

threatened by interference which can be called «socio-logical

» (according to P. Bourdieu).

In order to give all pupils access to mathematical culture

with its own specific features, it would be necessary, as for

physical culture, to offer to all the pleasure and the opportunity

to carry out exercises (here intellectual and abstract

ones) (see for example (11)). Above all, let us not forget

that we are talking about an integral part of the cultural

heritage to be transmitted and that theorems are also

works of art: Pythagora’s theorem is a classic on the same

plane as a play by Shakespeare or a painting by Leonardo

da Vinci, and there is in it an aesthetic wealth that we must

try to offer to all. Contrary to those who want to confine

underprivileged children to «useful mathematics» (in the

sense of creating minimal automatic responses with no

practice in deductive reasoning), I think that we must

attempt to allow all children to exercise a right to abstraction

(for which mathematics teaching offers the best opportunity),

the formative element in the loftiest activities of the

human mind.

Certain mathematicians, conscious of the collective responsibility

they bear for the harm done by misuse of their

discipline, recognise that it is their duty to react. Academic

failure at the present time is no longer solely the failure of

pupils, it is the failure of the whole educational system,

and, if teachers fail in their struggle against this failure, the

responsibility will fall on those whose mission it is to train

teachers, in other words, those working in tertiary education.

The mathematical community must conscientiously do

its duty towards the school system through the initial training

of teachers and inservice training (with

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special emphasis on recent research on mathematics teaching)

for practising teachers, as well as by the development

and improvement of the forms of support

necessary for all those for whom it cannot be provided by

the family context.

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**EQUALS: An Inservice Program to Promote the**

**Participation of Underrepresented Students in**

**Mathematics**

Sherry Fraser

Mathematics has been called the queen of the sciences.

She could also be called the gatekeeper to the job market.

Too often, students who might find job satisfaction in a

scientific or technical field are unable to enter that field

because of inadequate preparation in mathematics. Why is

it that students, especially female and minority students,

stop taking mathematics in high school, as soon as it

becomes optional to do so?

Many women and minority students don’t see the relevance

of mathematics to their future lives. This perceived

lack of usefulness of mathematics contributes to the high

dropout rate. If students don’t see the need for math they

do not take the elective mathematics courses and effectively

screen themselves out of many potential careers.

Another factor in students dropping mathematics is their

lack of confidence in their ability to be successful in doing

mathematics. Traditionally, mathematics has been seen as

a male domain. In the United States it is socially acceptable,

especially for girls, not to be good in math. Unless

the student feels competent and confident in doing mathematics

she or he will not continue on when the courses

become optional.

Students need support from their teachers, counselors,

parents, and peers if they are to continue on in their mathematics

education. Intervention programs that develop students’

awareness of the importance of math to their future

work, increase their confidence and competence in doing

mathematics, and encourage their persistence in mathematics

have the best chance of success. Thus, EQUALS

uses these strands - awareness, confidence and competence,

and encouragement in its programs.

EQUALS is a mathematics inservice program at the

Lawrence Hall of Science, University of California,

B e r k e l e y, serving teachers, counselors, administrators,

and others concerned with keeping women and minority

students in mathematics education. It focuses on methods

and materials for the kindergarten through twelfth grade

level that will help attract and retain underrepresented students

in mathematics. Since 1977, 10,000 educators have

participated in EQUALS programs. Six national sites have

been created to disseminate the program throughout the

country.

The EQUALS program includes multiple approaches to

the issues of access and retention. At first, EQUALS

mathematics instructors—all former public school teachers—

set out to sharpen the participants’ awareness that

disproportionate numbers of women and minority students

decide not to continue on in mathematics in high school

and are thus unprepared to enroll in vocational or college

programs requiring quantitative skills. To develop a commitment

to recruiting and retaining students in mathematics,

participants must have an investment in the issues. So

we ask participants to investigate specific conditions indicating

their schools’ performance in bringing women and

minorities into mathematics (such as comparing math course

enrolments of males and females or surveying students’

career aspirations). The participants then become the

experts on the situation in their schools. They begin to define

the problem and are ready to work on solutions with

others.

Secondly, EQUALS provides teachers with learning materials

and methods that engage them in doing mathematics

with competence and enjoyment. EQUALS participants

encounter logic, probability and statistics, estimation, geometry,

and nonroutine applications of arithmetic for problem

solving. These are areas of mathematics that are

relevant for mathbased occupations and in which women

and minority students tend to lag behind males and majority

students in achievement tests. The learning environment

must be one that is cooperative and non-threatening.

EQUALS models the kinds of teaching approaches we

hope to encourage in the classroom, such as providing

time for people to work together on math problems and

other new materials; minimizing the fear of failure and

encouraging risk taking; providing manipulative materials

to use in making abstract math concepts concrete; and

helping people develop a range of problem-solving strategies

that suit their style of teaching and learning.

Recognizing that a number of people must have a stake

in such a program of change, the EQUALS approach has

been to build, where possible, coalitions of administrators,

teachers, and parents who will work cooperatively to

spread EQUALS through their schools. EQUALS participants

are strongly urged to teach fellow teachers and

parents the math activities they’ve learned in the program,

as well as some of the startling statistics about women’s

and minorities’ disadvantage in employment and earnings.

They bring to their classrooms or schools women and

minority men who work in math-based professions or

skilled trades. These people serve as role models and

encourage students to think about their futures in terms of

necessary and realistic work.

These activities help teachers convince themselves and

co-workers at school of the importance of EQUALS goals

and generate support for the often difficult task of inserting

EQUALS into a text-and-test dominated math syllabus.

The activities also reinforce the idea that EQUALS participants

are collaborators in the effort to make mathematics

meaningful and productive for students who otherwise may

be filtered out of a wide range of occupational choices.

During the year-long program, EQUALS participants keep

journals of their experiences using EQUALS in the classroom.

The journals reveal that many EQUALS participants

identify strongly with the math-avoiding students for whom

the program is designed. Again and again the program is

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experienced as a breakthrough for the teachers themselves.

A feeling of personal achievement perhaps contributes

as strongly as the practicality of the curriculum and

the vitality of the workshops to the program’s unusually

high evaluations — mean scores of 4.5 and above on a

scale of 5 in teachers’ ratings of the workshops, and findings

that at least 84% of participants apply EQUALS

immediately and continually in their classrooms. Schools

sending teachers to EQUALS report that in two or three

years they observe increased enrollments of previously

underrepresented students in advanced mathematics

classes and more favorable attitudes about mathematics

among all students. Most recent pre- and posttest data

indicate that EQUALS teachers and their students are

improving in their problem-solving skills as well.

Because of the need expressed by teachers for more

experience with computers, and its usefulness as a tool in

the mathematics curriculum, EQUALS in Computer

Technology was developed and offered for the first time

this year. Whether they have participated in a math or computer

workshop, EQUALS teachers experience astounding

growth, particularly in leadership skills, because they are

encouraged to speak up, make presentations, and deliver

ideas. Small victories are quickly acknowledged. As the

person grows, his or her commitment to the program and

the people who fostered that growth remains. As a result,

we have advocates everywhere, whose support, in turn, is

crucial to us.

What we have learned from the thousands of teachers

with whom we have worked is renewed respect for the difficult

work they do each day without the public support they

so desperately need. Many elementary and secondary

schools are alienating environments where teachers are

placed in adversary roles to students, parents, and administrators.

When they come to a program like EQUALS,

which provides a non-threatening, supportive environment

where they can take risks, make mistakes, and learn new

skills, their response is one of gratitude and enthusiasm.

What this tells us is that there is little opportunity for cooperation

and creativity in their own schools.

Our task then, as math educators, is to remember and be

sensitive to the many difficulties facing teachers as they try

to strengthen their mathematics programs and to provide

them with the respect and resources they need to accomplish

their goals. As teachers grow in their confidence and

competence in mathematics, so will their students.

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**FAMILY MATH**

Virginia Thompson

*Background*

Several years ago, we were asked by teachers in our

EQUALS inservice program to think about ways to provide

parents with ideas and materials to work with their children

in mathematics at home. Many parents had expressed

frustration in not knowing enough about their children’s

math program to help them or in not understanding the

mathematics their children were studying. In 1980, the

EQUALS program received a three-year grant from the

Fund for the Improvement of Postsecondary Education

(FIPSE) of the U. S. Department of Education to develop a

FAMILY MATH program for parents and children to learn

math activities together that would reinforce and complement

the school curriculum. Although the activities are

appropriate for all students, a major focus has been to

ensure that underrepresented students — primarily

females and minorities — are helped to increase their

enjoyment of mathematics.

The 12- to 16-hour FAMILY MATH courses provide

parents and children (kindergarten through 8th grade)

opportunities to develop problem-solving skills and build

understanding of mathematical concepts with «hands-on»

materials. Parents are given overviews of the mathematics

topics at their children’s grade level and explanations of

how these topics relate to each other. Men and women

working in math-based occupations meet with the families

to talk about how math is used in many occupations; other

activities are used to demonstrate the importance of

mathematics to future fields of study and work.

*Course Content*

Materials for each series of FAMILY MATH courses are

based on the school mathematics program for those grade

levels and reinforce fundamental concepts throughout that

curriculum. Topics include: arithmetic, geometry, probability,

statistics, measurement, patterns, relations, calculators,

computers, and logical thinking. The activities included in

this FAMILY MATH sampler illustrate how math topics are

approached. A career activity is also included. In any given

class, four to six activities are presented for parents and

children to do together. They then talk about how they solved

the problems and how these activities help with school

mathematics. Families are then given these and other activities

to continue the help at home. Often, parents will bring

in new books and activities they’ve found to share with

other class members.

*Participants*

Parents learn about FAMILY MATH from their children’s

teachers or principals, at PTA or school site council meetings,

through newspaper articles or church bulletins, or

from radio announcements. Classes are offered in the

afternoon or evening at schools, churches, community centers,

community colleges, or the Lawrence Hall of Science.

*Impact of the Program*

Evaluation shows that families can and do use FAMILY

MATH activities at home and that they have become motivated

to continue their exploration of mathematics.

Teachers and principals find that FAMILY MATH creates a

positive dialogue between home and school and a way to

involve parents in their children’s education.

*The Future*

During 1983 — 84, the FAMILY MATH staff will be offering

workshops to help parents and teachers to learn how to

establish and conduct FAMILY MATH classes; the full curriculum

will be published; and a film will be made of the program

to help disseminate its philosophy and approach to

communities outside of the San Francisco Bay area.

If you would like to be on the FAMILY MATH mailing list to

receive notices of available materials and workshops, please

send your name and address

to :

Virginia Thompson and Ruth Cossey

FAMILY MATH/EQUALS

Lawrence Hall of Science

University of California

Berkeley, CA 94720

We welcome your comments and suggestions for

future FAMILY MATH activities.

*Appendix*

Growth of FAMILY MATH Classes

in San Francisco Bay Area

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Year No. of No. of No. of Total No. of

Offered Classes Sites FamiliesParticipants

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1981 -82 6 3 46 67

1982-83 11 8 136 197

1983-84 16 12 345 654

(to date)

Total 33 23 527 918

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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FAMILY MATH Trainer-of-Trainer Workshops

at Lawrence Hall of Science

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Year No. of % Educators% Parents (w/o

Offered Participants direct educational

responsibilities)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1983 115 77%

23%

1984 133 85% 15%

Total 248

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Evaluation of the courses, through observations and a follow-

up questionnaire, evidences a high level of math-related

activity undertaken by FAMILY MATH participants.

Over 90% of the 67 parents who attended classes regularly

during the first year have played math games with their

children and helped them with their math homework; over

80% have talked to their children’s teacher about their

mathematics progress. Parents have also taken actions for

themselves, including getting a math puzzle or game book

(50%); a math refresher book (27%); or taking another

math class (18%). These numbers compare favorably with

the implementation levels observed during the FAMILY

MATH sessions. It appears that the math-related activities

that are begun during the class are sustained.

In October 1983 and February 1984, we conducted two

2-day FA M I LY M ATH training sessions for interested

parents and teachers. The response to these workshops

was overwhelming: 140 applied to the October session and

164 to the February one. The logistics of handling that

many people for two days, 6 hours each day, was formidable.

Yet, because we could call on the entire EQUALS

staff of 9 mathematics educators, we were able to organize

people into groups of 30 and take them through the concepts

and activities of the 12-hour course. Evaluations of

these training sessions indicate extreme satisfaction and

high enthusiasm for the organization and conduct of the

program. Further, participants were asked how they would

use the training by means of a FAMILY MATH Planning

Sheet. From the October session, 51% indicated that they

would establish or team teach a FAMILY MATH course this

academic year and, according to our information, 22% of

them have already done so; an additional 16% have firmly

scheduled a class to begin this spring. In the February session,

53% stated they would offer a FAMILY MATH course

either over summer 1984 or during the 1984-85 academic

year. The majority of trainees who did *not* intend to conduct

classes said that they intended to use the materials in their

classrooms or at home with their children, at faculty and

school board meetings, at church, or at community meetings.

Project staff will conduct a follow-up of all trainees in

late spring 1984 to determine the extent of these dissemination

activities.

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**Mathematics for All is No Mathematics at All**

Jan de Lange Jzn.

Under the influence of Prof. Freudenthal’s Institute IOWO

(Institute for the Development of Mathematics Education)

Mathematics for ally has been a much discussed item in

the Netherlands for the last decade. Since 1971 lots of

materials were developed for primary education by the

Wiskobas department of IOWO and since 1974 Wiskovon

developed texts for secondary education.

It is not easy to characterise mathematics the way it was

developed during that period, but some of the much-used

slogans were:

Mathematics as a human activity - Everyday-life mathematics -

Mathematics in the world around you.

During the initial years it was not clear how big the influence

of IOWA was, but recent research (1984) carried out by

Rob de Jong showed that the influence of the

Wiskobas-group is very large as of this moment. As de

Jong stated:

«The lOWO-Wiskobas paradigm for math education can be characterised

as realistic which means among other things: connection with

informal strategies of children, using inspirmg contexts and aiming at

the comprehension of fundamental concepts. «

«The results of the research: Wiskobas characteristics have been

traced to a large extent and in correspondence with the original intentions

in five series of textbooks taking 35% of Dutch primary school,

and increasing.»

«Moreover: about 80% of the materials used in teacher training can

be characterised as IOWO-like.»

«Finally: when teachers are considering a new textbook, 4 out of s

take a ‘realistic’ method.»

This kind of primary mathematics for all may be illustrated

by the following examples:

*Examples*

1. The first example is meant for children of about ten

years of age.

Question: «Which camera presents the picture shown?»

2. Another one, for about the same age-group:

A map of part of the Island of Bermuda is presented:

Question: «Which drawing shows the situation as it is

seen on the Bermuda?»

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3. At a somewhat higher level are the following examples

(11— 13 years) about straight lines. A very simple problem:

«How can you place three cubes on a straight line»

is illustrated in the following way

A ship is finding its way on a river with numerous shallow

spots. To make navigation easy, a number of signs have

been placed on the border. Now you have to sail Jon the

straight line» formed by two of these signs that form a pair.

As soon as the next two signs are collinear, you change

course. This idea is also used when entering harbours.

Here we see a map of a harbour. Ll and L are lights. Ll is

much higher than L2.

The tugboat Constance and its tow are reaching the harbour

at Perry. The captain of the Constance estimates that

they will pull into port in about 15 minutes. He watches the

harbour lights very closely, especially Ll and L2.

During the last minutes he sees them like this:

Assignment: «Draw the route of the last few miles on the

worksheet.»

In a discussion it becomes apparent that the children are

capable of understanding the principle. Some of them are

even able to place themselves in the position at sea and

can translate horizontal information (the L1, L2 line) into

vertical information and make the right conclusions.

For lower secondary education quite a number of experimental

texts were developed by IOWO. Some 20 booklets,

mainly of a geometrical background, were the result of five

years of experimenting, observing and evaluating. Some of

this can be found back in one of the biggest and most

influential series of textbooks in the Netherlands. Some

continuation of the project — that had to stop when IOWO

terminated all activities in 1980 — takes place at the Foundation

of Curriculum Development, especially in the field of

global graphs.

It is interesting to note that the reactions of teachers to

IOWO-like material that is part of an established series of

books is at present more favourable than to the original

material eight years ago. Via those textbooks they sometimes

rediscover the original IOWO-material.

Most reactions are like «Mathematics can be fun» and this

seems to surprise teachers even more than students.

Some examples from the original IOWO-Wiskivon materials:

5. The closer you come, the less you can see. That is also

the problem of the lighthouse man.

The man walks towards the lighthouse. Behind the tower

rabbits are playing in the grass. At home the man tells his

children: «When I approach the light-thouse, I get closer to

the rabbits. Although they don’t run away I see less rabbits

when getting closer. Why?»

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6. Question: «Is the tower higher than the bridge or not?

Explain your answer!» Without proper preparation this is a

very difficult problem. Everybody knows the phenomenon,

but very few people are aware of what really causes it. The

designer hoped that a side view would arise more or less

spontaneously, but this was not the fact. But as soon as a

side view was suggested the pupils were able to say some

sensible things. Although these problems presented to 12-

to 13-year-olds still offer many difficulties.

7. Question: «How do you know the earth is a sphere?»

Answer: «Because when you are at the beach and a ship

is approaching the coast, you first see the upper part, and

only later the whole ship.»

Now this answer may not be completely correct, but the

next one is quite sophisticated: «When you see a picture of

the earth from a satellite, you see a circle.» The teacher:

«But then it can be a flat pie?!» «No, because wherever the

satellite flies, it always is a circle .»

8. Ratio and proportion, as well as the introduction of

angles can be done with shadows, as indicated above. But

there are, of course, other possibilities. One of them is

experimenting with flying model paper planes.

There are numerous plans to build a successful paper

model plane within 15 minutes. This activity alone has, of

course, some interesting geometrical aspects. But the

planes can be used for further experiments. It is necessary

that they fly reasonably, that means more or less in a

straight line.

Interesting is to compare this performance of the planes.

This can be done by observing how far each plane flies in

relation to the height it was thrown from.

It is obvious that for each student the height *h* will be different

(and more or less the same for one student) and that

the distance flown will vary. Let’s compare two planes:

*Plane 1 : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

*h* 90 90 90 90 90

*\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

*d* 450 400 360 500 480

*\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

*Plane 2: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

*h* 120 120 120 120 120

*\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

*d* 600 550 620 550 580

*\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

It looks like some more experiments with plane 1 are

necessary to make conclusions about *the* distance flown.

Plane 2 behaves very decent. One might say it flies

around 580 cm, when launched at 120 cm.

Some additional flights with plane I make it fair to say that

plane 1 flies 480 cm when launched at 90cm.

The question arises: «Which plane is the best?»

This leads right into numerous aspects of ratio, proportions,

fractions, angles and percentages.

A rather simple way of solving the problem in a geometrical

way is by making scale drawings, cutting them out, placing

them on to each other and comparing the «glide

angles».

The smallest angle gives the best plane! Why?

Finally, some remarks on mathematics for all at upper

secondary level. Since 1981 experiments were carried out

that will lead to a completely new curriculum for mathematics

from August 1985 on. Applications and modelling play

a vital role in this curriculum for Math A, aiming at students

who will need mathematics as a tool. Math B is for students

heading for studies in exact sciences.

Math A seems to be very successful: Applications and

useful mathematics starting in reality seem a fruitful

approach even for students who are poor at «traditional»

mathematics.

More than 90% of the students at present choose mathematics

at upper secondary level which is up from 70% in

previous years.

During the experiment 20 booklets were developed for

use by the students.

Many of the ideas from these books are to be found in the

well-known textbook series in the Netherlands: once more

the influence of OW and OC (the research group that can

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be considered being the successor of IOWO) proved to be

considerable.

As a matter of fact, the experiments with Math A were so

successful that the vice minister of education considered

making math compulsory for upper secondary level.

Some examples of the Math A program are as follows:

*Examples*

One of the subjects that is part of the Math A program is

*periodic functions.* And as a special case: *goniometric*

*functions.* The latter is not a very popular subject in most

courses as we all know. From the experiments we get the

impression that embedding goniometric functions in the

periodic functions, and in real-life situations, makes the

subject much more motivating to the students.

9. The book starts with the electrocardiogram (ECG). An

ECG, taken at 16 different spots on the body looks like this:

From this, one can take the average (over the whole body),

and finally make a mathematical model that look like this :

In an earlier version, it was not explained what exactly caused

the different peaks in the graph. When students asked

questions about the heart functioning, (math) teachers

were unable to explain. So now we explain the relation between

the pumping of the heart and the ECG.

This first periodic phenomenon offers ample possibilities

for further questioning. For instance:

This is a part — one period — of an ECG of someone suffering

from a heart attack. The P-top is identified.

Explain in what way this ECG differs from the ECG of a

healthy person.

10. An interesting phenomenon that is worth mentioning

within the framework of periodic functions is the biological

clock.

The fiddler crab is very active during low tide, and rests

during high tide.

When the crab is taken away from the beach, the graph

changes dramatically:

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In this case, we say the periodic activity of the crab is

caused by an external biological clock.

The same crab has another periodic phenomenon:

It is coloured darker at daytime and lighter at night. Again,

when taken away from its natural environment and placed

in a situation with constant light, the periodic colouring

remains.

The fiddler crab has an internal biological clock as well.

11. The prey-predator model usually is not found in curricula

for secondary education. Certainly not in mathematics.

We tried to introduce this model in a very simple way.

From a story the students have to draw a graph representing

the growth cycle of the predator (lynx) and prey (rabbit)

as well.

From:

and finally, because of the periodicity to:

This — not so realistic — story and the graphs are analysed

and confronted with real-life graphs, which look pretty

much the same and have the same characteristics.

Also questions are posed about a very simple model:

Nl (t) = 200 sin t + 400

N2(t) = 300 sin (t -2 :5p) + 500

Finally, the children are given another way of drawing

graphs of prey-predator models, which emphasises the

periodicity of the model:

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Very important within Math A is the activity mathematising

and modelling. This is a complex and difficult matter and

offers lots of discussion.

13. The yearly average tide graph of a coastal town in the

Netherlands (Vlissingen) is indicated in this graph:

Assignment : «Find a simple (goniometric) model to describe

the tidal movement.»

Initially, three rather different models were found by the students

(17 years of age):

f(x) = 2 sin 1/2x

g(x) = 190 sin 1/2x + 8

h(x) = 190 sin pi /6.2

Of course, a lively discussion was the result :

f(x), that was clear was a very rough model : the amplitude

was «more or less equal to two meters» and the period

was 4p or 12.56 which is not «far away» from 12

hours and 25 minutes.

g(x), as the girl explained, was better in respect to the

amplitude: The amplitude of 190 cm, together with a

vertical translation of 8 cm gave exactly the proper

high and low tides which was very relevant to her.

h(x), was more precise about the period. This boy considered

the period more relevant «because you have to

know when it is high tide». The period proposed by his

model was 12 hours and 24 minutes, which really is

very close.

After a long discussion it was agreed that:

k(x) = 190 sin pi/6.2 + 8

was a nice model although some students still wanted to

make the period more precise.

Many people think that this kind of mathematics is no

mathematics at all. When Math A was introduced some teachers

asked if they really had to teach this. After working

for a year or two with Math A most teachers change their

minds: Math A is full of mathematical activities of a very

high level. On the other hand, we have to consider that the

ultimate mathematics for all is no mathematics — as a

separate discipline — at all: It could be possible that especially

this kind of mathematics disappears when all other

disciplines that use mathematics teach their part of mathematics

integrated in the discipline involved.

And so daily-life mathematics for all may disappear in

daily-life sciences or general education.

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**Organising Ideas in the Focus of**

**Mathematics for All**

Roland J. K. Stowasser To Hans Freudenthal

on his 80th Birthday

*Summary*

The history of mathematics offers outstanding examples of

simple, and at the same time, powerful *ideas which orga -*

*nise their surroundings,* ideas connected with each other in

a transparent network. Rather Pascal’s «esprit de finesse»

rules the process of thinking, and to some extent, by analogy

learning, too.

An impressive example shows that even less able students

can take profit by such an organisation of mathematical

knowledge. There is more hope that a new quality of

mathematics teaching might result from the epistemological

and historical point of view rather than from the currently

flourishing empirical research, categorising and doctoring

merely the symptoms.

*About Organising Ideas*

There is a lot of lip-service and well-intended general advice

for the use of history in math teaching, but very few worked

out examples of the kind I would like to talk about.

In the history of mathematics, I was looking out for *ideas*

- influential in the development of mathematics;

- simple and useful, even powerful which at the same time

could act as

- «centers of gravity» within the curriculum;

- knots in cognitive networks.

In that sense I call them *organising ideas.*

In the course of history new central ideas developed by

reorganisation of the old stocks of knowledge allowing to

draw a better general map from those «higher points of

view».

«La vue synoptique» brings to light associations hitherto

hidden.

This does not work by «longues chaînes de raison»

(Descartes). Not Euclid’s «I’esprit de géométries» rules the

process of cognition and by analogy the process of learning,

but rather *a mode of thinking related to Pascal’s*

*«esprit de finesse» paving short ways from a few central*

*points to many stations.* \*

Pascal himself provides a splendid example. When 16

years old, he reorganised the knowledge about conics handed

over by Desargues unveiling the «mysterium hexagrammicum

», ever since called Pascal’s theorem: the high

point surrounded by a lot of close corollaries.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

«(...) il faut tout d’un coup voir la chose d’un seul regard, et non par

progrès du raisonnement, au moins jusqu’à un certain degré (...)

(Pascal, Pensées, ed. Lafuma 512 : the difference between l’esprit

de géométrie and l’esprit de finesse.)

Having cut organising ideas out of the historical context

one is left with the hard task to process them into more or

less comprehensive teaching modules (or even schoolbook

chapters). The products can be problem fields in

which very few organising ideas instead of dozens of theorems

operate as a means of problem solving.

Concentrating on a few simple, and at the same time,

powerful ideas which organise their surroundings and

which are connected to one another within a simple network,

offers help for the less able student, too. His inability

derives to a large extent from the fact that he is unable to

organise his thought with respect to a complex field in

which the connections are presented in the usual plain logical

systematical way and where teaching is used to administer

only spoonfuls of the subject matter, disconnected

and without depth.

*An Example on the Idea of Congruence*

In German schools, pupils have to learn and apply some

special divisibility rules, the end-digit-rules for 5, 25, 2, 4, 8

and the digit-sum-rule for 9 (not more!).

The organising idea behind the different looking types lies

hidden away as it was before Pascal’s paper about a generalised

digit-sum-rule (see appendix to [1]). He wrote the

paper in a mathematician’s fashion: describing the algorithm

by some simple examples (9-, 7-rule) and giving a

proof by recursion. His approach is not an appropriate proposal

for an interesting lesson on divisibility rules for 11

-year-olds.

Take my approach with the very familiar clock on the face

of which, so to say, Pascal’s idea, and even the more general

idea of congruence comes to light in a very simple way.

I quote from [1].

«In the 11-year-olds’ daily life experience the clock is just the right

thing to start with. I have a big cardboard clock without a minute

hand. The hour hand is on the 12.

A pupil is called to the blackboard. He is told to write down

the number of hours the hour hand should move. He writes

down an unpronounceable number of hours which goes

from left to right across the blackboard. Three seconds

after the last written figure I put the hour hand on the right

hour. For example, the pupil writes:

2045010223053123456789024681357902541403.

I put the hour hand on the 7. The pupils check by dividing

through by 12. One page is filled. Plenty of mistakes. In the

right hand corner is written the only interesting thing: the

remainder.

The pupils know that I am not a magician, especially that I

am not good at mental arithmetic. Of course I do not reveal

the trick. The pupils will work for it, discover it. A prepared

work sheet asking «what time is it», that means asking for

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the remainders regarding 12, shows a pattern: the remainders

of 12 (Zwölferreste) divided into the powers of 10

(Zehnerpotenzen) from 102 on, are all equal, thank God.

R12 (l0n) is constant for n 2.

Every 10th power pushes the hour hand 4 hours ahead.

I assume, otherwise it has to be dealt with, the pupils really

know what the abacus is, that they can see a decimal

number consisting of powers of 10.

Now my mystery trick in arithmetic is solved. No matter

how many digits there are in front, the hour hand simply

jumps to and fro among three positions (beyond the tens).

I prefer to do the rotation of the hand mentally instead of on the

actual clock. In the example R12 (2106437822) my mental arithmetic

looks like this:

The 22 hours at the end, being out of the routine, put the

hour hand in the final position:

The 11-year-olds even understand my enquiry whether the calculations

on the working sheet confirm that R(10000000000) = 4. The

reason why - hidden reasoning by induction because of the recursively

defined powers of 10 - can be found by 11-year-olds with a little

help.

Four hours remain from 102 hours after taking away the half days.

From 103 =10.102 hours remain 10 • 4 hours. From 103 hours remain

therefore again 4 hours after taking away the half days. We proceed

in the same way for 104 = 10.103(...)»

So far, the 11 -year-olds have discovered and fully understood

the quick method for remainders by watching the

familiar (Babylonian) clock. There will be no real difficulties

to transform this method so that it can be applied to arbitrary

non-Babylonian divisions of the day (e.g. 11-, 18-,

37-hour-clocks). Pascal’s idea has been grasped from the

paradigm (12-hour-clock) and transferred, not abstracted

from a lot of examples. As a teacher, I couldn’t but follow

Aristoteles’ advice for teaching, as opposed to research, to

use but one example, *the* paradigm, which embodies the

idea.

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prospective teachers from all over France, foreseen to form the

staff of local teacher colleges, presents a very simple but universal

method to solve geometrical construction problems. The infinite

halving procedure, similar to the metric system, used already

by Stevin to solve engineering problems, became later with

Bolzano a very simple proof method. One and the same proof

scheme works and can be successfully applied by even high

school students to all the fundamental theorems about real

sequences and continuous or differentiable functions (theorems

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a good chance for general practice and preparation of the harder

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**CSMP: Realization of a**

**Mathematics Program for All**

Allan Podbelsek

**Introduction**

*Focus and Direction from Damerow ‘s Paper*

In his paper, «Mathematics for All,»\* Damerow raises an

interesting dichotomy relative to mathematics education —

do we continue with a mathematics curriculum designed

essentially for a small, elite group or do we develop a program

designed to bring more of the essence of mathematics

to all students? Stated another way - do we keep the

highly selective framework and methods of traditional

mathematics education but give up the privileged position

of the subject as part of the core of general education; *OR,*

do we seek to keep mathematics at the core of the curriculum

but find a way of teaching the subject to all students?

In spite of 25 years in mathematics education, I still personally

believe in attempting to teach as much of the essence

of mathematics as possible to the general student population.

Therefore, it is issues related to the latter choice that

this paper will address.

*Basis for this Paper*

As a student, mathematics was presented to me in the traditional

manner. Because of some effective teachers and

my own desire to see things related to one another, I developed

an interest in and an appreciation of mathematics as

a unified body of knowledge rather than a set of individual,

isolated topics. Consequently, in the past 10 years, much

energy in my work has been devoted to implement a unified

or integrated mathematics program to a subset of students

in the school system in which I am employed. It is my

hope that someday mathematics programs in the United

States will be unified in nature. Perhaps this will happen by

the turn of the century along with metrication! Through thinking

about mathematics as a unified body of knowledge

and dealing with teachers, administrators and community

members who were often resistent to such ideas, I developed

a strong interest in the very topics that Peter Damerow

addresses in his paper.

In the past five years, I have had quite a lot of involvement

in the implementation. of an innovative,

integrated program at the elementary (K — 6) level.

This program is the CSMP (Comprehensive School

Mathematics Program). CSMP was developed over a

period of several years by mathematicians and mathematics

educators from several countries. I believe

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\* Paper presented to the ICMI Symposium at the International

Congress of Mathematics, Warsaw, August 1983. Zentralblatt fur

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that one of the motivating factors of its founders was the

work of the mathematicians and mathematics educators

whose ideas were published in the 1963 «Cambridge

Report.» From initial leadership by Burt Kaufman, the federally

funded project moved from Southern Illinois University

at Carbondale, Illinois to its home at the Central Midwest

Regional Educational Laboratory in St. Louis, Missouri

(CEMREL).

When Ian Westbury called me last fall to discuss the

paper written by Peter Damerow, we discussed the

concept of Mathematics for All.» Immediately, CSMP came

to my mind as an excellent example of a program designed

in the spirit that I thought Peter Damerow had in mind. In a

later letter from Peter Damerow I was surprised to learn

that he was surprised to know that CSMP was still in existence!

*Purpose of this Paper*

Because of my belief that CSMP does represent a realization

of a mathematics program for all, I proposed this

paper. *It is the purpose of this paper to discuss/ evaluate*

*CSMP as a possible realizatiorl of a mathematics program*

*for all.* In order to accomplish this task, it seemed reasonable

to enumerate some critical attributes of a program

designed around a mathematics-for-all philosophy. In the

next section of this paper, I will elaborate on the attributes

of such a program based on my study of Peter Damerow’s

paper, and my own reading, understanding of mathematics

and experiences in mathematics program development.

In the third section of this paper I will describe CSMP and

evaluate it with respect to the criteria outlined in the second

section. I will also discuss the implementation of CSMP in

Louisville, Kentucky, as well as perceived sufficient conditions

for successful implementation of this program in

general. The third section will be followed with some

conclusions and recommendations.

**Critical Attributes of a Mathematics-for-AII Program**

What should a mathematics program for all be like? How

should it differ from many of the various, current programs?

Is it the topics and content included that must differ or, is it

the sequencing of the topics that should change? Is the

classroom structure an inhibiting factor in teaching mathematics

for all? Is it the background of the personnel asked

to teach mathematics that serves as a hindrance to teach

mathematics for all? What about the community expectations?

What about the mindset of the teachers of

mathematics — how they have been trained and what their

perceptions of mathematics are?

It would take too much space and time to describe in great

detail the attributes of a mathematics program for all.

However, I believe that using some broad constructs, it is

possible to communicate the essence of what is meant by

such a program. Below is a list of possible goals stated for

the purpose of analyzing CSMP:

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1. Develop the capacity to understand and interpret numerical, spatial,

and logical situations which occur in the world in which one

lives

2. Develop a scientific, questioning, and analytic attitude toward

mathematical problems

3. Develop mathematical knowledge, skills, and understandings relevant

to one’s personal and vocational needs to include:

a. problem solving

b. application of mathematics to everyday situations

c. estimation and approximation

d. using mathematics to predict

e. reading, interpreting, and constructing tables, charts

and graphs

f. computer literacy

g. understanding and application of basic operations

4. Develop an awareness and appreciation of what is mathematics

by recognising and using the following features of the subject:

a. content dealt with in mathematics

b. types of thinking used within the discipline

c. methods of proof

d. orderliness of mathematics

e. beauty of patterns and structures of mathematics

f. power of mathematical processes, patterns, and structures

g. interaction of mathematics with other areas of human activity

h. every spiraling development of mathematics through the history

of pcople

i. balance between inductive and deductive reasoning

Most of the above stated goals are self-explanatory. For

any program which proposes to teach mathematics for all,

it is important to examine how the program would teacher

present each of the goals or Subgoals included above.

**Evaluation of CSMP as a Mathematics-for-AII**

**Program**

*Brief Description of CSMP*

In its recently published report on CSMP, entitled

«Conclusions and Recommendations of the Evaluation

Review Panel,» the Evaluation Review Panel be

gins with a rather concise statement of what CSMP is

all about.

The Comprehensive School Mathematics Program (CSMP) is a dramatic

curricular innovation in elementary school mathematics. During

its development, conscious decisions were made about how mathematics

should be taught. The most important of these were the follo -

wing:

- Mathematically important ideas should be introduced to children

early and often, in ways that are appropriate to their interests and

level of sophistication. The concepts (but not the terminology) of

set, relation and function should have pre-eminent place in the

curriculum. Certain content areas, such as probability, combinatorics,

and geometry should be introduced into the curriculum in a

practical, integrated manner.

- The development of rich problem-solving activities should have a

prominent place in the curriculum. These activities should generate

topics, guide the sequencing of content, and provide the

vehicle for the development of computation skills.

- The curriculum should be organized into a spiral form which would

combine brief exposures to a topic (separated by several days

before the topic appears again) with a thorough integration of

topics from day to day.

- Whole group lessons should occupy a larger and more important

role in mathematics class and teachers should be provided with

highly detailed lesson plans which lay out both the content and

pedagogical development of lessons. Furthermore, training in

both the content and pedagogy of the program should be made

available to the teachers.

These beliefs about the teaching of mathematics were translated

with remarkable integrity into the eventual curriculum materials.

CSMP is a model of one very distinctive way of teaching mathematics

and is one of the few that can be studied in detail by mathematics

education researchers and teachers. Its implementation and evaluation

in schools is, in a sense, an experimental test of these distinctive

features.

In this K— 6 program, the objects of mathematical study

are: numbers of various kinds, operations on and relationships

between numbers, geometrical figures and their properties,

relations and functions, and operations on functions.

Growth in the ability to reason is seen to play an

important role in the study of mathematics. CSMP developers

argue that in mathematics the development of the art

of reasoning goes hand in hand with the growth of imagination,

ingenuity and intuition. In this perspective, elementary

arithmetic takes on the form of «adventures in the

world of numbers. « Individual numbers can assume a personality

of their own in this world. Teaching arithmetic shifts

its emphasis from an obstacle race for mastery of basic

skills to the stimulation of exploring the world of numbers.

Skills such as balancing a checkbook have their place in

real life but are of little interest to a fifth or sixth grader and

are not realistic activities for goals of an elementary school

mathematics program. More genuinely rewarding are the

stretching of the powers of imagination and the challenging

of the mind.

CSMP believes that its program requires a special pedagogy

which they call «a pedagogy of situations.» A pedagogy

of situations is described as Zone which is based on

the belief that learning occurs in reaction to the experience

of confronting a situation (real, simulated, or imagined) that

is rich in consequences, is worthy of confrontation, and has

genuine intellectual «content.» As such, a pedagogy of

situations has the following properties:

1. involves children at a personal level,

2. presents an intellectual challenge that is accessible to a broad

range of abilities,

3. provides opportunities for creativity,

4. supports experiences that have mathematical content.

The CSMP curriculum presents a large number of varied,

yet interrelated situations that provide the experiences out

of which mathematics grows. The philosophy of the program

is based on the idea that there is no reason why very

young children should not have the pleasure of mathematical

thinking at an elementary level and of exploring

mathematical ideas.

CSMP uses three languages to express mathematical ideas at a

young child’s level. These languages are: (1) the language of strings,

(2) the language of arrows, and (3) the language of the Papy

Minicomputer. The language of strings is used to help children think

about classification of objects. Its structure is much like that of the

Venn diagram. The language of arrows helps the children think and

describe relations between objects. Its structure underlies the

concepts of function and vector to be studied at a later stage in the

mathematical development of the child. The third language is based

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on a simple abacus and is called the Papy Minicomputer. With this

concrete model, children explore and learn number concepts such as

place value and develop computation algorithms.

Four major content strands comprise the CSMP curriculum. These

are:

The world of numbers

The languages of strings and arrows

Geometry and measurement

Probability and statistics

Attached in the Appendix is a «Summary of the

Mathematical Content of the K— 6 Curriculum.»

*Some Conclusions of the Evaluation Panel*

Below is a summary of the comments made by the evaluation panel.

1. The most important conclusion about CSMP is that it does teach

problem-solving skills better than the standard textbook curricula.

2. The original CSMP belief that merely doing computations as part

of the problem activities will develop computational skills as well

as the traditional program does is not justified by test data.

(Modest supplementation removes differences.)

3. CSMP belief that emphasizing problems in a group setting and

posing problems directly in the CSMPlanguages will develop adequate

skills in word problems is justified by test data.

4. CSMP student effects should be appreciatively larger when more

experienced teachers use the revised program. (It was found that

often the teacher did not receive sufficient training.)

5. CSMP students probably know more mathematics than the evaluation

results indicate. (Tests given do not measure all of the

mathematics learned.)

6. CSMP has positive effects on students at all ability levels. (This is

important in a mathematics-for-all situation.)

7. The spiral feature of CSMP may be one of the most widely applicable

of all features of the program. (More research is needed to

determine how the mechanics of the spiral curriculum affect student

learning at different points in time.)

To embark upon the implementation of a program as innovative

as CSMP is a complex and difficult task. In the

United States, conditions are not usually conducive to easily

making the kind of changes in teaching mathematics

required by CSMP. The Evaluation Review Panel for CSMP

summarizes these conditions succinctly as follows:

The status quo of mathematics education makes curricular innovation

almost impossible. Content and sequencing of topics have

always been heavily influenced by the very traditional, computationally

oriented view of mathematics held by many school administrators,

principals, and teachers. Recent increased use of commercial

standardized tests, and state and locally mandated competency

tests, together with public dissemination of the results of these tests,

have narrowed the traditional focus further so that, to a large extent,

these tests effectively control the curriculum. (...)

This accountability movement has placed increased pressure on teachers

to have students achieve these goals, even to the exclusion of

other less well measured goals such as problem solving, or less well

understood content such as probability. In the future, successful curricular

innovations are likely to be limited to those which can provide

advance proof of those positive student effects which are valued by

the public as represented by school boards and administrators.

*Analysis of CSMP Using Criteria for Mathematics-for-AII*

*Program*

In the second portion of this paper (page 4), several goals

that attempt to characterize the essence of the content of a

mathematics program-for-all were listed. In this section,

CSMP is analyzed relative to these goals. Each of the

goals is restated below with comments relative to the

CSMP status with respect to the stated goal.

1. Develop the capacity to *understand* and *interpret* numerical, spatial,

and logical situations which occur in the world in which one lives.

CSMP: CSMP is rich in situations where students must develop this

capacity. However, the program’s authors are not so concerned that

these situations be in the «real» world from the adult point of view.

Their goal is to create situations for this goal which are in the «real»

world of the learner which depends on the age and experiences of

the learner. CSMP does this very well.

2. Develop a scientific, questioning, and analytic attitude towards

mathematical problems.

CSMP: The variety of pedagogical situations through which the

content of the program is developed does an excellent job of focusing

on this goal. The detailed dialogue provided for the teacher often

follows a discovery approach in which the students are pushed or

guided to question, analyze, predict or conjecture.

3. Develop mathematical knowledge, skills, and understandings relevant

to one’s personal and vocational needs includmg:

- problem solving,

- applications to everyday situations,

- estimation and approximation,

- using mathematics to predict,

- reading, interpreting, and constructing tables and graphs,

- computer literacy,

- understanding and applying basic operations.

CSMP: CSMP emphasizes problem solving, estimation and approximation,

using mathematics to predict, charts and graphs, and

understanding and application of basic operations. Again it takes a

different point of view relative to applications to everyday situations

because everyday situations are seen from the perspective of the

learner. Computer literacy is not included in the written program.

Emphasis on understanding the basic operations is strong. Less

emphasis on mastery of pencil/paper algorithms is predominant in

the philosophy of the program. It is worth noting, however, that great

stress is placed on mental arithmetic.

4. Develop an awareness and appreciation of what is mathematics

by recognizing and using the following features of the subject:

- content dealt with in mathematics,

- types of thinking used within the discipline,

- methods of proof,

- orderliness of mathematics,

- beauty of patterns and structure of mathematics,

- power of mathematical processes, patterns, and structures,

- interaction of mathematics with other areas of human activity,

- spiraling development of mathematics through history,

- balance between inductive and deductive reasoning.

CSMP: CSMP presents to the learner a broad picture of what is

mathematical content. Through teacher led discussions, student activities

and written work, the types of thinking associated with mathematics

is illustrated and practiced. Through string activities intuitive

arguments are practised. However, formal proof is not dealt with in

this program which terminates at the end of grade six. The stage is

set for pursuit of proof in grades seven and above. Through explora-

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tions and problem-solving situations, the orderliness of mathematics

is frequently acted out. In CSMPproblem-solving situations, patterns

and structures are prominent. Students are frequently asked to utilize

a pattern in order to determine a function rule. In addition, the

power of mathematics is experienced frequently as students use

various structures and models to understand relationships and solve

problems. Interaction of mathematics with other areas of human activity

is seen in some of the special workbooks. Again, these activities

are often those more realistic to the world of the child. The program

is spiraling but no emphasis is made relative to history of people.

There is a nice balance between intuitive/inductive reasoning and

checking guesses in a semi-formal manner.

As is evident in the analysis, CSMP fares very well in

terms of the goals stated in a mathematics-for-all program.

The greatest lack seems to be in the areas of computation

(particularly from a drill and practice/mastery viewpoint),

computer literacy, and history of mathematics.

*Implementation of CSMP*

In the past six or eight years, CSMP has had fairly extensive

implementation in the United States with considerable

success. To successfully implement CSMP at the

local level requires a lot of coordination and special attention

to several areas. These are outlined and discussed

below.

1. *Teacher Training.* CSMP indicates that 30 hours of training is

recommended to help teachers learn about the program and how to

teach it Much time is required to bring teachers to understand and

appreciate the philosophy of CSMP.

2. *Materials.* CSMP materials are mostly consumable; therefore,

there is considerable recurring cost which makes the program more

costly to maintain than a traditional one.

3. *Community Awareness.* Because their children will he bringing

home materials so different from what is brought home in the traditional

programs, there must be a well-articulated plan to acquaint the

parents with the program.

4. *Administrative Stagy Awareness*. Local school principals and central

office staff must know and understand some of the aspects of this

program because they are in positions where they often must explain

a program to community members.

In my own school system, Jefferson County Public

Schools, Louisville, Kentucky, initial implementation began

in 1979 for second and third year students. The initial

implementation at the sixth year was in 1982. Overall, the

teachers and parents are quite pleased with the program.

As was expected, teachers were concerned about students

developing a mastery of certain pencil/paper algorithms

especially those that are tested on the achievement tests.

It is very important in our community to show improvement

on achievement test scores and a great deal of the school

system’s image in the community is determined by performance

on these tests. Therefore, some instruction related

to computation was provided to students from traditional

texts.

The school system’s elementary specialist received over

30 hours of training in St. Louis before assuming responsibility

for training teachers for grades one through five. In

1982, I spent three days in St. Louis preparing to train the

sixth grade teachers.

Early months of implementation for teachers of grades

one through five were hampered by the fact that the teachers

had at most two days of training. A few teachers

received no training. Getting them out of the traditional textbook

was not an easy task. Because of these problems,

special care was taken to provide sixth grade teachers with

more training. Thus, I provided 60 hours of intensive training

to nearly all sixth grade teachers involved in teaching

the program. These teachers were much more confident in

their first year of implementation than were the elementary

teachers.

It was very helpful to have an expert from CSMP come to

our system to provide information to a large group of community

members. Generally, as parents learn and see what

the program can do, they are very supportive .

Getting and maintaining materials and supplies has been

a challenge in our system. The annual cost of consumable

materials threatens the continued use of the program in our

school system.

Recently, a traditional textbook which utilizes some of the

philosophy of CSMP was chosen as a supplement to the

CSMP materials. Perhaps in another five or ten years,

other traditional texts will incorporate some of the CSMP

philosophy.

*Sufficient Conditions for lmplementation of CSMP*

Based on my experiences over the past eight years, the

four areas cited above under Implementation of CSMP»

are absolutely essential. The Evaluation Review Panel for

CSMP strongly supports this claim. In addition, they indicate

that the role of the local coordinator in implementing

and managing the program in school districts is vital to the

success of CSMP. Without a skilled and influential person

at the helm, a solid implementation of CSMP is almost

impossible. Someone has to interpret the program and

help teachers, administrators, and parents realize the need

for and importance of the program.

***Summary***

*Some Conclusions*

The purpose of this paper was to provide evidence that

CSMP is a realization of a mathematics program for all

based on implications of the Damerow paper. According to

the criteria developed in this paper to characterise a

mathematics program for all, CSMP rates very high. The

program is effective with all ability groups and it does much

to foster problem solving through mathematizations.

Shortcomings seem to be in the areas of computer literacy

and history. In addition, CSMP places less emphasis on

computation with paper and pencil than is required by

school systems in order for students to perform well on the

achievement tests.

On the other hand, CSMP is compatible with some recent

trends in mathematics and mathematics education. Some

of these trends are listed below:

1. increased emphasis on problem solving,

2. increased mathematics requirements for highschool

graduation,

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3. need to provide teachers with more mathematics

training,

4. increased use of computers in schools,

5. increased interest in discrete mathematics and algorithmic

thinking in mathematics.

Some Recommendations

CSMP has much to offer as a mathematics program

for all. However, it must be scrutinized and updated

in several ways. Suggestions of areas in which change

should be considered are listed below.

1. The use of computers must be brought into the

program

2. Logo should be used in the text materials

3. More use of history of mathematics should be included

4. Programs should be developed at higher grade

levels to sense as an extension to the present

program which terminates at the end of sixth

grade

5. Ways should be explored to make the program

more cost effective

6. The ICME 5 Theme Group on Mathematics for All should

explore ways that CSMP and other similarIV. Geometry

programs can be supported A. Networks

7. Research should be planned to help develop CSMP. As

this theme group considers issues related to the transformation

of mathematics education from the training of

experts into an essential part of general education, I

hope that the contributions made by CSMP toward this

end will be examined and valued. I believe CSMP is

one of a few, if not the only, ex-emplification of a

mathematics program which is built Kin the spirit» of

what should be a mathematics program for all.

*Appendix*

**A Summary of the Mathematical Content in CSMP**

*Kindergarten*

I. The World of Numbers

A. Counting

1. Count dots in pictures.

2. Draw a given number of dots.

3. Find the dot picture that corresponds to a given numeral.

4. Play counting games.

B. Numeration

Recognize and write numerals for whole

numbers.

C. Order

1. Play a game in which whole numbers are located on

the number line.

2. Compare sets to determine which has more elements.

D. Addition and Subtraction

Interpret and draw dot pictures for

simple addition and subtraction stories.

II. Probability, Statistics, and Graphing Data

Collect and record data in bar graphs.

III. Problem Solving and Logical Thinking

A. Reasoning

1. Use clues to identify unknown numbers.

2. Recognize and represent the intersection of two sets.

B. Relations and Functions

Interpret and draw arrow pictures for

simple relationships.

C. Sorting and Classifying

1. Sort and classify objects such as attribute blocks and

centimeter rods.

2. Place dots for objects in Venn diagrams.

D. Patterns

Determine a rule for a sequence of objects.

IV. Geometry

A. Networks

Follow one-way roads from one point to another.

B. Taxi-Geometry

Draw taxi-paths from one point to another.

C. Measurement

1. Compare lengths of centimeter rods and

lengths of paths on a grid.

2. Use attribute blocks to compare areas.

D. Transformational Concepts

Work with mirror and «cut-out» activities

that involve reflective symmetry.

E. Euclidean Concepts

1. Recognize circles, triangles, squares, and

rectangles.

2. Do activities that involve spatial relationships and

perspective.

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**Mathematics for Translators Specialized in**

**Scientific Texts — On the Teaching of**

**Mathematics to Non-Mathematicians**

Manfred Klika

Mathematics is the unique art and

science that enables us to cope with the

complexity of economic, social and

technical problems in a rational, quantitative

way. The education and training of

students in this field is an international

concern. (Proceedings of ACME 4)

Comprehending that which can be comprehended

is a basic human right. (M.

Wagenschein)

*Introduction*

Perhaps you are surprised that I want to present a contribution

on a topic which seems to be very specific. Yet in

fact I deal directly with the main questions presented in the

paper of the Organizing Committee of this theme group [5]:

\* What kind of mathematics curriculum is adequate

to the needs of the majority, what modifications to

the curriculum are needed for special groups of learners?

In this paper, the extent to which experience gained in our

project could be transferred to the teaching of mathematics

to the majority will be discussed. Starting with specific

objectives, I will develop my argument to reflect on mathematical

education in the future.

*The Structure of the Course «Fachübersetzen «*

Because of a large amount of international co-operation,

the need for high quality translations has increased in

recent years, especially in the physical sciences and engineering.

A trend-setting degree program has been set up at

the Hochschule Hildesheim for training

«Fachübersetzer»—technical translators in specific technical

fields (at present limited to mechanical and electrical

engineering). The program is aimed at providing the students

with both a practically oriented and a theoretically

based foundation in linguistic attainments closely connected

with knowledge in technical fields.

The students should acquire the capability of seeing specific

interrelations within their future fields of activity, of working

independently, of working with problems, between disciplines,

with a scientific approach. The close integration of

linguistic and technological studies is achieved by using

the subjects covered in the technical courses of one year

as the basis for exercises in translating technical texts in

the following year of the program (see the table «Structure

of Coursework ‘Fachubersetzen’»).

In order for the students to gain the solid foundation

necessary for studying technical fields, compulsory lectures

are offered on the «fundamentals of technology»

during their first two semesters in which mathematics plays

an important role.

The program is addressed to students who have a special

inclination and ability for languages and who are also interested

in physical sciences and engineering. Employment

openings for the graduates include: translators in trade and

industry, national and international agencies, and publishing

companies, and linguists in related fields like terminology,

lexicography and documentation.

But there may be a problem in the future here. Far-reaching

changes are now also occurring in the field of linguistics

as a result of advances in electronics; computers

are being used more and more in the work of translation.

Already, future technical translators are being introduced to

the methods and problems of mechanical translation

during their course of study. In the next few years, this

component of their study will be undertaken in conjunction

with a study of computer science. Both «Fachübersetzen»

and computer science could meaningfully work together in

linguistic data processing.

Up to now research has provided some impressive

results, in particular in the translation of very specialized,

greatly standardized texts. Computers will be able to relieve

translators to an increasing extent of routine work, so

that they will be able to devote all of their time to more

demanding linguistic tasks. The linguistic and technical

demands on translators will therefore certainly increase in

the next few years. But I venture to make the following prediction:

computers will not replace translators.

*Experiences with «Fundamentals of Technology» and*

*Conclusions Relating to the Overall Aims of*

*«Mathematics for All»*

Because our lessons on technology and mathematics were

closely related, our major aims had to be these: team-work,

co-operation, and the introduction of alternative forms of

instruction.

The customary courses in university mathematics were

either not suitable or not available, so I had to construct a

new mathematics curriculum myself.

Some details about the framework:

- The mathematical qualifications (abilities, knowledge,

skills etc.) of our students are extremely varied. Tests

which have been carried out show that:

\* knowledge that can be examined is learnt only for examinations

(overemphasising the calculation component);

\* the abilities of imagination, estimation, and visualisation

are developed to a relatively low level;

\* graphs of functions and related equations of functions

are poorly associated with each other;

\* even many students who are very interested in technical

subjects lack confidence in their mathematical ability

to solve a problem in the technical field.

- It was not appropriate for me to teach pure mathematics,

for, as experience shows, linguistically

oriented students frequently have a weak mathe-

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matical background, and, correspondingly, have little

interest in (purely theoretical) questions in this field. (By

the way, the majority of our students are female. This

has given rise to very interesting new insights on the

topic of women and mathematics, e. g. [10]; our experience

up to now has not shown the existence of the attitudes

that are usually attributed to women concerning

technical things.)

The trend to modern technologies (calculators, computers)

appears to be reducing the amount of calculation;

therefore it makes no sense to perform calculations

without calculators and computers in our curriculum.

Furthermore, we cannot exclude the possibility that in

the future the relationship between linguistics and computer

science as described above may even change

attitudes towards mathematics and the learning of

mathematics.

There are a lot of important inquiries into the teaching of

mathematics and many critical papers have been written

which show clearly that the present secondary school

mathematics curriculum doesn’t achieve either its own

goals or those expected by others (e.g. [1, 6, 7]). The

remarks made in the section «Selectivity of the School

System» by the Organizing Committee of our theme group

[5] set us thinking, too.

Our own experiences with the first mathematical courses

for technical translators show the same results: the curriculum

of traditional school mathematics (that is primarily

aimed at formal education and computational skills)

doesn’t prepare the students for questions requiring comprehension

and sense, particularly those involving central

mathematical terms which are used in technology as

«hand tools.» Students have no idea how to use the disconnected

details, the significance of which they do not

understand.

A frequent question, Couldn’t we do some practical exercises

on this subject?» refers clearly to the problem touched

on above. Through a «retreat into calculation» perhaps

some students believe they come to understand the

facts. No wonder this is the case because at school they

have been led to make exactly the same assumption.

Bauer [2] shows that because only aspects such as

«memory,» «cognition,» and occasionally «production,»

are tested in school-leaving examinations, these are therefore

the very things that are expected in school mathematics.

(By the way, in our course formulae are used occasionally

and calculations made, naturally, but only to further consolidate

comprehension of formulae and concepts.)

But it would be worse to Renounce mathematics as a

substantial part of the core curriculum of general education.

»[5] The consequence of this would be that the comprehension

of mathematical interrelations would deteriorate

further, even more than appears to be the case now.

There is no reason to deny the significance of mathematics

for all in the future [6]. On the contrary, it is a very

important objective of mathematical education in the future

to raise the level of attainment to a higher degree.

How could this be done?

In a recent paper, R. Fischer [4] has argued that «one of

the functions of mathematics education within the official

educational system should be to contribute to a liberation

from mathematics.» This means that mathematics has

become independent in view of its richness and wealth of

material, which he calls its «second nature.» Men are running

the risk that mathematics will control them, and it will

be necessary for them to take steps to avoid this possible

state of affairs. «Mathematics education can fulfil this new

function mainly by emphasizing questions of sense in the

classroom and, thus, questions of the relation between

men and knowledge.»

I feel this is a very important aspect, as it improves the

mathematics curriculum for the teaching of all students.

With regard to the suggestions mentioned above and the

fact that the content of the curriculum has to be structured

in a new way, I see as particularly important the role of fundamental

ideas.

*Fundamental Ideas*

A possible answer to some of our main problems in this

theme group «mathematics for all» has to be to point out

«fundamental ideas. « The problems of general mathematical

education stem from courses which are too full and

which contain mathematical topics which are too much atomized.

Interpretations of the concept of «fundamental

ideas» are varied. My colleagues and I have looked into

the role of fundamental ideas in recent years [9].

What is my understanding of fundamental ideas?

I am going to offer you the following approach: to look at

fundamental ideas from different points of view, to clarify

why and in which way ideas are fundamental. For the purpose

of explanation and demonstration, the following complex

of questions will be used:

- Is the establishment of fundamental ideas (including the

mathematical topics) related to their position within the

mathematical context? (e.g., relation to theory, consideration

of relevant logical structure and the topic-specific

systematical hierarchy, taking into account the training

of mathematicians at a university level, «recruitment

problems» of up-and-coming university teachers.)

- How applicable, or even better, how usable are the fundamental

ideas and what is their significance in practice?

(e. g. relation to the real-life situation, to experience,

to the-description of the environment, to the demands of

school and work.)

- Which interrelations exist between fundamental ideas

and educational objectives such as mathematical

modelling, argumentation, acquisition of the ability to

use heuristic methods?

In the light of these questions, we divided the concepts

of fundamental ideas into the following aspects:

\* central ideas («guidelines») embodied in mathematical

terms and theorems which have importance within the

implied framework of a given theory by being the common

basis of numerous postulates based on that theory

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or through which a hierarchical development can be

achieved. These central ideas relate primarily to the

theoretical nature of mathematics. They have only a little

significance within our teaching of fundamentals of

technology. (Central ideas in the sense of didactics are

elementary forms of components in mathematical theories.)

\* major mathematizing models. These are mathematical

ideas (concepts, theorems, methods, etc.) which are

useful for explaining important facts of real life or which

are suitable to serne as the terminological framework for

the mathematical approach to a multiplicity of situations

outside mathematics.

\* field-specific strategies are central strategies for problem

solving, especially for establishing proofs, finding

relationships and concept formation in specific fields of

mathematics. These strategies can be characterized as

being suitable for a variety of different problems in a

field.

*Examples for major mathematizing models:* functions, differential

and difference quotients, integrals, differential and difference equations,

graphs, Cartesian and polar coordinates, systems of equations

and inequalities, vectors, matrices, events, distributions, stochastic

variables, chains, boxes, algorithms, . . .

*Examples for field -specific strategies:* approximation, linearization,

analogy between algebra and geometry, geometrization, estimation,

special algorithms (e. g. of Gaulb), analogy between plane and spatial

facts, transformation, simulation, principle of counting, testing,

special theorems (e.g. fundamental theorem of differential and integral

calculus), ...

My thesis is that major mathematizing models and

field-specific strategies are apt to structure the process of

mathematical learning with a lasting effect. They have to

involve:

- comprehensive relevance

- sense-creating significance

This is the «higher level» I talked about above before pointing

out my views concerning fundamental ideas. R.

Fischer [4] suggests that «it is not necessary and furthermore

not meaningful for the next generation to learn all

those things we have learned.» Indeed, there are a lot of

common outcomes which are taught in the present curriculum

which are not fun and (I suppose) make no sense any

more.

The majority of our future technical translators have not

continued taking mathematics in their last secondary

years, and there is only a little time to teach some mathematics

successfully (within a limit of about 50 hours!). But

there are contexts enough in order to teach mathematizing;

nearly all facts in our

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1 Original German in [4], translated by M. K.

coursework are concerned with fundamentals of technology

and technical subjects which include the teaching of

mathematizing models.2 We have seen that, apparently

because students were working in a context, they actually

became interested in mathematical concepts because they

became relevant to their studies.

There are a lot of difficulties when students have to interpret

graphs and especially when they have to use the

extensive symbolism in mathematics. Interpreting this

notation is necessary for grasping the underlying concepts;

there is no possibility of avoiding these difficulties because

notation occurs in scientific texts. In this respect, the teacher

has the task of helping over and over again, and for

the very reason that this may take a long time, it is necessary

to start at an early age. And, furthermore, it is necessary

to demand questions of sense in the mathematical

curriculum at all times and in all places.

To teach and to learn the process of concept formation is

not easy, but I am convinced that this way is better than the

one the present curriculum provides. This is my last message.

Perhaps this is a chance to help students lose their

widespread fear of mathematics and mathematical education.

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2 Examples are given in [9].

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**Part III:**

**Problems and Developments**

**in Developing Countries**

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**Having a Feel for Calculations\***

David W. Carraher, Terezinha N. Carraher, and Analucia D.

Schliemann

In studying the arithmetic of Liberian tailors Reed and Lave

(1981) proposed that there were two qualitatively different

modes of doing arithmetic. The unschooled tailors used a

«manipulation of quantities» approach — an oral,

context-based way of working with numbers — in contrast

to the «manipulation of symbols» approach employed by

their schooled counterparts. It is possible that such different

modes of doing arithmetic may be found within the

same individuals, especially if they use maths in everyday

work settings. If so, it could be instructive to look at and

compare these modes and to investigate possible relationships

between them.

The present report looks at these two ways of understanding

and doing maths. We will draw primarily upon

a study (Carraher/Carraher/Schliemann, 1982, 1985)

which we conducted among young street vendors in northeast

Brazil; but we should recognise, if only in passing,

the relevance, to our analysis, of cross-cultural studies,

particularly those of Gay and Cole (1967), Lave (forthcoming),

Lave, Murtaugh, & de La Rocha (1984), Scribner

(1984), and Saxe and Posner (1983).

The present study investigated the uses of mathematics

by young schooled vendors who use maths in their jobs in

the informal sector of the economy (Cavalcanti, 1978.) and

who belonged to social classes which characteristically fail

in grade school, often for problems in maths. The study

proposed to compare and contrast the quality of maths performance

among the same children in the market place —

the informal setting — and in a formal setting.

In the informal setting, interviewers were customers who

made purchases of fruits, vegetables, or popcorn from the

vendors. In the course of the transaction they posed questions

about real or possible purchases, such as «How

much would 6 oranges cost (at 15 cruzeiros each)?» or

«How much change will I receive if I pay for the oranges

with a 200 cruzeiro bill? « .

The vendors often worked out their calculations spontaneously

in an outloud fashion, as in the case below:

Customer: «How much is one coconut?»

Vendor (12 years old,3rd grade): «35.»

Customer: «I’d like 10. How much is that?»

Vendor: «Three will be 105, with 3 more, that will be 210

... I need 4 more ... that is ... 315 ... I think it is 350.»

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\* Support for the present research was received from the

Conselho Nacional de Desenvolvimento Cientifico e Tecnologico,

Brasilia.

In cases where the reasoning was not clear, minimal questioning

by the customer was sufficient for the vendor to

describe his steps.

In the above case, the question posed by the interviewer

may be formally represented as 35 x 10. The child’s elaborate

procedure consisted in the use of repeated chunked

additions for multiplying. The response of the child could be

formally represented in the following manner: (3 x 35) + (3

x 35) + (3 x 35) + 35 = 350, where the «chunking» is reflected

in the parentheses. Notice also that the vendor must

keep track of successive subtotals («I need four more») so

he knows when to stop.

We gave five vendors (mean age 11.2 years) who had

diverse levels of schooling (from 1 to 8 years) a total of 63

items in the market place. *They answered correctly, without*

*using paper and pencil, in 98.2% of the cases.*

Similar or formally identical problems were devised for

testing later in the child’s home under conditions which

were «formal» in the sense that testing was done with the

experimenter and child seated together at a table, paper

and pencil before them, engaged in a school-like task. In

this situation, word problems which involved calculating

with money and computation exercises (with no reference

to real objects or money) were given. *The success rates*

*were 73.4% for the word problems and 36.8% for the com -*

*putation exercises.* A Friedman 2-way ANOVA on ranks

showed the performance of the vendors to be significantly

different according to condition (p = 0.039).

No less dramatic than the quantitative differences were

the qualitative differences in performance depending upon

the condition. The following protocol is that of a 12-year-old

vendor who, in the market place shortly before, had correctly

figured out how much 4 coconuts cost at 35 cruzeiros

each.

*Interviewer* (in home situation): How much is

35 times 4?

Child writes: *Child* says:

2 «Four times five is 20;carry the

35 two (which is written above the

x4 three). Two plus three is

200 five . . . times four is 20».

(What happened is that the child added the two onto the

three before multiplying, rather than after.)

It is instructive to look at other contrasts of this sort. M,

aged 11 years, responded correctly and without any appreciable

pause when asked in the market place what 6 kilos

of watermelon would cost (at 50 cruzeiros per kilo).

Customer: «Let me see. How did you do that so fast?»

Child: «Counting one by one. Two kilos, one hundred.

Two hundred. Three hundred».

On the formal test, the child’s procedure is different:

Interviewer [reading test item aloud]: «A fisherman

caught 50 fish. The second one caught 6 times the

amount of fish the first fisherman had caught. How many

fish did the lucky fisherman catch?»

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Child [writes down 50 x 6, with 360 as the result. Then

answers]: «Thirty-six». [The examiner repeats the problem

and the child does the computation again, this time

recording 860 as the result. His oral response is 86.]

Examiner: «How did you calculate that?»

Child: «I did it like this. Six times six is thirty-six. Then I

put it there».

Examiner: « where did you put it?» [Child had not written

down the number to be carried.]

Child [pointing to the digit 5 in 50]: «That makes 86».

[Apparently adding 3 and 5 and placing this sum in the

result.]

Examiner [checking to see that the child had not forgotten

the original numbers]: How many fish did the first

fisherman catch?»

Child: «Fifty».

Another child, in the market place, is asked to give change

for a 500 cruzeiro bill on a 40 cruzeiro purchase. Before

reaching for the customer’s change he subtracts by adding

on: «Eighty, ninety, one hundred. Four hundred and twenty.

» In the formal test he must solve the problem «420 plus

80». He misaligns the 8 under the 4 and adds, getting 130

as the answer. Though the reasoning was not made explicit,

it appears that the child added the 8 to the 2, failed to

lower the zero, but carried the 1, then added the 8 again,

this time to the 4 carrying the 1. The child is once again

given the problem and proceeds mentally, getting the correct

answer.

In sum, then, we are faced with two basic facts: (1) the

performance of the vendors in the market place was substantially

superior to their performance on problems in the

formal setting; (2) the procedures were qualitatively different.

Procedures in the formal setting tended to involve

written, right-to-left computation. Procedures in the market

place were oral and used techniques which did not emerge

in the formal setting, such as chunked additions for

multiplication problems and subtraction by adding on.

Many issues were raised by these findings, questions such

as

- Are the differences in performance a matter of the

concreteness or abstractness involved?

- Is the important dimension the coral vs. written» continuum?

-How much can the results be explained on the basis of

poor teaching?

- How does school maths relate to this other knowledge of

computation displayed by the children?

Some of these issues are addressed by subsequent

research which is being reported in this Congress

(Carraher, 1984). Here we would like to address, in particular,

the issue of concreteness vs. abstractness.

It should be noted that there is nothing inherent to fruits

and vegetables which should make the calculations easier.

That is to say, there is nothing particularly mathematical

about produce. An inspection of the protocols does not give

much support to the idea that the vendors had memorised

the prices: their pacing and subtotals demonstrates that

they work out the problems as they go along. And it should

be recalled that they did the problems in their head, without

the benefit of pencil and paper for recording intermediary

steps.

Perhaps it is not so much a question of relative ease of

the market place problems as the relative difficulty of the

school problems. But one might ask: Why should arithmetic

be particularly difficult (except for the «weak», «dull», or

«deprived» child)?

An historical consideration of multiplication shows that

what schools teach today as Arithmetic is, in fact, one set

of concepts and procedures among several alternatives. In

modern Western societies, for example, we learn to multiply

by «column multiplication». Unknown to most users of

the system, column multiplication is really only one procedure

of several which have been invented throughout history.

In ancient Egypt, multiplication was performed by a

«halving and doubling» procedure (see Diagram 1). During

much of the Middle Ages in Europe counters and counting

boards were used for multiplying, as well as other computations

(Damerow/Lefevre,1981). Roman numerals were

used for recording answers but not for actually computing.

Even after the introduction of Indian or Arabic numerals

and new methods for computing, reckoning boards and

counters continued to be used for centuries and there was

resistance among many Europeans to learning to work out

numerical problems on paper. But even paper and pencil

methods varied considerably. In Venice hundreds of years

ago complicated lattices were used for multiplying (see

Diagram 2).

If we try to understand these systems today, we find them,

at least at first, awkward and strange. We begin to understand

what it means to say that numeric systems involve

arbitrary conventions for the manipulation of symbols.

When one uses a computational procedure not fully

understood one is likely to make errors through being out

of touch with what is going on. Even mathematics educators

may have an appreciation for this lack of touch when

they try to take square roots or to multiply determinants.

For most people, such procedures are not clear, and rote

memory must be relied upon. It should be recognised,

however, that some school procedures do begin to make

sense and one may begin to develop an understanding for

what were formerly strange conventions. How does this

happen? How is school mathematics related to the informal

types of maths described in the present report? What leads

to their integration? This remains an important theoretical

as well as a practical issue.

The present analysis strongly suggests that the errors

which the street vendors make when using school taught

procedures do not reflect a lack of understanding of addition,

subtraction, and multiplication but rather a difficulty

with the system of symbol manipulation conventionally

adopted in our societies for solving arithmetic problems.

Borrowing, for example, is a typical stumbling block for

maths as presently taught in schools. It should be recognised

that it is possible to subtract without borrowing, and the

vendors do subtract in this way, using regroupings to de-

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compose the problem into one with intermediate steps.

There appears to be a gulf between the rich intuitive

understanding which these vendors display and the

understanding which educators, with good reason, would

like to impart or develop. While one could argue that the

youngsters are out of touch with the formal systems of

notation and numerical operations, it could be argued that

the educational system is out of touch with its clientele.

Bridging this gap would require, it appears, a better knowledge

on the part of educators of the «spontaneous» procedures

and concepts which pupils bring into the classroom,

or perhaps, leave at the entranceway.

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Diagrarn l: The «Halving and Doubling» Method of

Multiplying

Explanation: The lesser of the multiplicands (13) is successively

halved, the result being written in the left-hand column. When there

is a remainder (always of one half), it is discarded. In the next

column, the multiplicand is successively doubled. Those members

of the column which stand opposite odd numbers in the left-hand

column are set aside and summed. 17 + 68 + 136 = 221, the correct

answer to 13 x 17.

Diagram 2: The Diagonal Lattice Method of

Multiplying 907 x 342

Explanation: All the possible multiplications, digit by digit, are made

and the results placed in the boxes. Summing proceeds from the bottom

right to the bottom left, then upwards to the top left. Each digit of

the final answer (310 194) is determined by adding the elements of

the corresponding diagonal.

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Example : 13 x 17 -> 17

6 34

3 68 -> + 68

1 136-> +136

Answer 221

**Can Mathematics Teachers Teach Proportions?**

Terezinha N. Carraher, David W. Carraher, and

Analucia D. Schliemann

When pupils learn new topics in mathematics in school —

say, ratio and proportions — one usually assumes that their

problem-solving ability has been expanded. Presumably,

they will be able to solve problems which they were previously

incapable of solving since they have at their disposal

the mathematical knowledge required for proper solution.

But how can pupils tell when this (or any) mathematical

knowledge is called for? And how do they choose which

information to plug into the mathematical routine when

there are many (irrelevant) facts available?

If mathematics is to be useful to everyone, mathematics

teachers must consider carefully issues related to the

transfer of knowledge acquired in the classroom to other

problem-solving situations. When pupils learn a general

problem-solving procedure in mathematics classes, it is

important that teachers concern themselves with ways of

turning these procedures into resources that their pupils

will, in fact, draw upon when actual problems arise.

Following the computational procedures appropriately in

the classroom in no way assures that they will be used

elsewhere when the lesson is over.

The Rule-of-Three is a simple procedure which mathematics

teachers present to pupils as a neat formal way of

solving problems involving ratio and proportions. When a is

to b as c is to *d,* one can find any unknown from the other

three values. The mathematics involved is quite straightforward.

However, the simplicity of the mathematics in

already-set-up problems may easily mislead one into treating

proportions as a topic which can be readily learned by

pupils in school. A problem must first be seen as one which

calls for a proportionality analysis before the Rule-of-Three

is considered a viable approach to its solution. Besides

general matters of transfer, cognitive development may be

another point to consider; several researchers

( P i a g e t / I n h e l d e r, 1951; Inhelder/Piaget, 1955;

Piaget/Grizel/Szeminska/Bang, 1968; Karplus/Peterson,

1970; Aguiar, 1980; Lima, 1982) have shown with different

contents that children adopt additive solutions to ratio problems

at earlier stages in development and that it is only

when the stage of formal operations is reached that proportionality

reasoning seems to appear.

In order to better understand how knowledge from mathematics

is deployed in other school-related subjects, we looked

in this study at how pupils solved three proportionality

problems from physics. We also investigated the tendency

to use the Rule-of-Three in two conditions: (1) when only

the essential information was given; and (2) when relevant

information was given along with information irrelevant for

solving the problem.

*Method*

Three problems involving proportions were presented in six

different forms each to 720 Brazilian pupils ranging in age

from 14 to 20 years and in level of schooling from 6th grade

to the last year of high school. This covered a range of six

years of schooling, with the lowest level corresponding to

the year at which proportions are taught in Brazil. Testing

was done collectively in written form and pupils were asked

to select one of six alternative answers to each problem

and justify their choice. Each pupil received a paper containing

one form of each of the three problems; order of problems

on the papers followed the Latin Square.

One problem involved judging the height of a building

from its shadow, when the height and a shadow of a pole

are known. The second involved determining the weight

necessary to balance a scale with unequal arm lengths.

The third problem type involved the size of shadows as a

function of the distance of an object from the light source

and the size of the object. Six different forms of presentation

of the three problems were designed to check the

influence of the following variations upon problem difficulty:

(1) verbal versus diagramatic presentation of the problem;

(2) specific, numeric versus general, algebraic response

form; (3) availability versus non-availability of a formula to

compute solution; and (4) applying a computation procedure

to given values versus identifying the relevant parameters

upon which the procedure should be applied. The

first two variations were orthogonal to the three ratio problems;

the last two were restricted to one or two specific

problems. In all, a total of 18 versions of the problems were

used.

*Results*

Even though the verbal presentation of the problems explicitly

mentioned that parameters were directly/inversely proportional

— which could have been used by pupils as a cue

to the choice of the Rule-of-Three — the form of presenta -

tion, verbal versus diagramatic, had no effect upon the difficulty

of the items. Further, computing with direct proportions

was consistently easier than computing with inverse

proportions; success rates varied between 30% and 46%

for the direct proportions problems and were around 12%

for inverse proportions problems. Success also varied

according to an interaction between response form - numeric

versus algebraic and problem type — direct versus

inverse proportions. Computing a response was easier

than indicating a formula when direct proportions were

involved while the reverse was true for inverse proportions.

The relative ease of inverse proportionality problems seemed

to result, however, more from a response bias in the

algebraic items than from a better understanding of the

problem.

Providing students with a formula for the solution rendered

solution significantly more likely but percentages of correct

responses still remained under 70.

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Identifying relevant parameters proved much more difficult

than computing a response for the same problem when the

necessary information had already been isolated. Pupils

were often unable to indicate the relevant parameters. This

type of error cannot usually be observed in mathematics

lessons.

The justifications provided by a sample of 220 pupils for

their answers (660 in total) were analyzed in order to identify

the distinct problem-solving routines used. The overwhelming

majority of responses was in fact not justified:

pupils either provided a linguistic account of their computations

(such as «I multiplied and then divided»), or claimed

not to know enough about the content of the problem (such

as «I have not yet studied this in physics»), or presented

rather vague justifications (e.g., «I followed the logic of the

problem»). When an explanation was clearly given, the

Rule-of-Three was observed with greater frequency (which

varied between 18.6% and 20.6% across the three problems)

than any other specific explanation. It was usually

associated with successful solution when direct proportions

were involved while the reverse was true with respect

to inverse proportions.

Several pupils (percentages varied between 7.1 and 32.2

across problems) used a functions approach to the problems

(e.g., «If the shadow of the pole is 3 times its height,

then the shadow of the building is 3 times its height, and

the building is one third the shadow»). This approach often

resulted in error because pupils used additive comparisons

between the measures (e. g., «The shadow is 6 m longer

than the pole, thus the building must be 6 m less than its

shadow»). This type of error can be related to trends observed

by Piaget and Inhelder (1951) and several others in

cognitive development. Some pupils observed still other

relations between the measures (e.g., «The shadow is the

square of the size of the pole. For the same reason, the

building is the square root of the size of its shadow»).

A scalar approach (e. g., «The shadow of the pole is one

fourth the shadow of the building. That means that the building

is 4 times the height of the pole») to the solution was

much less common (percentages varied between 3.9 and

7.1 across problems) than either a functions or a

Rule-of-Three approach; it tended to yield correct responses

with direct proportions and wrong ones with inverse

proportions. In summary, what both the functions and

the scalar approaches seem to reflect, in general, is an

attempt on the pupils’ part to relate one set of two numbers

in some way and then transfer this relationship to the

second set without much analysis of what type of relationship

may in fact hold.

*Conclusions*

Four main conclusions will be stressed here. First, teachers

seem to be somewhat successful in teaching pupils

how to use formulas to solve problems and significantly

less successful in teaching them how to use the

Rule-of-Three. The very low success rates in problems

with inverse proportions uncover the pupils’ difficulties with

this algorithm. Second, analyzing problems and rendering

them amenable to solution by using the Rule-of-Three is

even more difficult for students. This difficulty appears if

pupils must simply point out which information is crucial

and also if they must indicate an algebraic formula for solving

the problem. This result underlines the issues related

to the type of knowledge acquired by pupils in mathematics

classes and those related to transfer of training. Third,

pupils cannot be said to have truly learned proportions if

their competence is restricted to their performance in

mathematics lessons. Hart (1981), working with the daily

life problem of decreasing quantities in a recipe, found

even less indication of transfer than that which was observed

in our study. Finally, it is necessary to turn back to the

theme of this group’s work: If mathematics is to be useful

to everyone, issues related to the transfer of knowledge

from the classroom to other problem-solving situations

must receive a much more systematic treatment both by

researchers and teachers.

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**Mathematics Among Carpentry Apprentices:**

**Implications for School Teaching**

Analucia D. Schliemann

One possible source of children’s difficulties when dealing

with problem solving at school may lie in the discontinuity

between formal school methods and the natural strategies

they develop in their daily activities (see

Carraher/Carraher/Schliemann, 1985). Alternative proposals

to minimize this gap require a deeper analysis of how

problem-solving skills relate to specific experiences and

how arithmetic training contributes to an improvement in

problem-solving ability in and out of school. Scribner (in

press) has shown that, compared with students, dairy

employees show more variability and more effort-saving

strategies when solving problems related to their job activities.

Lave (in preparation, see Reed/Lave, 1979), working

with Liberian tailors, found that problem-solving procedures

are closely related to practical and school experiences:

tailors who learn arithmetic in the shop understand

the general principles of problem solving but have

difficulties with large numbers- those who have been to

school can easily deal with large numbers by means of

school-taught algorithms but more often make absurd

errors that are overlooked. These data provide evidence

for the context-specific approach (see The Laboratory of

Comparative Human Cognition, in press) and suggest that

cognitive skills are closely linked to specific experiences

and practice. However, concerning problem-solving abilities,

clearer data, such as those gathered by Scribner and

Cole (1981) on literacy, are still required. In the present

study, carried out in Recife, Brazil, data on problem solving

among a group of professional carpenters and a group of

carpentry apprentices, with different educational backgrounds,

are analysed.

The group of professional carpenters was made up of 12

adults who had had from none to five years of formal

schooling. They learned their profession while working as

assistants to the owner of the shop, in most cases their

own fathers. Their verbal reports suggest that this process

of instruction closely followed the pattern described by

Greenfield and Lave (1982) for informal education.

Naturalistic observation of the daily work of these professionals

revealed that arithmetical problem solving often

occurs when a customer brings to the carpenter a drawing

or a photo of a piece of furniture to be made. The carpenter

has then to calculate how much wood he needs to buy

and how much he will charge for the finished product. He

buys wood from large shops already cut into standard

pieces from which parts are to be cut.

The group of carpentry apprentices was composed of 18

adolescents from poor backgrounds, aged 13 to 18 years,

who attended a three-year course of instruction in carpentry.

All of them were also attending the formal school system

and had at least four years of school instruction in

mathematics. Naturalistic observation of the activities in

the carpentry school revealed that: (a) carpentry apprentices

start their practical training by performing simple tasks

such as cleaning and polishing; (b) teaching is mostly done

by demonstration with few verbal explanations that could

help to improve performance in more difficult tasks; (c)

after one year of the course apprentices begin to build

pieces of furniture; (d) instructions for building each piece

are accompanied by a drawing or a three-dimensional

model of the piece and by a list of all the parts needed,

each one specified in terms of length, width and thickness;

(e) wood is available in blocks, from which each part is cut

with the aid of powered tools; (f) only at the end of the

three-year course are apprentices trained how to make up

a list of the parts required for building a particular piece of

furniture; (g) parallel to practical training, formal classes on

language, arithmetic, geometry and drawing are regularly

offered with great emphasis laid from the outset on measurement

and how to calculate area and volume.

In this study, in order to analyse how the two groups differ

in the way they deal with a problem related to their daily

work, each of the carpenters and apprentices was asked to

find out how much wood he would need to buy if he were

to build five beds like the one shown in a drawing (see

Figure 1). They were told that they could use paper and

pencil, if they so wished. While they were trying to solve the

problem, the examiner talked to them and discussed

details of the drawing as well as the steps they followed in

order to find a solution. The sessions were tape-recorded

and were run in the shops during working hours or, for the

apprentices, in a school classroom. An observer took

notes, which were used in the analysis, together with tape

transcripts and written material produced by the subjects.

*Results*

Results were analysed in terms of arithmetical operations

performed, strategies used to perform operations, dimensions

taken into account, and final result.

Tables 1 to 4 show the answers of the first-year apprentices

compared to those in their second and third year, and

to those of the professional carpenters, in each of the

a b o v e-mentioned items. Only one apprentice did not

attempt to perform the task. Two professional carpenters

who had never been to school gave a final answer without

explaining how it was obtained. These three cases were

not included in the analysis that follows.

As shown in Table 1, more than half of the first-year

apprentices preferred to use addition even when multiplication

could have been applied as a short cut. Among

second- and third-year apprentices, multiplication was

used by 70% of the subjects and, among professionals by

90%. The correlation between the level of mastery of carpentry

(considering that professionals are at the highest

level) and the use of multiplication as opposed to addition,

although not very high (Kendall’s p= 0.37), was very significant

(z = 2.70, p = 0.0028).

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The strategies to solve addition and subtraction operations

were classified into three categories: (a) mental computation,

when the answer was immediately given without

the use of paper and pencil; (b) school algorithms, when

paper and pencil were used and the answer was found by

working initially with units, then with tens, followed by hundreds,

and so on, with carrying from one column to the

next, whenever needed; (c) mixed strategy, when mental

computation was used for the simpler operations and

school algorithms for the harder ones. Table 2 shows that

mental computation occurred more often-among professionals

and that school algorithms were preferred by apprentices:

9 out of 10 professionals used head computing in isolation

or combined with school algorithms, while only 4 out

of 17 apprentices did the same. The Fisher Exact

Probability Test shows that such distributions differ significantly

(p = 0.005).

Table 3 shows that more than half of the first-year apprentices

considered only the length of the parts in their

attempts to solve the problem. Second- and third-year

apprentices considered both length and width or, in most

cases, length, width, and thickness. Professional carpenters

always worked with the three dimensions. The correlation

between the number of dimensions considered and

the level of mastery of carpentry was very significant

(Kendall’s t = 0.59,Z = 4.13, p < 0.0001).

The final answers given to the question «How much wood

do you need to buy if you have to build five beds like the

one in the drawing?», were classified into four categories.

In the first kind of answer, where 9 apprentices were classified

(see Table 4), the dimensions considered were all

added up and a final result was inadequately given as the

number of meters, or square meters, or cubic meters

necessary to build the beds. A second sort of answer, given

by 6 apprentices, consisted of a specification of the length,

the width and, in some cases, the thickness of a huge block

of wood. The length of such a block was obtained by

adding up the length of each part of the bed, the width by

adding up the width of each part, and the thickness by

adding up the thicknesses. The third category of answers

consisted of a list of all the parts with a specification of how

many of each was needed to build one or five beds. Only 2

second-year apprentices gave such an answer. Finally, the

fourth kind of answer, where 8 professionals were classified,

consisted of the compilation of two lists, the first specifying

the parts as seen in the third category, and the

second listing the standard parts usually found in the market

from which the parts could be cut. Two professionals,

not included in Table 4, after computing the size of certain

pieces gave a final answer in terms of how much money

the five beds would cost. Correlation between the degree

of mastery of carpentry and the kind of answer, considering

that the fourth category was the best of all, was very high

(Kendall’s t = 0.85) and significant (z = 5.95, p < 0.0001)

An analysis of the relationship between the answers given

by professional carpenters and the number of years they

had been to school did not reveal any clear trend, the only

noteworthy feature being that the two subjects who declined

to explain how they arrived at a final answer were illiterate.

*Discussion*

The results obtained in this study suggest first of all that,

when faced with a problem-solving task, individuals try to

find an answer that is closely related to their daily experience:

while professional carpenters seek a list of standard

pieces to buy, apprentices try to find the measures of a

block of wood from which parts could be cut. What is most

striking in these attempts is the suitability of professional

carpenters’ strategies to find a solution when compared

with the unsuitability of the apprentices’ a p p r o a c h .

Although the apprentices had had formal teaching on how

to calculate volume, their attempts were unsuccessful and

the results obtained were absurd. However, they did not

seem to perceive the absurdity. It seems that the task was

approached by the apprentices as a school assignment

and they did not try to judge the suitability of the answers.

For the professionals it was taken as a practical assignment

and the solution sought was a feasible one. That difference

between a school approach and a practical

approach, as noted by Lave (personal communication),

seems to change the nature of the problem.

Computing strategies, although different, were equally

effective in both groups, for hardly any mistakes were

made. This is an unexpected result bearing in mind that formal

school attendance was very different between the two

groups and among the individuals in the professional

group.

Of special importance for education is the fact that, despite

receiving special teaching on how to calculate area and

volume, and how to solve formal problems involving these,

apprentices were not able to use this formal knowledge to

solve a practical problem. This fact is even more striking if

we consider that the elements of the problem were part of

their daily experience. It seems then that problem solving

at school has to be taught differently if it is to have any use

out of school. One possible suggestion arising from the

data presented here is to provide, in addition to formal teaching,

opportunities for problem solving in practical

contexts. This may improve comprehension and lead to the

discovery of new and more economical strategies and

solutions.

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Table 1: Number of Subjects in Each Sub-group

According to Operations Used While Trying to Solve the

Problem

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Sub-groups Addition Addition Total

and Multiplication

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1 st-year

Apprentices 4 3 7

2nd- and 3rd-year

Apprentices 3 7 10

Professional

Carpenters 1 9 10

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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Table 2: Number of Subjects in Each Sub-group According to Strategies

Used to Solve Arithmetical Operations

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Sub-groups Mental Mixed School Total

Computation Strategy Algorithms

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

lst-year

Apprentices 0 1 6 7

2nd - and 3rd - year

Apprentices 0 3 7 10

Professional

Carpenters 0 0 10 10

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Table 3: Number of Subjects in Each Sub -group According to Dimensions

Considered When Trying to Solve the Problem

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Sub-groups Length Length and Length, Width Total

Width and Thickness

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

lst-year

Apprentices 4 1 2 7

2nd- and 3rd-year

Apprentices 0 4 6 10

Professional

Carpenters 0 0 10 10

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Table 4: Number of Subjects in Each Sub -group According to

Kind of Final Answer Given

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Sub-group Addition of Block from List of List of

all dimensions adding up parts standard

considered each dimension parts

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

lst-year

Apprentices 6 1 0 0

2nd- and 3rd-year

Apprentices 3 5 2 0

Professional

Carpenters 0 0 0 8

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**The Relevance of Primary School Mathematics in**

**Tribal Aboriginal Communities\***

Pam Harris

**Introduction**

Up until fairly recently (i. e. within the last ten years) it has

usually been the policy of Departments of Education to

expect all schools to use the same mathematics syllabus in

basically the same way, irrespective of the cultural, linguistic,

and mathematical background of the pupils. In

Aboriginal schools in the Northern Territory this led eventually

to a fairly strong backlash from teachers who claimed

that wit is not relevant to our situation», the Bite referring to

either mathematics in general, or to the particular syllabus

they were required to follow, and specific requirements of

that syllabus.

Relevance of curriculum content has long been an issue

in Aboriginal schools and is bound to remain so for as long

as Aboriginal people lack control over the education their

children receive, and non-Aboriginal teachers come from

outside to teach in a situation they do not understand.

Some education administrators are inclined to dismiss the

issue, saying that teachers are using irrelevance as an

excuse for their own laziness and incompetence, and that

good teaching of the set syllabus without adjustments is all

that is needed for Aboriginal pupils to succeed in maths.

Whilst it is no doubt true that some teachers may too easily

give up on teaching maths, rationalising that it is «not

relevant» to their Aboriginal pupils so they Won’t do its, the

question cannot be so easily dismissed. Many experienced

and hard-working teachers have questioned the relevance

of the mathematics curriculum in Aboriginal communities in

such comments as «You do your best, but you wonder

what it’s all for», «It’s not worthwhile to teach them what

they’re not going to use», and «What’s the use of teaching

topics that are not needed?».

The question of whether the same syllabus and set

of aims for teaching primary mathematics is equally

suitable for both Aboriginal and non-A b o r i g i n a l

schools—and whether it would even be practicable to

have different guidelines—is one which is decided

separately in each state department, and it is not proposed

to discuss it here. In this paper I will look at the

broader question of whether primary mathemat-

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\* This paper is part of a larger publication entitled Te a c h i n g

Mathematics in Tribal Aboriginal Schools, which is one of four

publications in the Mathematics in Aboriginal Schools Project

series. The Mathematics in Aboriginal Schools Project was a

national research project jointly funded by the Curriculum

Development Centre in Canberra and the Northern Territory

Department of Education during 1980-81.

ics is relevant for tribal Aboriginal children living in tradition-

oriented communities.

First I will outline some of the factors which discourage

people who are teaching maths in remote Aboriginal communities

and which often lead to the protests that it is not

relevant. Then I will consider factors that make mathematics

a particularly difficult subject, not just for one group,

such as Aborigines, but for many people in any population.

That should help to put the problems frequently encountered

in Aboriginal schools into perspective. Finally I will

give reasons why maths *is* relevant in Aboriginal communities,

though the aspects which are most relevant and useful

may be different from those which are most obviously

relevant in other types of communities.

**1. The Feeling that Mathematics is Not Relevant in**

**Aboriginal Communities — How it Arises**

The feeling which some teachers have that maths — or

most of it — is not relevant in the remote Aboriginal communities

where they are teaching seems to come from

three main sources:

1. Negative expectations passed on by other people.

2. The teacher’s own observations of lack of reinforcement

of maths in the pupils’ home life.

3. The cultural and linguistic bias of teaching materials.

4. Discouragement because of difficulties teaching maths

and the pupils’ generally low level of achievement.

If these influences on the classroom teacher’s attitude can

be appreciated by mathematics advisers and those controlling

curriculum decisions, and if the teachers themselves

can understand some of the forces at work, this should

help to put the question of relevance into perspective and

enable freer communication between all levels of those

concerned with primary maths education in Aboriginal

schools.

*1.1 Negative expectations of teachers*

It has been reported previously (P*.* Harris, 1980, p. 19) how

teachers often receive negative attitudes from other people

to the extent that they go to an Aboriginal community

expecting that their pupils will not be able to do mathematics.

A stereotype very common in the wider Australian

community is that «Aborigines can only count one, two,

three, many» followed by the conclusion, self-evident to the

speaker, that therefore they cannot do maths. This stereotype

is based on a mixture of fact, ignorance, and over

-generalisation.

The fact is that most Aboriginal languages do have very

few words for cardinal number. It is common to have separate

words only for one and two, and perhaps three, and

then words that refer to a few and many, or, as it is often

colloquially said «little mob» and «big mob». However,

there is ignorance about what this really means for

Aboriginal children learning to count (which they most often

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do in English), and it is a gross over-generalisation to assume

that some lack of counting vocabulary in their own language

could be taken as evidence of a general lack of ability

to cope with all areas of mathematics. (The lack of number

words in Aboriginal languages has been briefly discussed

in P. Harris, 1980, p. 13.)

Contrary to the popular myths, one prominent linguist who

is familiar with Australian languages has argued that the

gap in number vocabulary does not indicate that counting

itself is lacking in the culture. He suggests that it is there

«in the sense that the principle of addition which underlies

the activity of exact enumeration is everywhere present»

(Hale, n. d.).

Although new teachers going to Aboriginal schools may

not consciously espouse the common myths and stereotypes,

they are often confronted with them and, without any

information to the contrary, it is not surprising that many of

the teachers take up their new appointments with low

expectations of what their pupils might achieve in mathematics,

and a feeling that there is probably not a great deal

that they personally can do to change the situation.

*1.2 Lack of reinforcement of maths in the pupils’ home life*

On arriving in an Aboriginal community, the teacher is often

greatly impressed by the difference in lifestyle and living

conditions — differences which they see have implications

for their teaching of mathematics in that place.

Living conditions vary of course, and there are some communities

where the pupils are living in conditions not too

different from the teachers’, but in many of the more distant

tradition-oriented communities with which the Mathematics

in Aboriginal Schools Project has been particularly concerned,

the teacher will find that their pupils live in makeshift

humpies, beach camps, one-roomed tin huts, or maybe in

nothing at all, just sleeping behind a windbreak. This reality

comes as a shock to some.

As the teacher begins to see the many aspects of

mathematics which do not appear to be reinforced in

the pupils’ home life—aspects which are constantly

reinforced in the home lives of most A n g l o-A u s t r a lian

children—the word «irrelevant» soon comes to

mind. It does not seem relevant, for example, to

teach young children to tell the time on a clock when

the teacher knows that very few, if any, of the pupils

have a clock in their home, their parents rarely mention

clock time, and, in fact, about the only time they

see a clock or are expected to use one is in the

classroom. Telling the time seems to be a schoolbased

activity with neither reinforcement nor usefulness

in the child’s home life,\*\* in contrast to the situ-

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\*\* Notice that these examples are of things that *seem* irrelevant to many

teachers coming into Aboriginal communities — the validity of this

conclusion will be discussed later. To follow up the question of teaching

clock time in Aboriginal schools, the reader is referred to a publication in

the Mathematics in Aboriginal Schools Project series entitled *Teaching*

*about Time in Tribal Aboriginal Communities,* Pam Harris, 1984 published

by the Northern Territory Department of Education, Darwin.

ation of most children brought up in the Western-European

tradition where the child has usually been surrounded by

clocks and had its daily activities regulated by the clock

almost since it was born, and where the parents often

consciously encourage the child or infants of lower primary

age to learn to tell the time, and assist it in its efforts,

thus reinforcing what the teacher does at school.

Many more examples could be given of areas of mathematics

where the non-Aboriginal teacher is often frustrated

in attempts to teach skills and knowledge in contexts that

will be meaningful to the child and will be used and reinforced

outside of the classroom. Fractions, for example, are

difficult to teach, and appear to be rarely used, even in

employment; and how meaningful and motivating is it to

teach measurement of mass (weighing) and measurement

of capacity through cooking activities using written recipes,

when the child never sees a recipe being followed or a

standard measuring instrument being used at home? How

does the teacher teach division in a meaningful context

when the pupils and their families customarily divide and

share in unequal portions according to kinship obligations?

These and many more such questions daily confront the

teacher and require decisions which the newcomer may

not feel qualified to make, not yet having had a chance to

think through his or her own philosophy of education in a

bicultural situation.

Apart from the lack of home reinforcement during the child’s

schooling, there is often also an absence of the motivation

which some people have to learn mathematics

because of its uses in employment. In many communities,

both teachers and pupils are well aware that school leavers

have little chance of finding a good job — or any kind of job

at all. For example, the magazine of one large Aboriginal

community, in reporting that a certain young man had started

work with the Housing Association, noted that this was

his first job since leaving school five years before, and that

out of all the young men who had left school over the past

six years, only six were employed at the moment (Junga

Yimi 2 :3).

*1.3 Cultural and linguistic bias of teaching materials*

Some teachers, sensitive to the different lifestyle, interest,

and aspirations of their Aboriginal pupils, also consider

many of the commercially available teaching aids unsuitable.

The pictures and examples given in work-books

often seem to portray very little that is a familiar part of the

Aboriginal child’s daily life. And of the hundreds of rhymes

and songs available to introduce number and counting to

preschoolers and infants, the majority seem to talk about

subjects that are rather meaningless to Aboriginal children.

While it is possible for individual teachers to overcome this

problem to some extent by making their own worksheets,

using materials in the environment, and adapting the wording

of the number rhymes, the fact that these adaptations

are necessary can be a nagging reminder that the maths

materials were not intended for Aboriginal pupils.

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The fact that all materials are presented only in the

English language seems to be another indication that

mathematics is strictly «whitefella» business, not a part of

Aboriginal culture or current lifestyle.

*1.4 Beaching difficulties and low level of pupil performance*

Whether the teacher comes with low or high expectations

of their pupils, they are often discouraged by the apparent

low level of performance of many of their pupils in mathematics,

and the reports they have heard seem to be confirmed.

The pupils often do not seem to understand or

remember certain things no matter how many times they

are taught or how clear the explanation seems to be.

Some teachers who have previously taught Englishspeaking

children may find that the methods that worked

before do not seem to work with their Aboriginal pupils, and

so may tend to think that the problem lies with the pupils,

perhaps with some difference in thinking processes which

prevents Aborigines from «catching on» to maths. Other

teachers may have tried hard to bridge the gap between

the home experiences of their Aboriginal pupils and the

background experiences that seem to be assumed in the

maths syllabus, to «make mathematics more relevant in

the home and community» as suggested in the Northern

Territory’s 1974 Infants Curriculum, but find they are fighting

an uphill battle with little support. In such circumstances

it is easy to come to a general conclusion,

consciously or unconsciously, that the children in that and

similar Aboriginal communities «can’t do maths» mainly

because «it’s irrelevant» — the mismatch between the syllabus

requirements and the community’s requirements

seems to be too great.

Before looking at the counter arguments that the study of

mathematics *is* relevant in Aboriginal communities, we

should first question the extent to which the difficulties in

teaching and learning mathematics which are often experienced

in Aboriginal schools are actually peculiar to those

schools.

**2. The Problem in Perspective**

*2.1 Mathematics is a difficult subject*

The fact is that a great many people everywhere, including

many living in sophisticated Western societies, find mathematics

much more difficult than other subjects, question its

relevance for themselves, dislike it, and feel that they

«can’t do it».

This frequent rejection of mathematics by otherwise well

educated people has been pointed out (and accepted) by

leading mathematics writers and mathematicians. The first

sentence in Skemp’s *Psychology of Learning Mathematics*

(1971) talks about «Readers for whom mathematics at

school was a collection of unintelligible rules (. . .)», and

Kline (1962) begins his *Mathematics: A Cultural Approach*

with the words «One can wisely doubt whether the study of

mathematics is worth-while (...)». And the great French

mathematician René Descartes (1596 — 1650) told how,

after doing some study in Arithmetic and Geometry, he

found the «hows» and «whys» of the subjects not sufficiently

clear, and consequently «was not surprised that many

people, even of talent and scholarship, should (...) have either given

them up as being empty and childish or, taking them to be difficult

and intricate, been deterred at the very outset from learning them (.

. .)» (from leading quotation in Kline,1953, *Mathematics in Western*

*Culture).*

There are a number of factors which make mathematics

more difficult than other subjects for both school children

and adults, and these factors apply just as much for

Anglo-Australians and other Westerners as they do for

Aborigines.

(1) Mathematics is very abstract, much more abstract than

any other subject introduced in the primary school.

(2) Mathematics is more sequential than other subjects.

(3) Mathematics learning is more teacher dependent than

other subjects — there is not so much that can be «discovered

» by the student working alone.

(4) Mathematics is often taught in a dull, uninteresting way

without any meaningful context or examples.

(5) In some areas of mathematics, especially number

work, it is possible to perform well without the understanding

that will enable the learning to be used later;

thus problems are often not detected by the teacher.

(6) There is less support for remedial work in mathematics

(e.g. compared to the facilities provided for remedial

reading).

(7) There are more teachers who lack confidence in their

own grasp of the subject and their ability to teach it than

there are in the other basic subjects. (For example, an

article in the Arithmetic Teacher, May 1981, states that

in one teacher training program in Ohio, two-thirds of the

students counted over a nine-year period have named

mathematics as their least favourite and most feared

subject.)

In addition to these, there are two more major factors

which affect the teaching and learning of mathematics in

traditional Aboriginal communities.

(8) Learning mathematics and adopting a mathematical

way of thinking is like learning and adopting a second

culture, and,

(9) when this is done in English, then the second culture

has to be learned in a second or foreign language.

These last two points need some further explanation.

*2.2 Learning Mathematics is like learning another culture*

When an immigrant child, whose family speaks for example,

only Greek at home, enters an Australian primary

school and is required to learn mathematics in English, this

is not as difficult a task as when a vernacular-speaking

Aboriginal child is required to learn mathematics in English.

The Greek child is already part of a culture which has a

rich tradition of mathematics going back for hundreds and

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thousands of years. It is part of that child’s way of life and

way of thinking, and the child’s task on entering the

English-speaking program is merely to transfer what he or

she *already knows* from his or her own language into another

language which is closely related.

The Aboriginal child’s task is different, and much more difficult.

He also has a rich cultural heritage, but it does not

include much of the Western mathematics which is taught

in primary schools. Western mathematics is a new way of

thinking, a new way of ordering the world which is in many

respects at variance with Aboriginal ways. In a sense, it is

another culture. For the Aboriginal student, learning mathematics

in English is not a case of transferring ideas from

one language to another; old ideas must be reorganised

and a whole range of new ideas must be learned and

appreciated.

*2.3 Aboriginal children often have to learn the*

*«second culture « of mathematics in a second or*

*foreign language*

Many Aboriginal children have to learn the second culture

in a second or foreign language - English. A N. T.

Department of Education linguist, who is herself bilingual in

English and French, is of the opinion that number and

mathematics are among the most difficult areas of learning

in a second language. This applies even to «those of us

who have all the conceptual knowledge at our fingertips»

(Mary Laughren, pers. comm.). Ways of expressing mathematical

ideas such as comparison are very language specific

and the differences between languages are great,

even between closely related languages such as French

and English.

If these language differences and learning difficulties are

so significant for highly educated people who have a similar

mathematical background and speak a closely related

language, how much more significant and potentially

constraining must they be for Aboriginal people whose

mathematical background is quite different and whose own

language is quite unrelated to English or any of the other

languages which have contributed to the growth of formal

mathematics, such as Greek, Hindu, and Arabic?

Having looked at some of the differences which impress

and often discourage those involved in mathematics education

in tradition-oriented Aboriginal communities, and

tried to put them into clearer perspective, we now turn to

look at the positive side — the assertion that mathematics

is relevant in Aboriginal communities.

**3. The Assertion that Mathematics is Relevant in**

**Aboriginal Schools**

*3.1 Mathematics is relevant because . . .*

Mathematics is relevant and necessary in tradition-oriented

Aboriginal communities as it is in other Australian communities,

for the following reasons —

*3.1.1 Mathematics is needed in everyday life, in*

*employment, and in the conduct of community affairs*

People tend to think that the more Aborigines move back to

their homelands and assert their right to live in an

Aboriginal way, as many are doing these days, the less

they will need or want what Western-style education provides,

including mathematics. The practical reality is exactly

the opposite. In order for an Aboriginal community to

exist independently and run its own affairs according to the

wishes of its people, there must be at least some in the

group who are fluent in English and competent in mathematics

and thus able to communicate confidently with

government officials and other white Australians in the

wider community. The greater the desire for independence,

the more urgent is the need for Aboriginal people to acquire

for themselves skills in English literacy and Western

mathematics.

(By stressing the need for skills in Western mathematics

and literacy in English in this context, I am not at all questioning

the value of bilingual/bicultural education programs

in which the early emphasis is on acquiring literacy in the

vernacular and understanding mathematics concepts

which are a part of the traditional tribal way of life. These

vernacular programs, apart from their other advantages,

provide a sound basis for improved performance when the

student must later tackle English literacy and primary

school mathematics taught through the medium of English

as a second language.)

*3.1.2 Aboriginal people have requested mathematics*

Whenever tribal Aborigines have stated what they want

from education, they have always (in my experience) included

high on the list of priorities (a) ability to speak, read,

and write English, and (b) Knowing numbers The reasons

given for wanting to «know numbers» are very practically

oriented to managing their own affairs. See, for example,

comments recorded by H. H. Penny in the report of his

investigation into the training of Pitjantjatjara teachers in

South Australia (1976, p. 18).

*3.1.3 Mathematics is necessary for secondary and*

*most tertiary education*

If Aborigines are to achieve their aims of being teachers,

doctors, mechanics, etc. then they must have a good basic

understanding of maths, and the option to choose to do it

at higher levels beyond primary schooling.

*3.1.4 Mathematics is a major clue to understanding*

*the way Anglo-Australians think (in line with*

*their Western-European cultural heritage)*

Morris Kline begins his important book *Mathematics in*

*Western Culture* thus:

«(.. .) mathematics has been a major cultural force in Western civilisation.

Almost everyone knows that mathematics services the very practical

purpose of dictating engineering design (v.) It is (...) less widely

known that mathematics has determined the direction and content of

much philosophic thought, has destroyed and rebuilt religious doctrines,

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has supplied substance and economic and political theories, has

fashioned major painting, musical, architectural, and literary styles,

has fathered our logic, and has furnished the best answers we have

to fundamental questions about the nature of man and his universes

(Kline,1953)

To understand the Anglo-Australian culture by which they

are surrounded, Aboriginal people need to have some

understanding of mathematical thinking.

*3.2 Some mathematics topics are more relevant*

*than others*

Nevertheless, although mathematics as an area of study is

relevant in both Aboriginal and non-Aboriginal societies, it

soon becomes evident to the teacher in an outback

Aboriginal community that some maths topics (like money,

for example) are much *more relevant* than others, some

which are accepted without question in other schools

appear to have *very little relevance* (e. g. fractions, division),

and yet others appear to be *relevant and motivating*

*only if they are introduced at a different stage from that*

*recommended in the syllabus* and with different emphasis

(examples are the introduction of standard units of measure

and learning how to tell the time on a clock).

The teacher soon sees a need to adjust the syllabus to

meet local requirements. This does not imply that the aims

for the endpoints to be reached by the end of primary

school should be changed, but that in Aboriginal schools

these endpoints may be more effectively reached through

a primary maths program that has different emphases, different

sequencing, and different teaching methods from

those recommended for Anglo-Australian children.

*3.3 Some mathematics topics are more useful than*

*they at first appear*

Adjustment to the syllabus are necessary to meet the

needs of differing local conditions, but newcomers especially

should be wary of making changes, particularly any

which involve not treating a topic because it seems irrelevant.

Such decisions need to be made only after careful

consideration of the future needs of the child and the relation

of that topic to other parts of the syllabus, and are best

made in consultation with an adviser, where one is available.

*3.3.1 For example, why teach fractions?*

One Northern Territory teacher, an experienced and

conscientious person, once wrote and asked the mathematics

curriculum unit in Darwin to give her some good

reasons why she should teach fractions, because she said

she could not see the use of them and there seemed to her

to be more important things on which to spend one’s teaching

time.

The questions «Why teach fractions?» and «Should we

teach fractions at all?» are often asked in Aboriginal

schools, so I will give here some reasons for teaching fractions

and these will serve as an example of the various

aspects to be considered in regard to topics which at first

seem irrelevant.

Aboriginal children, like any others, need an elementary

understanding of fractions because:

(1) Fraction terms such as «half» and «quarter» are an

integral part of everyday English speech.

(2) Decimal fractions cannot be properly understood if the

idea of fractions (equal parts) is not understood.

(3) Fractions are used in employment, for example in the

hospital (half dose of medicine for a child), when making

out time sheets (time-and-a-half pay for working after

hours), and when stock-taking in the store.

(4) Common fractions cannot be entirely replaced by decimals

— not all situations involving fractions can be

handled in decimal form.

In addition, work on equivalence of fractions and simple

addition and subtraction of fractions provides older students

with extra practice in the four operations which does

not look like the same old «sums» being dished up yet

again. That is important for slower students who have not

achieved in the four operations but are not motivated by

the methods used with younger pupils. One principal in a

large Aboriginal secondary boys’ school reported that he

had found work on fractions very helpful in increasing students’

skills in the four operations. This idea is also supported

in some teachers’ guides, see for example page 152

of *Mathematics - A Way of Thinking* by Robert

Baratta-Lorton. Here I have presented linguistic, cultural,

practical, mathematical and motivational reasons for retaining

some work on fractions. These are just some of the

aspects which will have to be considered in every question

of adjusting the content and sequencing of the mathematics

syllabus to suit a particular situation or group.

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**Bicultural Teacher Training in Mathematics**

**Education for Aboriginal Trainees from**

**Traditional Communities\***

Kathryn Crawford

**Introduction**

This paper will discuss the challenges facing an educator

in the development of a bicultural, bilingual teacher training

programme in mathematics curriculum for Aborigines from

traditional communities in Central Australia.

Although many of these challenges stem in particular

from the characteristics of the communities involved and

their particular culture, there are also many aspects of this

educational task that are paralled in any country where

efforts are made to cater for indigenous groups of people

in an education system that has been derived from

Anglo-European cultures.

The course described forms part of the Anangu Teacher

Education Programme (ANTEP) an accredited teacher

training course intended for traditionally oriented Aboriginal

people currently residing in the Anangu communities who

wish to take on greater teaching responsibilities in South

Australian Anangu schools. The course will be directed

from the South Australian C.A.E. but most teaching will be

carried out on site by a lecturer residing within the

communities. Pukatja (Ernabella) will be the host community

for the project.

The programme as a whole represents a significant

departure from conventional teacher education in a number

of ways. Perhaps the most striking difference between

this teacher training course and many others is that from

the beginning, development of the curriculum has been a

co-operative venture between lecturers and educators on

the one hand, and community leaders and prospective students

on the other.

The extent of this co-operation is indicated by

French-Kennedy’s (1984) description of the aims of the

curriculum design workshop held in April 1984:

«The general aim (...) was to bring together prospective ANTEP students;

interested Anangu; non-Anangu with demonstrated expertise

in the area; the relevant ANTEP lecturers in charge and the on-site

lecturer for the purpose of considering, in detail, the initial offering of

units. « (p. 3)

The first group of students will commence the course in

August 1984.

The tone of the early negotiations with the A n a n g u

communities indicated that an interactionist perspective

on bicultural education and on mathematics

education and curriculum development in particular

would be most appropriate. The rationale for the design

of the two units Teaching Mathematics I and II

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\* This is a revised version of a paper presented at the 1984

Conference of the South Pacific Association of Teacher

Education.

that form the mathematical component of the course has

evolved from the urgent need to provide experiences that

will enable students to negotiate the complex interacting

factors from the known in their own culture to a competence

in the use of mathematical ideas from Anglo-European

cultures. The perceived community needs in Western

mathematics were eloquently stated by one member of the

community as follows:

«Our children need to know enough maths so they don’t get ripped

off.»

Initial discussions with community leaders and prospective

students suggested the following general aims for the course:

1. Development of student awareness of their cultural

expertise in

a) the Anangu ways of thinking about relationships and

patterns to do with the locations, qualities and quantities

of objects and people in the environment;

b) the needs of the Anangu community to

- affirm Anangu culture and Pitjantjatjara language;

- develop new strategies and mathematical knowledge

to meet the need for dealing with Anglo-Europeans

and their culture;

- explore traditional ways of teaching young children and

the modification of these methods as necessary to

accommodate new knowledge.

2. Widen student awareness of and ability to apply elementary

mathematical knowledge (S.A. Curriculum K—

8) to solve community problems.

To enable students to develop a rationale for teaching

behaviour and methods that are appropriate to the

needs of the children of the community.

**Negotiating Meanings Between Two Cultures**

Gay and Cole (1967) examine the teaching of mathematics

in a cross-cultural situation. They suggest:

«(...) in order to teach mathematics effectively, we must know more

about our students. In particular we must know about the indigenous

mathematics so that we can build effective bridges to the new mathematics

we are trying to introduce.» (p. 1)

The need to build conceptual bridges from the known to the

unknown is not of course an educational problem restricted

to the context of bicultural education. The mathematics curricula

in most primary and secondary schools are notably

dissociated from the everyday concerns of the student

population. This has been a cause for considerable

concern among mathematics educators in an increasingly

technological society. In a bicultural context the situation is

made more serious by the fact that different cultures

emphasise different conceptual schema. Thus, temporal

sequences and quantitative measurement are dominant

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themes in industrialised Western cultures but largely irrelevant

in traditional Aboriginal cultures. Scientific and technological

thought, the appropriate registers of language

and mathematics as an abstract discipline have developed

over hundreds of years within Anglo-European culture and

reflect these dominant themes particularly at the elementary

level. Experience suggests that for many Aboriginals

from traditional communities the content of the elementary

mathematics curriculum is perceived as incomprehensible

and often irrelevant.

There has been a great deal of valuable descriptive

research by J. Harris (1979), Dasen (1970), Kearins

(1976), Rudder (1983) and others about the kinds of classification

systems arid the rate and order of development of

concepts related to mathematics in differing Aboriginal cultures.

The work of Parm Harris1 provides an account of the

effect of these differing cultural perspectives on mathematics

learning. Her detailed and historical accounts of

Aboriginal attitudes and beliefs about such topics as

money, measurement and number provide a valuable

resource for teachers. Her work is important in combatting

the negative expectations expressed by teachers in such

statements as: Aboriginal children do not generalised

The clear accounts of how Aboriginal children *do* generalise

and why they find school mathematics difficult are instructive.

Her explanations of how these difficulties may be

overcome redirect the focus of the problem from the «failings

» of Aboriginals and Aboriginal culture to the inappropriateness

of many teaching practices for children from traditionally

oriented communities.

Most recently published curriculum materials such as

those devised by Western (1979), Northern Te r r i t o r y

Department of Education (1982) and Guy (1982), intended

for use by Aboriginal teacher trainees or as resources for

teachers in Aboriginal schools have acknowledged the

implications of J. Harris’s (1979) statement:

«As the child matures learning to label and order his experiences it

is inevitable that his cognitive development will be very strongly

influenced by Aboriginal systems of knowledge.» (p. 143)

Thus, these materials take care to acknowledge language

difficulties, use materials from a familiar context for illustration

and take particular care in topics such as time and

measurement to provide experiences to facilitate conceptual

development. However, a closer analysis of such

materials suggests that the teaching procedures and the

content are still culturally biased to the extent that

Aboriginal people are likely to have difficulty relating school

experiences in mathematics to community needs and problems.

In a community-based teacher training course it seems

that it is possible for the first time to develop procedures for

negotiating meanings between the two cultures. With this

in mind the lecturer’s notes at the beginning of the first

module in the course state:

«A co-operative exchange of knowledge is particularly important in

the tutorial sessions because mathematics by its

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1Personal correspondence and CDC Mathematics in Aboriginal

Schools Project Series (in press).

nature involves the use of higher order cognitive skills and problem

solving requires confidence. There is considerable research evidence

to suggest that egalitarian relationships foster these skins better

than authoritarian directions. It is important that student/teacher interaction

and role play used in the sessions provide a suitable model for

interaction with children.» (Teaching Mathematics I, Module 1)

The course has been developed based on a model designed

to maximise the possibility of interaction between

the world view expressed by Anangu culture and that of

Anglo-European culture as evidenced in school mathematics.

This is achieved by placing an emphasis on the student

expertise *and* contribution in providing information

about Anangu world views *as a necessary part of the cour -*

*se.* The course is constructed in such a way that students

are invited to participate in co-operative decision-making

about appropriate methods for negotiating meanings from

one culture to another.

Group interaction and co-operation in arriving at solutions

to the problems set by tasks in each module are an essential

component of course experiences. This is a necessary

process if an understood social concensus about the multiple

realities, perceived by Anangu and non-Anangu tutorial

group members is to be achieved. Such cognitive interaction

is the basis for the development of a synthesis of

world views and of the clearly understood and generalised

universal premises that will form the basis for future group

decision-making. The outcomes of such a synthesis are in

the changed perspectives that the participants take away

with them.

Figure I below illustrates some of the contexts in which

this approach will be used.

It is expected that in Teaching Mathematics I the mathematical

content will be that of the South A u s t r a l i a n

Department of Education Syllabus’s Early Childhood section

(Modules 1 — 10). In the second year of the course the

emphasis will shift to the mathematical content of the upper

grades of primary school. The community has expressed

the desire that teacher trainees should function as a

resource of useful mathematical knowledge within the

community. To meet this need there will be a mathematics

component in the Work Skills Unit that is also included in

the ANTEP programme.

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A system of *clustered* modules has been used in the

course so that conceptual construction links between related

topics are made as explicit as possible.

Figure 2

The experiences provided for students during the course

have been chosen so that the links between mathematical

ideas as expressed in traditional Anangu culture, the settlement

community, Anglo-European culture and the school

curriculum are emphasised.

The diagram below illustrates the first 20 modules of

Teaching Mathematics I. See appendix for a more detailed

description of the form of a particular module.

More modules are constructed to promote an exchange of

ideas between the distinctly different conceptual frameworks

of the two cultures. The diagram below shows some

of the ways in which this occurs.

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Figure 3

At all points emphasis is placed on practical experimentation

and application of ideas in the community and

the local school. The teaching staff at Ernabella school has

been most supportive of the programme and is enthusiastic

about allowing students to expand their practical experience

within the school as their skills increase.

**Communication Between Cultures**

During the development of this course it has been necessary

to pay considerable attention to the communication

difficulties of a bilingual programme. For some of the reasons

expressed in the previous section, language difficulties

are *particularly evident* in the mathematics area.

Barbara Sayers (1983) suggests that language difficulties

are a particular problem in the teaching of mathematics.

She goes on to describe her experience at the

Wik-Mungkan community at Aurukan:

«I have understood what was said in terms of understanding the linguistic

aspects of the language, but I have not understood the message

they encoded. Such messages are incomprehensible because

I did not understand the presuppositions on which they were built,

nor the Aboriginal concepts which were involved. To sum up, I could

understand what was said but not what was meant.» (p.3)

The reverse situation occurs all too often when mathematics

is taught in Aboriginal schools.

Some proficiency in English has been required for selection

to the ANTEP course. However, on-site experience at

Ernabella suggests that:

1. Bilingual presentation of materials is essential.

2. Articles and workbooks should include English and

Pitjantjatjara translations.

3. Participant responses were always in Pitjantjatjara with

the exception of one person.

4. Difficulties will be experienced in translating some

English words into Pitjantjatjara particularly in the field of

mathematics.

The course has been designed with considerable emphasis

on an activity-based process/discovery learning design

to maximise the possibility of concensus about the *mea -*

*ning* of language generated by students (in either language).

It has also seemed appropriate to maximise the superior

visual/spatial skills of the Anangu people, and the precision

of the Pitjantjatjara language in this respect, by illustrating

concepts by role play or diagram as far as possible. For

example, in Module 7 of Teaching Mathematics I, posture,

which is an effective and important means of communication

within the Anangu communities, is used extensively in

an action-based activity to convey ideas about seriation.

During the development of the course students will be

encouraged to develop techniques for using graphic displays

to convey relationships.

The Ernabella school has an Apple computer. Work has

already been done by Klich2 in converting spatial

games known to community elders into a form suitable

for presentation on a video screen. The prospective

students have already expressed much in te r -

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

2 Personal correspondence.

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est in learning to use this computer. The development of

cultural symbolism through the use of Apple Logo software

seems an extremely promising medium for the introduction

of Euclidean notions of geometry and sequenced procedural

skills.

It has become evident that there is insufficient information

available about the use of Pitjantjatjara language in relation

to mathematical concepts. In any case there has been

some suggestion from members of the community that language

development in the vernacular is somewhat depressed

among settlement children. It seemed advisable to

include in the course some action-based research for students

directed at the collection of information about the

ways in which Pitjantjatjara language is used to describe

mathematical ideas.

This seemed an important aspect of their learning because:

- On-location experience with Aboriginal teacher aids suggests

that their assistance of children even when

expressed in the vernacular consists of, often incomplete,

attempts to translate Anglo-European ideas rather

than the out-of-school modes of expression. Confusion

results.

- The procedure seems likely to provide experiences that

will heighten student awareness of mathematical ideas

and concepts within their own culture.

- Eagleson et al. (1982) suggest that Aboriginal English is

a restricted code. It is restricted, not in the sense that it

is inferior but because it has been developed as a

means of communicating ideas derived from Aboriginal

world views. The syntax of this fomm of English often

follows that of the vernacular. Without further development

and clarification it is usually not an appropriate

means of communicating mathematical ideas from

AngloEuropean cultures.

- The collection of information of this kind seems likely to

heighten the awareness of both students and educators

of points where confusions about mathematical

concepts is likely to arise.

The inclusion of these types of exercises in the course

was confirmed as a useful innovation by David Wilkins, linguist,

of Yipirinya school in Alice Springs. The teachers at

that school have found a similar approach most useful,

especially for mathematics. Prospective students were

enthusiastic about the idea since it affirms their cultural

expertise and provides opportunities for consultation with

community leaders about precise vocabulary.

The procedure as it has currently been developed

involves collecting taped information of children and adult

language usage to describe certain situations. For

example, a child may be blindfolded and directed through

a maze of carefully placed obstacles by the rest of the

group to elicit information about vocabulary and syntax

connected with location and direction. The school linguist,

on-site lecturers and students then use the collected information

as a basis for language development in the vernacular

and as a source of information about conceptual differences

between cultures. For example, Anglo-Europeans

tend to describe direction in terms of Left and Right,

many Aboriginal groups use the four points of the compass.

There does not appear to be a set policy for bilingual teaching

in the Ernabella school. In general, the linguistic

resources available and the language competence of children

in either language govern the level of verbal discourse.

In a bicultural context it is necessary to actively affirm

language registers that are appropriate for discourse about

science, technology and mathematics. This register of

English language is not normally evident in the language-

arts curricula of Australian primary schools. The teacher

training experience provided in the courses described

above, emphasises the use of small group co-operative

tasks and elaborated ideas as logical and meaningful communication.

It is hoped that these will provide students with

the necessary skills to allow for needed curriculum change

and a rationale for the appropriate use of first the vernacular

and then English as a useful medium for instruction in

mathematics.

**Conclusion**

The emphasis in the teacher training course described

above has been in the development of a rationale for

connecting the conceptual frameworks (with respect to

mathematics) of two very different cultures.

Experiences have been provided to increase student awareness

of the mathematical ideas within Anangu culture.

Strategies have been suggested and opportunities provided

for the development of a rationale for teaching procedures

that will assist children as they move from one cultural

context to another.

To this end, the course has been constructed on a process

model where information is collected and students are

encouraged to play an active role in the decision-making

about outcomes. Language development in both English

and the vernacular will be an important factor in this process.

The visual/spatial knowledge and the heightened awareness

of relationships that are characteristic outcomes of

Anangu culture will be used in teaching procedures that

emphasise these aspects of mathematics rather than

demanding prior mastery of incomprehensible algorithmic

procedures.

It is to be expected that there will be some problems to be

negotiated as the course proceeds. At this stage, however,

it seems that success, in terms of student competence as

teachers and a more appropriate learning environment for

the mathematics curriculum in Anangu schools, may well

depend on the extent to which students are enabled to actively

participate in building bridges between the two cultures

for themselves. Only then will they become truly competent

as teachers in a bicultural context.

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*Appendix*

**Module 7**

Comparison II — (comparing people)

*Lecturer’s Notes*

The purpose of this exercise is to expand direct comparison

according to one attribute to seriation strategies.

The use of people appears appropriate since limits on

behaviour and location between different members of the

community are likely to be familiar ideas.

*You will need:* large sheets of paper and felt pens.

*Student activity:* Explain to students that the following activity

is one way of *ordering* by comparison of an attribute

using personal orientation to show a *relationship.*

*Tutorial Session*

1. a) All students to compare heights in the following way.

As two people approach, the taller person turns side on

and put hand on hip, the shorter person turns side on

and puts hand in air

e.g.

b) Practice this until all possible pairs have been tried.

c) After two students have approached each other, as in

the above diagram, a third approaches. If he is shorter

than A and B he stands next to B side on with hand in

air. B places hand on hip facing C.

e.g.

If he is taller than A the following arrangement is made:

d) Students should try different patterns using the rule that

one can only have one person on either side, e. g. a person

with both hands on hips can only be approached by

two shorter people, and a person with both hands in the

air can only be approached by two taller people.

e) Change the rule so that when both hands are used one

should be up in the air and one on the hip.

e.g.

The resulting arrangement should be as above. A fifth

person approaching the group may:

- join at the tall end if he/she is the tallest;

- join at the short end if he/she is the shortest.

Find a position between two people in the line so that

he/she stands appropriately with one hand in the air and

one hand on hip.

2. Discuss the activity. Note the similarity of hand on his

shape to > symbol used in mathematics. Seek suggestions

from students for other actions or postures that

may be suitable. How else might people be compared/

ordered? Age?, kinship?, totem?, weight?

3. Devise suitable ways of recording results or ordering

activities. How can we show the relationship between

people?

4. Discuss ways of using this activity and this type of experience

when teaching children to order objects according

to attributes such as smoothness, length, weight.

How can the relationship between objects be shown?

*Practicum*

The group should devise an ordering lesson suitable for

young children (seek modification and improvements). This

activity should be carried out with a small group of children

(< 10). The results should be reported next session.

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**Mathematics for All**

**Problems of cultural selectivity**

**and unequal distribution of mathematical education**

**and future perspectives**

**on mathematics teaching for the majority**

Report and papers presented in theme group I,

‘Mathematics for All’

at the 5th International Congress on Mathematical Education,

Adelaide, August 24-29, 1984

Edited by

Peter Damerow, Mervyn E. Dunkley, Bienvenido F. Nebres and Bevan Werry

**Division of Science**

**Technical and Environmental**

**Education**

UNESCO

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**Preface**

This resource document consists of twenty-two papers prepared by authors from all regions and

presented at the Fifth International Congress on Mathematical Education (ICME 5). Over 2000

mathematics educators from sixty-nine countries gathered in Adelaide, Australia, in August 1984,

to discuss problems in their field. This document is one outcome. Its purpose is to continue the

dialogue to assist nations in their search for a mathematics programme for all students.

*Mathematics for All is* the first document in mathematics education in Unesco’s Science and

Technology Education Document Series. This, coupled with Unesco’s publications *Studies in*

*Mathematics Education* and *New Trends in Mathematics Teaching,* was initiated to encourage an

international exchange of ideas and information.

Unesco expresses its appreciation to the editors, Peter Damerow, Mervyn Dunkley, Bienvenido

Nebres and Bevan Werry for their work, to the Max Planck Institute for Human Development and

Education for preparing the manuscript, and to the ICME 5 Programme Committee for permitting

Unesco to produce this report.

The views expressed in the text are those of the authors and not necessarily those of Unesco,

the editors, or of ICME 5.

We welcome comments on the contents of this document. Please send them to: Mathematics

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Introduction:

Report on the Work of Theme Group I

«Mathematics for All» at ICME 5

**1. Introduction**

Many factors have brought about a change in the overall

situation of mathematics education. These include the

move to universal elementary education in developing

countries, the move to universal secondary education in

industrialised countries (where there have also been growing

demands for mathematical competence in an increasingly

technologically and scientifically oriented world) and

from the experience gained with worldwide curriculum

developments such as the new mathematics movement.

The tacit assumption, that what can be gained from mathematics

can be gained equally in every culture and

independently of the character of the school institution and

the individual dispositions and the social situations of the

learner, turned out to be invalid. New and urgent questions

have been raised. Probably the most important ones are:

- What kind of mathematics curriculum is adequate to the

needs of the majority?

- What modifications to the curriculum or alternative curricula

are needed for special groups of learners?

- How should these curricula be structured?

- How could they be implemented?

A lot of work has already been done all over the world in

attempts to answer these questions or to contribute to special

aspects of the problem.

- ICME 4 yielded several presentations of results concerning

universal basic education, the relationship of

mathematics to its applications, the relation between

mathematics and language, women and mathematics,

and the problems of teaching mathematics to special

groups of students whose needs and whose situations

do not fit into the general framework of traditional

mathematics education.

-The Second International Mathematics Study of the

International Association for the Evaluation of

Educational Achievement (IEA) dealt much more than

the first one with the similarities and differences of the

mathematics curriculum in different countries, and the

different conditions which determine the overall outcome

in mathematical achievement. The IEA collected

data on both the selectivity of mathematics and the differences

between countries in the way they produce

yield levels of mathematical qualification. Although final

reports on the Second International Mathematics Study

are not yet available, preliminary analyses of the data

have already produced useful results.

- In several countries national studies have been concerned

with the evaluation of the mathematics education

system. An important recent example is the Report of

the Committee of Enquiry into the Teaching of

Mathematics in Schools in England and Wales (commonly

known as the Cockcroft Report) in 1982.

- Last, but not least, there are many detailed studies, projects

and proposals from different countries dealing with

special aspects such as:

- teaching the disadvantaged;

- teaching the talented;

- teaching mathematics to non-mathematicians;

- teaching mathematics in the context of real life situations;

- teaching mathematics under atypical conditions, etc.

At ICME 5, papers were presented on a variety of topics

related to the theme Mathematics for All. Taken as a whole,

these contribute to a better understanding of the problems

of teaching mathematics successfully, not only to very able

students, but teaching worthwhile mathematics successfully

to all in a range of diverse cultures and circumstances.

**2. Summary of Papers Presented to the Theme Group**

The first group of papers dealt with general aspects of the

theme Mathematics for All.

Jean-Claude Martin, Rector of the Academy of Bordeaux

in France, analysed in his paper, A *Necessary Renewal of*

*Mathematics Education,* the special selectivity of mathematical

education as a result of symbolism and mathematical

language. The teaching of mathematics seems to have

been designed to produce future mathematicians despite

the fact that only a very small percentage of students reach

university level. This general character of mathematical

education causes avoidable, system-related failures in

mathematical learning and often results in a strong aversion

to mathematics. Martin proposed a general reorientation

of mathematical education aiming at a mathematics

which is a useful tool for the majority of students. The teaching

of mathematics as a means of solving multidisciplinary

problems by using modelling methods should restore

student interest, show mathematics as being useful, enrich

students knowledge of related subjects and so enable

them better to memorise mathematical formulas and

methods, encourage logical reasoning and allow more students

access to a higher level of mathematics.

Bienvenido F. Nebres in his paper, *The Problem of*

*Universal Mathematics Education in Developing Countries,*

discussed the same problem of the lack of fit between the

goals of mathematical education and the needs of the majority

in the special circumstances of the situation in developing

countries. He offered a conceptual framework for discussing

the specific cultural dimensions of the problem in

these countries by using the distinction between vertical

and horizontal relationships, i.e. the relationships between

corresponding institutions in different societies and the rel

a t i o n ships between social or cultural institutions within the

same country. The history of social and cultural institutions

in developing countries is that their establishment and growth

has been guided more by vertical relationships, i.e. an

adaption of a similar type of institution from the mother colonial

country, rather than by horizontal relationships. The result

is a special lack of fit between mathematical education

and the needs of the majority of the people.

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There is a tremendous need for researchers in mathematics

education in developing countries to look at the

actual life of urban workers, rural farmers and merchants

and to identify the mathematics in daily life that is needed

and used by people. Then it is necessary to compare this

needed mathematics with what is provided in the curriculum

and to search for a better fit between the two. A cultural

shift must be brought about in these countries.

Mathematical educators, together with other educators and

other leaders of society, should take up the need for the

social and cultural institutions to be better integrated with

one another and to develop together in a more organic

manner than in the past.

In a joint paper *Mathematics for All: Conclusions Drawn*

*from the Experiences of the New Mathematics Movement,*

Peter Damerow of the Max Planck Institute for Human

Development and Education in Berlin, West Germany, and

Ian Westbury of the University of Illinois at

Urbana-Champaign, United States, examined the problem

of designing a mathematics curriculum which genuinely

meets the diverse needs of all students in a country. They

argue that, by continuing to ignore the needs of all except

a small minority of students, the curricula developed within

the new mathematics movement proved to be no more

satisfactory than their predecessors. Traditionally, mathematics

curricula were developed for an elite group of students

who were expected to specialise in the subject, and

to study mathematics subsequently at higher levels in a

tertiary institution. As education has become increasingly

universal, however, students of lesser ability, and with

more modest vocational aspirations and daily life requirements,

have entered the school system in greater numbers.

A major problem results when these students are

exposed to a curriculum designed for potential specialists.

This same type of traditional curriculum has frequently

been transferred to developing and third world countries,

where, because of different cultural and social conditions,

its inappropriateness for general mathematical education

has only been compounded. So called reforms such as

new mathematics did little to resolve the major problems in

that they merely attempted to replace one specialist curriculum

by another.

The question addressed by Damerow and Westbury is

how to cater both for the elite and also for the wider group

of students for whom mathematics should be grounded in

real world problem solving and daily life applications. One

suggestion is that the majority would achieve a mathematical

«Literacy» through the use of mathematics in other

subjects such as science, economics, while school mathematics

would remain essentially and deliberately for specialists.

This is effectively to retain the status quo.

Alternatively, mathematics must be kept as a fundamental

part of the school curriculum, but ways of teaching it effectively

to the majority must be found. The majority of students

will be users of mathematics. Damerow and

Westbury concluded that a mathematics program which is

truly for all must seek to overcome the subordination of elementary

mathematics to higher mathematics, to overcome

its preliminary, preparatory character, and to overcome its

irrelevance to real life situations.

The findings of the Second International Mathematics

Study (SIMS) were used by Howard Russell, Ontario

Institute for Studies in Education, Canada, in his paper

*Mathematics for All: SIMS Data,* to argue that mathematics

is already taught to all pupils at the elementary level in

many countries. At the senior secondary level, however,

the prevailing pattern in most countries is for mathematics

to be taught only to an elite. At the lower level, the SIMS

data suggest that promotion by age, rather than by performance,

does not violate the concept of mathematics for all.

The SIMS data also appear to provide support for the

Cockcroft hypothesis that the pace of mathematics education

must be slowed if sufficient students are to be retained

in mathematics courses at the higher levels for it to be

accurately labelled mathematics for all. Alternatively, the

content of the curriculum could be trimmed down as suggested

by Damerow. Russell proposed a market-oriented

rationale to construct such a core of material, particularly to

meet the needs of the middle level students who will be

required to use mathematics in their chosen work in the

market place.

Afzal Ahmed was a member of the Committee of Inquiry

into the Teaching of Mathematics in Schools in England

and Wales (Cockcroft Committee), and is now the director

of the Curriculum Development Project for Low Attaining

Pupils in Secondary School Mathematics. In his paper, *The*

*Foundations of Mathematics Education for All,* he discussed

implications of the Cockcroft Report, published in January

1982, concerning the major issues of the theme group. He

pointed out that a suitable mathematics curriculum for the

majority assumes greater importance as societies in the

world become more technological and sophisticated. But at

the same time, the evidence of failure at learning and

applying mathematics by a large proportion of the population

is also growing. The Cockcroft Report proposes a

Foundation List of Mathematical Topics that should form

part of the mathematics syllabus for all pupils. In his discussion

of the Cockcroft Report, Ahmed focussed

particularly on the classroom conditions which facilitate, or

inhibit the mastery of these fundamental topics.

In a paper on *Universal Mathematics Education,*

Achmad Arifin from the Bandung Institute of Te c h n o l o g y

in Indonesia, described how community participation

should be raised in carrying out universal mathematics

education through looking at the aspect of interaction

within and between social and cultural institutions. He

asked the questions which parts of mathematics can

function as a developer of an individual’s intelligence and

how should those parts that have been chosen be presented?

Any program to answer these questions has to

take into account three components of interaction. Firstly,

depending on its quality, social structure through interaction

can contribute to the improvement of peoples’ a b i l ities,

especially by making them appreciate mathematics.

S e c o n d l y, a special form of social interaction, which he

called positive interaction, can motivate mathematics

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learning and create opportunities to learn. Thirdly, school

interaction itself can inspire, stimulate, and direct learning

activities. In developing countries, local mathematicians in

particular are able to understand their cultural conditions,

the needs, the challenges and the wishes of their developing

nation. Taking into account the three components of

interaction, they have the ability and the opportunity to

spread and share their knowledge and to translate and utilise

the development of mathematics in universal

mathematics education for their nation.

In many countries, there is one mathematics syllabus for

each year of the education system. Andrew J. C. Begg, in

his paper, *Alternative Mathematics Programs,* questioned

this practice and argued for the introduction of alternative

mathematics programs which will meet the varied needs of

all students in a range of circumstances and with a range

of individual aspirations. All such courses should contribute

towards general educational aims such as the development

of self-respect, concern for others, and the urge to

enquire. Thus, mathematics courses should provide an

opportunity to develop skills of communication, responsibility,

criticism, and cooperation. Such an approach has

implications for the way in which students are organised in

mathematics classes; for the scheduling of mathematics

classes; for the choice of teaching and learning methods;

for the extent to which emphasis is placed on cooperation

as against competition; for the use of group methods of

teaching; and for the provision that should be made for students

from diverse cultural groups. In this way, mathematics

programs for all students should assist not only the

achievement of mathematical objectives, but also the

attainment of personal, vocational and humanistic aims in

education. By matching mathematics programs to the

needs of students, the development of the self-esteem of

every student becomes central in the mathematics curriculum.

The second group of papers was concerned with particular

concerns related to Mathematics for All in industrialised

countries.

In their paper, *Arithmetic Pedagogy at the Beginning of the*

*School System in Japan,* Genichi Matsubara and

Zennosuke Kusumoto traced the introduction of the teaching

of western arithmetic to Japan in the late nineteenth

century. At a time when universal elementary education

was only just approaching reality in Japan, the government

declared a policy of adopting western-style arithmetic in

order to enable the country to compete more successfully

internationally. This move faced obstacles in its implementation

because of the traditional use of the abacus and the

widespread lack of familiarity with the Hindu-Arabic notation.

Further, in a developing national system of education,

teachers were in short supply and little attention could be

given to teaching methods in the training courses. The

paper emphasised the need to make such changes slowly

and to take into account the situation of those closely involved

with the changes if they are to be successful in modifying

the curriculum for mathematics for all.

The extent to which the mathematics learnt at school is

retained and used in later life is the subject of research

reported in a paper by Takashi Izushi and Akira Yamashita

of Fukuoka University, Japan, entitled *On the Value of*

*Mathematical Education Retained by the Social Members of*

*Japan in General.* A study in 1955 was concerned with

people who had learnt their mathematics before the period

in Japan in which mathematics teaching was focussed on

daily life experience and before compulsory education was

extended to secondary schools. Although it was found that

most people retained the mathematics skills and knowledge

well, rather fewer claimed that this material was useful

in their work. A second more limited study in 1982 confirmed

these general findings in relation to geometry. It showed,

broadly speaking, that younger people tended to use

their school mathematics more directly while older people

relied more on common sense. The study covered a further

aspect, the application of the attitudes of deductive thinking

derived from the learning of geometry. The thinking and

reasoning powers inculcated by this approach were not forgotten

and were claimed to be useful in daily life, but not in

work. Izushi and Yamashita conclude that the inclusion of

an element of formal mathematical discipline in the curriculum

is supported by Japanese society.

Another attempt to create a modern course in advanced

mathematics which is also worthwhile for those students

who don’t intend to proceed to university was reported by

Ulla Kürstein Jensen from Denmark in her paper titled

*Upper Secondary Mathematics for All? An Evolution and a*

*Draft.* The increase from about 5% in former years to about

40So in 1983 of an age cohort completing upper secondary

education with at least some mathematics brought about

an evolution toward a curriculum concentrating on useful

mathematics and applications in daily life and mathematical

modelling. This evolution led to the draft of a new curriculum

which will be tested under school conditions, beginning

in autumn 1984. The origin of this development is

based on new regulations for mathematical education for

the upper secondary school in the year 1961. It was

influenced by the new mathematics ideas and designed to

serve the needs of the small proportion of the students

passing through upper secondary education at that time,

but soon had to be modified for the rapidly increasing number

of students in the following years. So the mathematics

teaching, particularly for students in the language stream of

the school system, was more and more influenced by ideas

and teaching materials of a further education program

which was much more related to usefulness for a broad

part of the population than the usual upper secondary

mathematics courses. In 1981, this development was legitimated

by new regulations and, by that time, even mathematics

teaching in classes concentrating on mathematics

and physics became more and more influenced by the tendency

to put more emphasis on applications leading ultimately

to the draft of the new unified curriculum which is

now going to be put into practice.

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The central topic of a paper entitled *Fight against School*

*Failure in Mathematics,* presented by Josette Adda from the

Université Paris 7, was an analysis of social selective functions

of mathematical education. She reported statistical

data showing the successive elimination of pupils from the

«normal way at each decision stage of the school system

until only 16% of the 17 year age cohort remain whereas

all others have been put backward or relegated to special

types of classes. These eliminations hit selectively

socioculturally disadvantaged families. Research studies,

particularly at the Université Paris 7, have been undertaken

to find out why mathematics teaching as it is practised

today is not neutral but produces a correlation between

school failure in mathematics and the sociocultural environment.

They indicate the existence of parasitic sources

of misunderstanding increasing the difficulties inherent in

mathematics, e. g. embodiments of mathematics in pseudo-

concrete situations which are difficult to understand for

many pupils. On the other hand, it had been found that children

failing at school are nevertheless able to perform

authentic mathematical activities and to master logical operations

on abstract objects.

Two papers were based on the work of the EQUALS

program in the United States. This is an intervention program

developed in response to a concern about the high

dropout rate from mathematics courses, particularly in the

case of women and minority students. The program aims

to develop students’ awareness of the importance of

mathematics to their future work, to increase their confidence

and competence in doing mathematics, and to

encourage their persistence in mathematics.

In the first of these papers, *EQUALS: An Inservice*

*Program to Promote the Participation of Underrepresented*

*Students in Mathematics,* Sherry Fraser described the way

in which the program has assisted teachers to become

more aware of the problem and the likely consequences

for individual students of cutting themselves off from a

mathematical education. By working with teachers and

providing them with learning materials and methods, with

strategies for problem solving in a range of mathematical

topics, together with the competence and confidence to

use these, EQUALS has facilitated and encouraged a

transfer of concern to the classroom and attracted and

retained greater numbers of underrepresented students in

mathematics classes. Since 1977, 10,000 educators have

participated in the program.

Although the main focus of activity in the EQUALS program

has been on working with teachers and administrators,

needs expressed by these educators for

materials to involve parents in their children’s mathematical

education led to the establishment of *Family Math.* Virginia

Thompson described how this project has developed a curriculum

for short courses where parents and their children

can meet weekly to learn mathematical activities together

to do at home. This work reinforces and complements the

school mathematics program. Although the activities are

suitable for all students, a major focus has been to ensure

that underrepresented students, primarily females and

minorities, are helped to increase their enjoyment of

mathematics. The project serves to reinforce the aims of

the EQUALS program.

The move over the past ten years or so towards applicable,

real world and daily life mathematics in the

Netherlands, inspired by the work of Freudenthal, was described

by Jan de Lange Jzn. of OW and OC, Utrecht, in his

*paper Mathematics for All is No Mathematics at All.* Textbooks

have been published for primary and lower secondary

schools which reflect this view of mathematics, and research

shows that the reaction of teachers and students has

been very favourable. De Lange illustrated the vital role

played by applications and modelling in a newly-introduced

curriculum for pre-university students. Many teachers

apparently view the applications-oriented approach to

mathematics very differently from the traditional mathematics

content. The ultimate outcome, de Lange suggested,

may be that science and general subjects will absorb the

daily life use of mathematics and consequently this type of

mathematics might disappear from the mathematics curriculum.

That is, the ultimate for all students as far as mathematics

is concerned could in reality become no mathematics

as such.

Roland Stowasser from the Technical University of West

Berlin proposed in his paper, *Problem Oriented Mathematics*

*Can be Taught to All,* to use examples from the history of

mathematics to overcome certain difficulties arising from

courses based on a single closed system, which increase

mathematical complexity but do not equally increase the

applicability to open problems. He stated that mathematics

for all does not necessarily have to be directly useful, but it

has to meet two criteria: The mathematical ideas have to

be simple, and on the other hand, they have to be powerful.

He illustrated these criteria through a historical

example. Regiomantus formulated the problem to find the

point from which a walking person sees a given length high

up above him (e. g. the minute hand of a clock if the person

walks in the same plane as the face of the clock) subtending

the largest possible angle. The solution with ruler

and compasses in the framework of Euclidean geometry is

somewhat tricky. But according to Stowasser the teaching

of elementary geometry should not be restricted to Greek

tricks. For problem solving he advocated free use of possible

tools, and the solution of the problem is very simple if

trial and error methods are allowed. So the solution of the

historical problem represents the simple but powerful idea

of approximation.

What are the characteristics of a mathematics program

suitable for all students, and do any such programs exist?

These questions were addressed by Allan Podbelsek of

the United States in his paper, *Realization of a Mathematics*

*Program for All.* Podbelsek listed a number of criteria for

such a program covering not only content knowledge and

skills but also attitudes towards, and beliefs about, mathematics

and the process skills involved in its use. Mathematics

must be seen to be a unified, integrated subject, rather

than a set of individual, isolated topics. T h e

Comprehensive School Mathematics Program

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(CSMP) developed over several years in the United States

for elementary (K-6) level classes is found to meet these

criteria successfully in almost every respect. Practical problems

involved in the introduction of such a program as

CSMP to a school were discussed by Podbelsek. These

problems centred on the provision of adequate teacher

training for those concerned, meeting the cost of materials,

securing the support of parents and the local community,

and ensuring that administrative staff were aware of the

goals of the program.

Those translating mathematical, scientific or technical

material should have a basic knowledge of mathematics to

do their job satisfactorily, yet because of their language

background they are not likely to have studied mathematics

to any great extent at school. This is the experience

which led Manfred Klika, of the Hochschule Hildesheim in

West Germany, to a consideration of the nature and adequacy

of present school mathematics programs in his

paper *Mathematics for Translators Specialised in Scientific*

*Texts - A Case Study on Teaching Mathematics to*

*Non-Mathematicians.* Conventional school programs, he

claimed, do not prepare students to comprehend and make

sense of mathematical ideas and terminology. The solution

is to construct the mathematics curriculum around fundamental

ideas. Two perspectives on this notion are offeredmajor

anathematising models (e. g. mathematical

concepts, principles, techniques, etc.) and field-specified

strategies suitable for problem solving in mathematics (e.g.

approximate methods, simulation, transformation strategies,

etc.). A curriculum based on such fundamental ideas

would result in more meaningful learning and thus a more

positive attitude to the subject. A course based on this

approach has been established at the Hochschule

Hildesheim within the program for training specialist translators

for work in technical fields.

The major concern of the preceding contributions to the

topic “ Mathematics for All ” were problems of designing a

mathematics curriculum which is adequate to the needs

and the cognitive background of the majority in industrialised

countries. The organising committee of the theme

group was convinced that it is even more important to discuss

the corresponding problems in developing countries.

But it was much more difficult to get substantial contributions

in this domain. To stress the importance of the development

of mathematical education in developing countries,

the work of the theme group terminated with a panel discussion

on *Universal Mathematical Education in Developing*

*Countries,* with short statements of major arguments by

Bienvenido F. Nebres from the Philippines, Terezinha N.

Carraher from Brazil, and Achmad Arifin from Indonesia,

followed by the reactions of Peter Towns and Bill Barton,

both from New Zealand. The discussion concentrated on

the relation between micro-systems of mathematical education

like curricula, textbooks and teacher training and

macro-systems like economy, culture, language and general

educational systems which, particularly in the developing

countries, often determine what kind of developments

on the level of micro-systems are possible. Bienvenido F.

Nebres expressed the common conviction of the participants

when he argued that, in spite of the fact that often it

is impossible to get a substantial improvement of

mathematics education without fundamental changes in

the macro-systems of education, micro-changes are possible

and are indeed a necessary condition to make people

realise what has to be done to get a better fit between

mathematical education and the needs of the majority. This

result of the discussion highlights the importance of the

papers submitted to the theme group dealing with special

aspects of mathematical education in developing countries.

Three reports were given by David W. Carraher,

Terezinha N. Carraher and Analucia D. Schliemann about

research undertaken at the Universidade Federal de

Pernambuco in Recife, Brazil. David W. Carraher prepared

a paper titled *Having a Feel for Calculations* about a study

investigating the uses of mathematics by young, schooled

street vendors who belong to social classes characteristically

failing in grade school, often because of problems in

mathematics, but who often use mathematics in their jobs

in the informal sector of the economy. In this study, the

quality of mathematical performance was compared in the

natural setting of performing calculations in the market

place and in a formal setting similar to the situation in a

classroom. Similar or formally identical problems appeared

to be mastered significantly better in the natural setting.

The reasons were discussed and it was stated that the

results of the analysis strongly suggest that the errors

which the street vendors make in the formal setting do not

reflect a lack of understanding of arithmetical operations

but rather a failing of the educational system which is out

of touch with the cognitive background of its clientele.

There seems to be a gulf between the intuitive understanding

which the vendors display in the natural setting and

the understanding which educators try to impart or develop.

Terezinha N. Carraher reported in her paper *C a n*

*Mathematics Teachers Teach Proportions?* results of a

second research project. Problems involving proportionality

were presented to 300 pupils attending school in

Recife, in order to find out whether a child already understands

proportions if it only follows correctly the routines

being taught at school. The results indicate characteristic

types of difficulties appearing in certain problems, some of

which can be related to cognitive development. It is suggested

that teachers’ awareness of such difficulties may

help to improve their teaching of the subject. For if mathematics

is to be useful to everyone, mathematics teachers

must consider carefully issues related to the transfer of

knowledge acquired in the classroom to other problem solving

situations.

The third paper, presented by Analucia D. Schliemann,

*Mathematics Among Carpentry Apprentices: Implications for School*

*Teaching,* highlighted the discontinuity between formal school

methods of problem solving in mathematics and the informal

methods used in daily life. This research study contrasted the

approaches to a practical problem of quantity estima-

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tion and associated calculation taken by a group of experienced

professional carpenters without extensive schooling,

and a group of carpentry apprentices attending a formal

school system and with at least four years of mathematics

study. The results showed that apprentices approached

the task as a school assignment, that their strategies

were frequently meaningless and their answers absurd. On

the other hand, the professional carpenters took it as a

practical assignment and sought a feasible, realistic solution.

Very few computational mistakes were made by either

group but the apprentices appeared unable to use their formal

knowledge to solve a practical problem. Schliemann

concluded that problem solving should be taught in practical

contexts if it is to have transferability to daily life situations

out of school.

Pam Harris from the Warlpiri Bilingual School discussed

in her paper, *Is Primary Mathematics Relevant to Tribal*

*Aboriginal Communities?,* the problem that, in the remote

Aboriginal communities of Australia, teachers often get the

feeling that mathematics is not relevant. Several reasons

can be identified. Teachers often receive negative attitudes

from other people so that they go to an Aboriginal community

expecting that their pupils will not be able to do mathematics.

Furthermore, they observe a lack of reinforcement

of mathematics in the pupils’ home life. Teaching materials

mostly are culturally and linguistically biased. Teachers feel

discouraged because of the difficulties of teaching mathematics

under these conditions. Nevertheless, Pam Harris

stressed the importance of mathematics, because

Aboriginal children have to get an understanding of the

Second culture» of their country. They need mathematics

in their everyday life, in employment, and in the conduct of

community affairs. But to be successful, mathematics

teaching in Aboriginal communities has to allow for and

support local curriculum development. Individual schools

and language groups should make their own decisions on

the use of the children’s own language, the inclusion of

indigenous mathematical ideas, priorities of topics, and

sequencing the topics to be taught.

Kathryn Crawford, from the College of A d v a n c e d

Education in Canberra, presented a paper on *Bicultural*

*Teacher Training in Mathematics lEducation for Aboriginal*

*Trainees from Traditional Communities* in Central Australia.

She described a course which forms part of the Anagu

Teacher Education Program, an accredited teacher training

course intended for traditionally oriented Aboriginal

people currently residing in the Anagu communities who

wish to take on greater teaching responsibilities in South

Australian Anagu schools. The most important difference

between this teacher training course and many others is

that it will be carried out on site by a lecturer residing within

the communities and that, from the beginning, development

of the curriculum has been a cooperative venture between

lecturers and educators on the one hand, and community

leaders and prospective students on the other. The

first group of students will begin the course in August 1984.

The course is particularly designed to meet the fact that different

cultures emphasise different conceptual schemes.

Thus, temporal sequences and quantitative measurement

are dominant themes in industrialised Western cultures but

largely irrelevant in traditional Aboriginal cultures. To overcome

these difficulties, the focus of the problem is redirected

from the “failings” of Aboriginals and Aboriginal

culture to the inappropriateness of many teaching practices

for children from traditionally oriented communities.

The course has been developed based on a model designed

to maximise the possibility of interaction between the

world view expressed by Anagu culture and that of

Anglo-European culture as evidenced in school mathematics.

This is achieved by placing an emphasis on the student

expertise and contribution in providing information

about Anagu world views as a necessary part of the course.

In this community based teacher training course, it

seems that it is possible for the first time to develop procedures

for negotiating meanings between the two cultures.

**3. Conclusions**

The presentations given at the sessions of the theme

group summarised above can be considered as important

efforts to contribute to the great program of teaching

mathematics successfully not only to a minority of selected

students but teaching it successfully to all. But in spite of all

these efforts it has to be admitted that the answer to. the

question, What kind of mathematics curriculum is adequate

to the needs of the majority?», is still an essentially open

one. However, the great variety of the issues connected

with this problem which were raised in the presented

papers makes it at least clear that there will be no simple

answer. Thus the most important results of the work of this

theme group at ICME 5 may be that the problem was for

the first time a central topic of an international congress on

mathematical education, and that, as the contributions

undoubtedly made clear, this problem will be one of the

main problems of mathematical education in the next decade.

As far as the content of these contributions is concerned,

the conclusion can be drawn that there are at least three

very different dimensions to the problem which contribute

to and affect the complex difficulties of teaching mathematics

effectively to the majority:

- the influence of social and cultural conditions;

- the influence of the organisational structure of the school

system;

- the influence of classroom practice and classroom

interaction.

*Cultural Selectivity*

One of the major underlying causes of the above problem

is the fact that mathematical education in the traditional

sense had its origins in a specific western European cultural

tradition. The canonical curriculum of «Tr a d i t i o n a l

mathematics» was created in the 19th century as a study

for an elite group. It was

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created under the conditions of a system of universal basic

education which included the teaching of elementary computational

skills and the ability to use these skills in daily life

situations. There is a clear distinction between the aims

and objectives of this basic education and the curriculum of

traditional school mathematics which was aimed at formal

education not primarily directed at usefulness and relevance

for application and practice. This special character of

the canonical mathematics school curriculum is still essentially

the same today in many countries.

The transfer of the European mathematics curriculum to

developing countries was closely associated with the establishment

of schools for the elite by colonial administrations.

Under these circumstances it seemed natural to simply

copy European patterns. It is quite another problem to

build a system of mass education in the Third World and

embed mathematics education in both the school situation

and the specific social and cultural contexts of that world.

The papers summarised above point clearly to some of

the problems. Curricula exist which encourage students to

develop antipathies towards mathematics; this is commonly

the case in Europe. Further, such curricula have sometimes

been transferred to countries where the social

context lacks the culturally based consensus that is found

in Europe, namely, that abstract mathematical activity is

good in itself and must therefore be supported, even if it

seems on the surface to be useless. It has been proposed

on the one hand, that a sharp distinction should be made

between applicable arithmetic in basic education and essentially

pure mathematics in secondary education, and on

the other hand, that mathematics should be integrated into

basic technical education. This argument raises the question

of the relation between mathematics and culture which

may be the first problem to address when the idea of

mathematics for all is raised as a basis for a program of

action.

*Selectivity of the School System*

While the particular curricular patterns of different societies

vary, the subject is still constructed in most places so that

few of the students who begin the study of mathematics

continue taking the subject in their last secondary years.

The separation of students into groups who are tagged as

mathematically able and not able is endemic. Curricula are

constructed from above, starting with senior levels, and

adjusted downwards. The heart of mathematics teaching

is, moreover, widely seen as being centered on this curriculum

for the able, and this pattern is closely related to

the cultural contexts indicated above. However, we must

consider the problem of conceiving, even for industrialised

societies, a mathematics which is appropriate for those

who will not have contact with pure mathematics after their

school days. Up to now we have made most of our students

sit at a table without serving them dinner. Most

attempts to face the problem of a basic curriculum reduce

the traditional curriculum by watering down every

mathematical idea and every possible difficulty to make it

feasible to teach the remaining skeleton to the majority.

There is only a limited appeal to usefulness as an argument

or a rationale for curriculum building to avoid the

pitfalls of this situation. Students who will not have to deal

with an explicit area of pure mathematics in their adult lives

but will face instead only the exploitation of the developed

products of mathematical thinking (e. g. program packages),

will only be enabled by mathematics instruction if

they can translate the mathematical knowledge they have

acquired into the terms of real-life situations which are only

implicitly structured mathematically. Very little explicit

mathematics is required in such situations and it is possible

to survive in most situations without any substantial mathematical

attainments whatsoever.

Is the only alternative to offer mathematics to a few as a

subject of early specialisation and reject it as a substantial

part of the core curriculum of general education? This

approach would deny the significance of mathematics. To

draw this kind of conclusion we would be seen to be looking

backwards in order to determine educational aims for

the future. The ongoing relevance of mathematics suggests

that a program of mathematics for all implies the

need for a higher level of attainment than has been typically

produced under the conditions of traditional school

mathematics — and that this is especially true for mathematics

education at the level of general education. In other

words, we might claim that mathematics for all has to be

considered as a program to overcome the subordination of

elementary mathematics to higher mathematics, to overcome

its preliminary character, and to overcome its irrelevance

to life situations.

*Selectivity in Classroom Interaction*

Some of the papers presented in this theme group support

recent research studies which have suggested that it

is very likely that the structure of classroom interaction

creates ability differences among students which grow

during the years of schooling. In searching for causes of

increasing differences in mathematical aptitude, perhaps

the simplest explanation rests on the assumption that

such differences are due to predispositions to mathematical

thinking, with the further implication that nothing can

be done really to change the situation. But this explanation

is too simple to be the whole truth. The understanding

of elementary mathematics in the first years of primary

school is based on preconditions such as the acquisition

of notions of conversation of quantity which are, in their

turn, embedded in exploratory activity outside the school.

The genesis of general mathematical abilities is still little

understood. The possibility that extra-school experience

with mathematical or pre-mathematical ideas influences

school learning cannot be excluded. Furthermore, papers

presented to the theme group strongly suggest that the

d i fferences between intended mathematical understanding

and the understanding which is embedded in normal

classroom work is vast. We cannot exclude the possibility

that classroom interaction in fact produces growing differences

in mathematical aptitude and achievement by a

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system of positive feedback mechanisms which increase

high achievement and further decrease low achievement.

It is clear that to talk of mathematics for all entails an

intention to change general attitudes towards mathematics

as a subject, to eliminate divisions between those who are

motivated towards mathematics and those who are not,

and to diminish variance in the achievement outcomes of

mathematics teaching. This, in its turn, involves us in an

analysis of social contexts, curricula and teaching. It is

these forces together which create a web of pressures

which, in turn, create situations where mathematics

becomes one of the subjects in the secondary school in

which selection of students into aptitude and ability groups

is an omnipresent reality almost from the time of entry.

**Part I:**

**Mathematics for All –**

**General Perspectives**

11

**A Necessary Renewal of Mathematics**

**Education**

Jean-Claude Martin

Mathematics for all must not only be accessible mathematics,

but interesting mathematics for all - or for the majority.

Such a theory leads one, in the case of the teaching of

mathematics in France, to raise problems of objectives and

curriculum organisation, but also of methods more than of

the content of the curriculum.

**1. The General Characteristics of Selective Education**

*(i) The Fundamental Teaching of Mathematics*

*for Mathematics Sake*

Mathematics as they are known today may be considered,

if not as a whole, as a system. The training of the highest

level of generalist mathematicians may a priori be defined

as leading to knowledge of this system.

Dividing the system of mathematics into parts going

from the simplest element to the most complicated may

represent, as a first approximation only, but quite logically,

a curriculum of study for the training of mathematicians.

That is what we shall call, to serve as a reference for our

later discussions, the teaching of mathematics for mathematics

sake. Its organisation in the form of a continuous

upward progression implies that each level reached will be

a prerequisite for the level immediately following.

Such a curriculum does not exist in the pure state but it

appears to be the foundation, the skeleton of most programs

of general mathematical training in many countries,

being a reflection of the European rationalist cultural tradition.

Adaptations of this consist essentially in heavier or lighter

pruning, stretching to varying degrees the progression,

or illustrating it to some extent by an appeal to real-life

experience (either in order to introduce a notion or to

demonstrate some application of it).

The first question raised then is whether such teaching

is a suitable basis for mathematics for all.

On the level of objectives, the reply is obviously negative:

The training of mathematicians can interest only a

minute portion of students.

*(ii) Selection by Means of Mathematics*

In France, statistics show that of any 1,000 students entering

secondary education, fewer than 100 will obtain seven

years later a scientific baccalaureat (including section D)

and a maximum of five will complete tertiary studies in

mathematics or related disciplines (computer science in

particular).

Referring again to statistics indicates that only about

one successful candidate at the baccalaureat in six holds

one of the types of baccalaureat (C or E) in which mathematics

are preponderant. That fact, together with other

indications concerning class counselling, brings out sufficiently

clearly the importance of selection — a well enough

known phenomenon anyway — by mathematics in the

secondary school. This selectivity appears moreover to be

relatively stronger than at university. This situation makes

mathematics a dominant subject. French, which formerly

shared the essential role in selection, is now relegated to a

secondary position.

This selection is manifested most often by a process of

orientation through failure for students at certain levels. But

in fact, this sanction is usually only the deferred result of an

ongoing selection process which takes effect cumulatively.

From the primary school, or as early as the first years of

secondary school, the classification between «Maths» and

«non-maths» students becomes inexorably stratified.

In recent years, the idea that selection through mathematics

is equivalent to a selection of intelligent students

has made some progress, even if it is only very rarely

expressed in such a clear way.

This function as the principal filter of the education system

has considerably harmed the prime constructive function

of mathematics as a means of training thought processes

by the practice of logical reasoning. Just as a filter

naturally catches waste, so mathematics produce academic

failures inherent in the system, in other words, not due

to intrinsically biological or psycho-effective causes but to

the teaching process itself.

The type of evaluation used is not unconnected. It has

the general fault of all standardised evaluation as is still too

widely practised.

It supposes a definition of the child’s normality that

pediatricians and psychologists contests1,2 : ranges of

development, differences in maturity are just as normal

and natural as differences in height and body weight. The

same applies to the formation and development of abstract

thought, which one must expect to be facilitated by the teaching

process and not measured and sanctioned by it.

Aptitude for abstraction seems to be generally considered,

with intelligence, as having an essentially innate character,

whereas it is admitted by researchers that the share

acquired in the social and family milieu and then at school

is probably preponderant.

The demands of «Levels of intelligence» are also judged

excessive for the teaching situation (first years of

secondary school). They would necessitate2 a clearly

above average IQ.

On this subject we may note a very important lack of

coordination between the quite reasonable programs and

instruction of the Inspectorate General of Teaching and the

contents of textbooks.

We shall see later some questions concerning the vocabulary

used, but where the program considers only arithmetic

or operations on whole numbers or rational numbers,

it can be seen that in fact a veritable introduction to algebra

is carried out.

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*(iii) Emotional Responses and Mathematics*

All teaching is obviously subject to emotional responses:

the student likes this, doesn’t like that, prefers this, and so

on. As far as mathematics is concerned, successful students

acquire an assessment based on a harmonious relationship,

but those who have difficulties feel strong emotions

that can induce suffering and anguish.4

**II. Problems of Language, Symbolic Writing**

A mathematical apprenticeship requires the acquisition of a

special language which is characterised by the interlocking

of a conventional language (with nevertheless its own

semantics and syntax) and a symbolic language.

If, beside their communicative aim, all languages serve

as a medium of thought — according to Sapir: The feeling

that one could think and indeed reason without language is

an illusions — the language of mathematics, more than any

other, is adapted to that very end. The sentence (containing

words) and the formula with its symbols are vehicles

of logical reasoning. In this area, symbolic writing is considerably

more powerful than conventional writing: one can

say that it is a motive force driving thought ahead more

rapidly.

*(i) The Power of Symbols*

On the occasion of the 4th International Congress on the

Teaching of Mathematics (ICME 4), Howsons5 clearly showed

the power of symbols, which one could have thought

in the first analysis to be only tricks of abbreviation, whereas

they do generate new meanings.

As essential elements of mathematics, they permit the

discipline to develop without its being necessary to burden

our thought processes with all the meanings with which

they are charged. A language open to independent development,

symbolic writing lends itself to operations the

automatic nature of which, once it is acquired, saves

conscious thought or at the very least permits considerable

economies in the process of reflexion.

An apprenticeship in symbolic writing and the attendant

operational procedures is therefore essential in the teaching

of mathematics.

*(ii) The Importance of Language Acquisition*

The nature of symbolic writing being a capacity for

self-development, if what has been learned in this area is

already considerable, the student will have no major difficulty

in acquiring the language necessary if he is to pass to

the next stage. Thus his difficulties will reside rather in the

structures of reasoning than in a knowledge of symbols. It

may be considered that this is the case of students in the

upper classes of secondary school.

On the other hand, at the beginning of this apprenticeship

(notably when algebra is introduced), the change

from the natural language to symbolic language, because

it is a prerequisite, no doubt has a special place in the hierarchy

of difficulties.

*(Iii) The Difficult Changeover to Symbolism*

The changeover from natural language to symbolic language,

as well as the problems caused by too rapid or too

early an introduction (poorly adapted to the development of

the thought processes of the student and his maturity) carries

with it some more technical difficulties, which in our

view have not been satisfactorily solved.

Symbolic formulation is more than mere translation. The

physicist is well aware of this, considering as he does

today this operation, called (mathematical) modelling, as

being of prime importance in the analysis of complex phenomena

or systems. In the same way, the return from the

formula to realist is an exercise that is not self-evident and

a table of correspondences and a dictionary will not suffice.

Symbolism introduces first of all a complication.

Afterward, naturally, when the obstacle is overcome, one

profits as a result of a simplification of procedures (automatic

responses in operations and their reproduction).

If one can solve a problem in ordinary language with a

level of difficulty N1, to use for its solution a poor knowledge

of symbolic language makes it more difficult (level N2).

On this subject the tests of C. Laborde7 seem significant.

Confronted with solving concrete problems or describing

mathematical objects, students do not use the codes they

have learned. But once the symbolism is better known, the

level of effort to attain the same goal is less. Level N3 is for

example the level of effort required of the master mathematician.

This summary demonstrates at the same time the

advantage of learning mathematics and the difficulty there

is, starting with the concrete description of a problem to formulate

it in mathematical terms. It also shows that the teacher

should give considerably more attention to lessening

the difficulty of acquiring the mathematical metalanguage

than accumulating purely mathematical knowledge.

*(iv) The Necessity of Introducing Stages Useful for*

*Conceptualisation*

G. Vergnaud8 has demonstrated that, when solving problems,

students used Faction theorems» or implicit theorems,

which were simply the products of their personal

conceptualisation revealing the workings of individual

thought processes. Several researchers have noted that

such processes did not follow the shortest path of the

mathematics taught nor the best method from the point of

view of logical rigour.

Because of this, it is often considered by the teacher to

be bad reasoning - to be done away with as quickly as possible

in favour of classical mathematical reasoning — whereas

it is rather logical reasoning in the process of developing.

The act of teaching, instead of ignoring or indeed rejecting

the representation constructed by the pupil, his own

personal mechanisms of thought, should consist, on the

contrary, in revealing these, understanding them and using

them.

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As long as educational research does not provide practical

ways of accomplishing this development, it is no doubt

right to give to the acquisition of symbolic logic a more

important share in the teaching process.

Inspiration may come from the evolution of symbolism in

mathematics through the centuries.

Howson alludes to this5 and the analogy of the evolution

of the individual’s knowledge according to Piaget’s theory.

Those noted in physics are arguments in the same direction;

if one begins with the hypothesis that human logic can

exist, it is likely that there are similarities.

But above all, so that the student finds his way naturally,

we should propose to him varied representations of the

same thing: “a supple and changing, suggestive and logical

formalism ” according to Lowenthal.9 We come back to

the recommendation of Howson and Brandsond: «no symbol

or contraction should be introduced if the student is not

ready fully and reasonably to appreciate the advantage it

offers».

We consider that the use of natural language along with

symbolic language can not only better guarantee the

acquisition of the symbolic language5,6 but above all serve

as a better basis or guide for the logical reasoning associated

with mathematical development.

*(v) Avoidable Difficulties*

As well as the intrinsic difficulties in the acquisition of the

symbolic language of mathematics, there exist difficulties

that one could avoid, growing out of the language used to

mediate between natural language and symbolic language.

This is the language used by teachers or school text to

give definitions, enunciate properties and theorems and to

provide the necessary explanations for beginners.

The language used by teachers is obviously very diverse

and varied, and there is no doubt that large numbers

of them know how to adapt as is necessary. In France the

General Inspectorate of Education encourages them to do

so. It recommends in particular that they avoid the introduction

of too many new words.

But if one considers school textbooks, one can ponder

whether these instructions have been taken into consideration.

The intellectual worth of the authors is not in question,

and one must seek the reason in an insufficient realisation

of the importance of the linguistic vehicle. We have

used a textbook for the level known as «5e» where, exceptionally,

a first chapter is devoted to helping in understanding

the terms used in the body of the text. So as to draw

a conclusion «a fortiori» we subjected this chapter to a test

for the «classification of texts according to the difficulty of

the approach required for understanding them» used in

technical education to select documents for students

according to their academic level.

This test has no pretentions to scientific perfection but

the results achieved demonstrate its pertinence.

The result is edifying: With respect to the French used,

this test should be given only to students three or four

years older. The analysis of difficulties shows essentially:

1. that the vocabulary used includes too many words

which are not part of the everyday language of the

student;

2. that certain known words are used in different

senses (paronyms);

3. that there is a supposition of certain references of

experience (not only mathematical but also of a

cultural nature);

4. that the structure of typical phrases aimed at

mathematical precision causes ambiguities on the

level of the French language.

As for the first two of these four observations, we carried

out a summary evaluation of the vocabulary requirements

of five of the most widely used textbooks. With respect to

the first level of the basic French vocabulary (representing

between 1,300 and 1,500 words) the comprehension of

the French used as a vehicle for mathematics teaching

(not including symbols) requires the knowledge of 100 to

150 new words or expressions.

In this body of material, the words that seem to be

known but which are used with a different meaning created

a doubly negative effect; they are not passive obstacles

to comprehension, but introduce confusion.

Several researchers1 0 have demonstrated this undesirable

effect of the most common of these: if (and only

if), then, and, or (exclusive), all ... These fundamental

words should be introduced with the same care as symbols

for they are not stepping stones to symbolic language,

they are merely its image.

Elements supporting reasoning, they need to be perfectly

assimilated so that the correct reasoning may be carried

out. But they are not the only ones that cause the specialised

language of mathematics to be in fact very different

from natural language. Other less frequent uses, as

well as syntax, increase the difficulty.

On this subject, one can raise questions (ii) concerning

the origin of language difficulties in mathematics. Do they

come simply from an insufficient mastery of the natural language?

Such a deficiency obviously introduces a handicap.

But the quite widespread existence of students classified

as «Literary» and «non-mathematical» shows clearly

enough that it is not sufficient to know French better in

order to understand mathematics.

Does not the difficulty of access to formal language also

reside in the incapacity of natural language to translate it

without weighing it down or even deforming it? This is

obvious for the initiate, to whom the formula offers a richer

meaning than the theorem that attempts to express it. The

connection, indeed the interdependence between the

mechanisms of formation of thought and of formal language,

still insufficiently known (cf. different hypotheses of

Piaget, Bruner, etc.) also lead to questions about an

influence of one upon the other (and vice versa).

But the fact that the question is so open to discussion

does not free the teacher from considering the more down

to earth problems of vocabulary and syntax. One should

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not write an introductory manual of algebra (or of other

areas in mathematics that make considerable use of symbolic

writing) without having the French corrected by a specialist

in reading.

Thus one would create the most direct contact possible

between natural expression and symbolic expression. And

if it were realised—which is likely— that short-cut explanations,

by means of Typical mathematical discoursed are not

practical, perhaps one would attain, at the cost of an apparent

waste of time, better comprehension.

**III. Proposals for the Teaching of Mathematics for A l l**

**Students**

From this analysis of faults and difficulties result some

paths that may lead to improvements in teaching.

*(i) Restore the Role of Mathematics as a Tool*

In our highly technological age, everybody no doubt needs

some background in mathematics.

That is why the teaching of mathematics as a tool ought to

be of interest to the majority on condition that, by taking

certain precautions, it is made sufficiently accessible. On

this condition it appears to us the only viable basis on

which to found the structure of mathematics for all.

But what form can such instruction take? It could not be

limited to a curriculum adapted to professional ends

constructed on a basis comparable with that of basic teaching.

For example, although the successive introduction

to algebra and then differential and integral calculus can

furnish a tool for the solution of problems of mechanical

physics, if it does not gradually reveal its concrete basis

and its applications we shall not consider it as instruction in

mathematics as a tool.

The pedagogical procedure too often used consists of

asking the student to acquire numerous prerequisites and

to await the whole construction piece by piece of the cognitive

edifice in order to perceive at last the end to which it

can be put means the teacher is avoiding his responsibilities

and it kills the student’s motivation .

Teaching mathematics as a tool means giving permanent

priority to the solving of problems and not to learning formal

aspects of the discipline. Pedagogically there are two

great advantages in this:

- we have seen that excessive formalism or too early an

introduction of symbolism was an obstacle in the early

stages;

- it is now allowed9 that the development of logical reasoning

is carried out essentially on the basis of experience

in problem solving.

The basic notion is to replace the upward progression in

mathematics isolated by its formalism by a spiral progression

dependent on other disciplines. This presupposes

undertaking at each stage of initiation an adaptation of teaching

methods inspired by research on language and

conceptualisation5,6,7,9,12.

- arouse interest in a problem ‘set the right type of problem);

- bring the student to pose it in logical terms, to translate

it into already familiar mathematical terms (modelling)

and thus bring home the practical application of mathematics;

- give practice in the corresponding operations;

- show the polyvalence and indeed the universality of

methods of logical reasoning, the utility of formalism;

- let the student measure from time to time the resultant

enrichment of his capacities in the area to which the

subject of the problem belongs;

- bring the student finally to a higher mathematical level.

*(ii) The Place of Mathematical Modelling*

One point that seems to us fundamental is the introduction

of «modelling». Here again one can see the fruit of the physicist’s

experience, but such a procedure is in our view

necessary more as a result of our earlier pedagogical

considerations concerning the difficulty of acquiring symbolic

language. «Modelling» or translating the concrete

problem into pertinent mathematical terms does not come

easily. The teacher must make a special study of the question:

- how does one, when faced with a more or less complex

system, observe it, identify it, express its workings in formal

relationships?

- how, when faced with a concrete problem, does one

describe it, translate it into equations?

- how, thanks to the tools of mathematics, does one progress

in one’s understanding, one’s solution of it?

- how finally, at the more sophisticated stages, does one

iterate identification and modelling to the limits of one’s

own knowledge?

Thanks to such a process, the teacher will be able to facilitate

the student’s conceptualisation. As a result of special

attention to the problem and the development of an open

educative process, the teacher will be able to follow the

student’s «natural» principles of reasoning, reveal the formation

of «theorems in action» mentioned above, and thus

facilitate by an appropriate pedagogical method the development

of these into true theorems.

Modelling thus leads into creativity and technological processes.

*(iii) The Necessity of an Interdisciplinary Approach*

An intensive use of modelling requires the mathematics

teacher to have a good knowledge of the applications of

mathematical tools in a variety of areas and makes it

necessary for the development of his teaching to be kept in

line with other disciplines using mathematics. This supposes

not only a basis in team work, but also in a national

curriculum and general interdisciplinary planning. A better

solution would no doubt be the creation of true multidisciplinary

subjects, an added advantage of which would be to

link up again areas of knowledge that the division into disciplines

has fragmented or simply overlooked.

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*(iv) Mathematics as a Tool and Mathematical Culture*

There is no inherent conflict between mathematics as a

tool and mathematical culture, one being able to lead to the

other and vice versa. Restoring the teaching of mathematics

as a tool will allow us to interest students and to offer

them greater possibilities of success and self-development

in the modern environment.

This is the path that seems the most certain to lead to

mathematics for the majority, through a process of success

and not of lowering standards.

*(v) Teaching Through Goals with a Differentiated*

*Progression*

The working out of such a system of teaching would imply

avoiding a drop in standards through an evaluative process

based on objectives that clearly marked out the development

of the curriculum, the chronology of which would be

subject to modification and would permit the most gifted

students to advance more quickly and those in difficulty to

follow at a different pace.

It is accepted that between the beginning of secondary

school and the baccalaureat the majority of students repeat

a year once or twice, and this gives room in the curriculum

and the means of attaining a differentiated progression.

The abrupt and penalising nature of repeating a year

when one begins everything over again, even the things in

which one has been successful, would be attenuated and

greater consideration would be given to the timing of initiation,

work and development.

It has always been accepted for a diploma like the baccalaureat

that differences of level should be tolerated in

various disciplines. Would it be fatal to experience failure in

mathematics between preschool kindergarten and the A or

B baccalaureat? Would it not be better to reach this stage

by a well organised progression and natural orientation

rather than in fits and starts with futile intermediate sanctions,

since in the end students will reject or avoid mathematics

if they cannot succeed.

Would it not be better to provide for success in slow

stages, or related to more limited objectives, rather than to

suffer failure so fully and so prematurely internalised that it

leads numerous students and then adults to a veritable

lack of mathematical culture?

**Conclusion**

Underlying the no doubt imperfect proposals presented

above is a deeper question of objectives.

Will mathematics, rather than being a filter of the elite,

recover their principal function of being the most wonderful

of tools (albeit an immaterial one), of being the way of teaching

logical reasoning?

By laying aside the attributes that make them forbidding

(language, abstraction), by capitalising on their interest and

power, mathematics could be accessible and interesting for

the majority of students, who would all reach their appropriate

level.

Finally, even in the event of only relative success, one

could restore the supply of scientists and mathematicians

that has dried up radically in recent years. Statistically one

would no doubt also achieve a better quality elite.

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**The Problem of Universal Mathematics**

**Education in Developing Countries**

Bienvenido F. Nebres, S. J.

In his paper «Mathematics for All— Ideas, Problems,

Implications,» Peter Damerow underlines two c e n t r a l

concerns. The first is that the canonical school curriculum

for mathematics was designed for an *elite* and so there are

serious adjustment problems when it is made universal.

The second is that it was designed for a *European* elite and

so the adjustment problems become even more serious

when it is introduced into the mass educational system of

a developing country.

«I think firstly we have to consider the fact that mathematical educa -

tion in the traditional sense has its origin in a specific Western

European cultural tradition, where the canonical curriculum of traditional

school mathematics was created in the course of the l9th century.

The transfer of this curriculum to developing countries in most

cases has been closely linked with the institutionalization of schools

by colonial administrations in these countries. It is well known that

these schools generally were attended only by an elite, which adopted

the Western European culture and often studied afterwards at

European universities. Under these conditions it seemed natural simply

to copy the curriculum of higher education. But it is quite another

problem to build up a system of mass education in the countries of

the Third World and to embed mathematical education into the specific

cultural contexts of these countries. «1

In a paper entitled *Problems of Mathematical Education in*

*and for Changing Societies: Problems in Southeast Asian*

*Countries,* which was presented at the October, 1983

Regional Conference in Tokyo, Japan, I tried to classify

mathematical education problems into two types,

micro-problems and macro-problems.

«*We can classify problems of mathematical education into two types:*

*the* first *we might can* micro-problems *or problems internal to mathe -*

*matical education. These would relate to questions of curriculum,*

*teacher-training, textbooks, use of calculators, problem-solving and*

*the like. The* second *we might can* macro-problems. *These are pro -*

*blems affecting mathematics education because of pressures from*

*other sectors of society: economy, politics, culture, language, etc.*

*One of the features of a* developed *society is a reasonable differen -*

*tiation of sectors and functions of society. While given sectors are, of*

*course, interdependent and affect one another, they also have some*

*reasonable autonomy. School budgets may increase or decrease,*

*but they have some stability and so it is possible to plan. Teachers*

*get a sufficient {though not high} salary so they can concentrate on*

*their teaching chores. But in contrast, structures in developing socie -*

*ties are not sufficiently developed to provide (for example) education*

*and culture with sufficient freedom from the pressures of politics and*

*economics Teachers may be called upon to perform many civic*

*duties - to the detriment of their classroom work. Their salaries may*

*nor be sufficient for them to be able to concentrate on their work.*

*Budgets may be unstable and information and opinion tightly control -*

*led ,* «*2*

In that paper I discussed the problem of universal mathematical

education for developing countries, mainly in terms

of economic constraints.

«The problem I would like to concentrate on here is that of the great

number of students who are in school only for four to six years. One

must, therefore, give them functional numeracy within severe

constraints. The time constraint is obvious. There are also problems

of scarcity of textbooks, not-so-well-trained teachers, language. We

might focus the question on only one aspect of the problem: curriculum.

In the Philippines, at least, the curriculum is the same whether

a student goes on for ten years through high school (or even beyond

to university) or whether the student stops after four to six years. I

propose the following questions. From a study of the curriculum and

from experience, at what point is functional numeracy realistically

achieved? After four years? After six years? After eight years? If one

were to look at the curriculum from the point of view of best helping

a student who will stay only for four to six years, would one redesign

the curriculum?»3

**However, on further reflection it seems clear that the** deeper

problem is, as is noted by Peter Damerow, cultural. «So

I think the relations between mathematics and culture is

the first and maybe the most general question which

arises when mathematics for all is taken as a program.

«4 For developing countries, the problem of mathematics

education and culture may best be understood by reflecting

on the history of the school system in these countries.

«All of the countries of Southeast Asia, with the exception of

Thailand, went through a prolonged colonial period. During the colonial

period, the school system was patterned exactly after that of the

colonising country. The norms of fit between school and society were

quite precise: the school system was to come as close as possible to

that of the mother country. It should produce graduates that would fit

into the civil service and who would do well in universities in the

mother country. With independence the above norms of fit between

school and society were seen with mixed feelings. Leaders became

conscious that a school system developed according to such norms

would, among other things, simply contribute to the brain drain. They

also became conscious that the school system had to respond to different

cultures and classes in the country: a westernized elite, a growing

lower middle class, urban workers, a traditional rural sector. The

aspirations for progress and equality led to new questions about the

role of the school system in society:

- Can the school system provide functional literary and

functional numeracy to the great number who attend

school only for four to six years?

- Can the school system provide the scientific and mathematical

skills for different levels in the agricultural, commercial,

and industrial work force?

- Can the school system train sufficiently well the small but

important number needed for leadership in the scientific

and economic sectors?

These are, of course, very difficult tasks. The specific problem faced

by the school system in many developing societies is that the society

at large expects it to fulfill the society’s dream of progress and

equality. These place unrealistic pressures on the school system.»5

1 Damerow, P. (1984): Mathematics for All - Ideas, Problem,

Implications. In: Zentralblatt für Didaktik der Mathematik. No. 3,

pp. 82-83.

2 Nebres, B. (1983): Problems of Mathematical Education in and for

Changing Societies - Problems in Southeast Asian Countries. In:

Proceedings of the lCMl-JSME Regional Conference on

Mathematical Education, Tokyo, p. 10.

3 lbid., p. 16.

4 Damerow, P., loc. cit.

5 Nebres, B., op. cit., p. 12.

18

**I. The Lack of Fit Between School Mathematics**

**and the Socio-Cultural Context of Developing**

**Countries**

There have been some very interesting examples in the

papers presented for the theme group «Mathematics for

All» at Adelaide regarding the lack of fit between school

mathematics and socio-cultural context.

In the paper «Having a Feel for Calculations,» several

examples are given of young street vendors using

non-school algorithms to do fast and accurate calculations.

«Customer: How much is one coconut?

Vendor (12 years old, 3rd grade): 35.

Customer: I’d like 10. How much is that?

Vendor: Three will be 105, with 3 more, that will be 210,

l need 4 more ... that is 315 ... l think it is 35O.»6

Yet these same young people did very poorly in the same calculations

in the school setting. The paper concludes:

«There appears to be a gulf between the rich intuitive understanding

which these vendors display and the understanding which educators,

with good reason, would like to impart or develop. While one could

argue that the youngsters are out of touch with the formal systems of

notation and numerical operations, it could be argued that the educational

system is out of touch with its clientele.»7

In another paper on «Mathematics Among Carpentry

Apprentices,» Analucia D. Schliemann compares the performance

and computational methods of professional carpenters

with apprentices. What was most striking was the

fact that the apprentices insisted on following «school» procedures

even when a little reflection would have shown

them that these were, in practice, absurd. fit seems that the

task was approached by the apprentices as a school assignment

and they did not try to judge the suitability of the

answers.»8

In an earlier discussion on universal primary education, I

had noted our failure to respond to an immediate need of

farming communities throughout the Philippines.9 T h e

introduction of high-yield varieties of rice brought in, of

course, greater productivity. However, it also demanded

higher inputs in terms of fertilisers, pesticides, labor or

machinery for weeding. Farmers had to take out loans to

avail of this new technological input. The farmers were lost

in the new economics of the system. As many of them put

it, «I know I am getting bigger harvests. But I also know I

am sinking deeper into debt.» Our school system in the

rural areas continued happily teaching sets and

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6 Carraher, D., Carraher, T., and Schliemann, A.: Having a Feel for

Calculations. lCME 5 Mathematics for All Collection, p. 2.

7 lbid., p. 6.

8 Schliemann, A.: Mathematics Among Carpentry A p p r e n t i c e s :

Implications for School Teaching. ICME 5 9 Mathematics for All

Papers, p. 7.

9 Barcellos, A. (1981): Universal Primary Education. Te a c h i n g

Teachers, Teaching Students. Steen & Albers eds., Birkhäuser, p.

123.

commutativity, oblivious of the need for simple bookkeeping.

The plenary address of Ubiratan d’Ambrosio on

*Ethno-Mathematics* places this discussion in an even more

fundamental setting. Can we develop mathematics in

countries with different cultural traditions, which may be

quite different from the mathematics developed in Greece

and Western Europe? Would such a mathematics serve

the needs of other cultures better?

There are other things we could do to understand better

the gap between school mathematics and our socio-cultural

context. I have, for example, studied the textbooks

generally in use in the Philippines. They are either direct

copies or relatively mild modifications of textbooks in the

Western world. There is little awareness that there is a different

context outside. I have also analyzed test items

given in an assessment study of sixth graders throughout

the Philippines. There were 40 items, 10 on computational

skills, 12 on concepts such as place notation, 10 on routine-

type applications, 8 on analysis of data. They are the

usual types of exercises we put in textbooks to develop

manipulative skills. The problem is that most of the

concepts or skills developed would have no relevance for

the young person dropping out of school after six years.

**II. The Historical Development of Institutions in**

**Developing Countries**

The above analysis highlights the serious gap between

school mathematics and the socio-cultural context of developing

countries. I would like to locate the primary cause of

these problems in the history of our social and cultural institutions.

The first I would call *vertical,* that is, the relationship

between *similar* institutions, like schools, in *different*

*societies.* The second I would call *horizontal,* that is the relationship

between *different* institutions in the *same country.*

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I would also add a third relationship of *rootedness,* that is,

the insertion of these institutions in the socio-economic-cultural

matrix that underlies the given society.

To understand the situation, we should note that the history

of social, industrial, educational institutions in most

developing countries has been guided mainly by *vertical*

relationships. For example, to understand the school system

in the Philippines, it is less necessary to understand

the local social and cultural situation as it is to understand

the American school system and to note the adaptations

that have been made. If one were looking at Malaysia, one

would turn to the British school system. The same can be

said regarding the system of hospitals and health care,

financial institutions, etc.

If we were to picture the development of institutions like

that of a tree, where the institutions represent the leaves,

branches, fruits (the visible developments in society), then

we would picture our institutions like upside-down trees.

They are rooted not so much in the socio-cultural matrix of

the country, as in the socio-cultural matrix of model countries

abroad. This whole pattern of development of institutions

according to *vertical* relationships has produced what

is usually called the modernized sector, which includes the

best of the educational system. This modemized sector is

not a product of the socio-cultural matrix of the country, but

is very much a foreign transplant. Much of the air it

breathes is imported air, whether this be imported curricula,

imported talent, or imported management techniques.

What are the consequences of this development guided

mainly by vertical relationships?

(1) The development of our institutions is directed by their

sources abroad, not by complementary institutions or

needs in our society. Thus a leading educator is invited to

an international conference and is introduced to the new

mathematics or to computer assisted learning or distance

educational techniques and he comes back and wishes

immediately to implement what he has learned. Because it

is the latest and the best, whether or not it has serious relevance

to the country. The norms for judging the value of our

schools are often in terms of how well our students do in

graduate school in England or the United States, rather

than in how they fit and contribute to the larger society.

(2) The analysis in the earlier part of the paper shows that

*horizontal* relationships and *insertion* into the local culture

are weak. The mathematics classroom is totally unaware of

the «street» mathematics of the young pupils inside. There

is no linkage between the needs of a rural community for

better bookkeeping and the new mathematics being taught

to its children.

These results define for me a crucial task for the future:

how to develop better horizontal fit and better rootedness

in the socio-cultural context.

**III. Tasks that We Might Attempt to Improve the**

**Situation of Universal Mathematical Education**

**in Developing Countries**

I would like to propose *two tasks.* One is in the area of bringing

about a cultural shift in our countries. The second is a

more specific task of working towards a better integration

between universal mathematical education and the outside

world to which our students will go.

*(1) Bringing about a cultural shift in our countries*

I would propose that mathematics educators, together with

other educators and other leaders of society, take up this

need of having the social and cultural institutions of the

country be better integrated with one another and be better

inserted into the culture. This is not to deny the importance

of linkage with other institutions in the Western world or in

the other developing countries. It is simply to accentuate

the need to have these imported developments be integrated

into the social and cultural milieu of the country. It

is important for us to accentuate the high cost to development

of this lack of integration. Whether the cost be in

terms of brain-drain or in terns of graduates who cannot

find jobs in our society. Or a young population without the

skills for a productive life.

*(2) How to proceed concretely to bring about a better*

*integration between the universal mathematics*

*curriculum and the world into which our students*

*will be going*

In the proceedings of the Fourth Intentional Congress on

Mathematical Education, Shirley Frye has a suggested

mathematics curriculum for students who leave school at

an early age.

«The particular goals of a minimal mathematics education include

having:

1. a sense of number;

2. the ability to quantify and estimate;

3. skills in measuring

4. usable knowledge of the basic facts

5. the ability to select the appropriate operation to find a solution;

6. the ability to use a calculator to perform operations

7 . a ‘money-wise’ sense.

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The last skill relating to being ‘money-wise’ is most important since

an individual should have the ability to decide whether wages are

being paid correctly and if purchasing transactions are fair. «10

The proposal is that mathematics educators in our country

attempt the following tasks:

(1) Study the actual articulation in the curriculum of

the seven goals stated by Shirley Frye, that is,

how are they translated into mathematics con-

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10 Frye, S. (1983): Suggested Mathematics Curricula for Students

Who Leave School at Early Ages. In Praceedings of the Fouth

International Congress on Mathematical Education. Zweng et al.,

eds., Birkhäuser, p.32.

cepts, into mathematical skills, into problems in the textbooks.

(2) Look for how these goals appear in the social environment

of the student. That is, what tasks or events

bring about a sense of number or require the ability to

quantify and estimate and so forth (cf. Carraher and

Schliemann papers).

(3) We should then compare the two, that is, the curriculum

inside the school and the appearance of these goals

and concepts in the outside world and see how we can

bring about a better integration between the two.

(4) Share results with one another and detemmine to show

progress in ICME 6.

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**Conclusions Drawn from the Experiences of the**

**New Mathematics Movement\***

Peter Damerow and Ian Westbury

**Introduction**

«Mathematics for All» is the title of our theme group. The

group’s implicit goal is the consideration of an issue which

is fundamental to the idea of general education - and to be

«successful» the theme has to be engaged in a way that

goes beyond the limitations which appear to be endemic to

contemporary curriculum studies: preaching the necessity

of a new program and then arguing in hindsight that widespread

adoption was, from the beginning, an unrealistic expectation.

Needless to say, no such goal can ever be

achieved in one discussion, but the issue which the group

canvasses does not go away because it is a difficult one to

address.

It is easy to make pointed comments on the emptiness of

the kind of conclusion from curriculum research we parodied

above. But at the same time, there is a sense in which

such a conclusion is an inevitable result of a web of problems

that all educational reformers, and particularly subject-

based reformers, have faced as they think about the

scope of their work and their agendas. *There is a need to*

*find ways of changing the contents and conditions of general*

*education as part of a larger concern for changing the fit bet -*

*ween the work of the schools and the rapidly changing scien -*

*tific and social demand for qualifications,* but how this is to be

done is clearly totally elusive.

Within mathematics education the history of the so-called

Knew» mathematics is one instance of this larger issue.

There was a world-wide movement to introduce a Knew»

mathematics—but we know that the effect of these efforts

was negligible: Little has changed in classrooms and the

change that has occurred bears little relationship to the

goals of the original reform movement. This fact defines the

parameters of our problem. Changing the fundamentals of

general education in a goal-oriented, systematic, time-limited

way poses innumerable unsolved issues. It may be true

that the needs of rapidly-changing societies do not allow us

to base the contents and the practices of general education

on tradition and the inner experience of the school. It may

be true that it is the task of our times to move the school

system from being a relatively autonomous, developing

social system into a guided institution which is continually

adapted to changing needs by planning decisions and

administrative action. But the fate of the new math

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\* An enlarged version of this paper is published in the Jour nal of

Curriculum Studies, 17, 1985, pp. 175 -184.

ematics movement shows that so far we only know what

the task is. We have not created the means needed to

address it.

*Mathematics for AID - As a Program of Reform*

The reason for the presence at ICME 5 of a group discussing

mathematics for all and the absence of a group

addressing how new mathematics might be introduced into

the school is intimately related to the experience of the new

mathematics movement. At the height of that movement it

was commonly assumed that the new mathematics *was* a

mathematics for all. It was, for instance, the first addition to

traditional arithmetic and so the first alternative to the Folds

curriculum of the universal primary school for about 200

years. But as Damerow et al. have shown, the claims that

were sometimes made for the real-world relevance of the

new mathematics, the claims that would have been needed

to establish a belief that the new mathematics had a

real-world relevance, were often even more difficult to sustain

than those associated with traditional mathematics.1

More important, the «new mathematics» showed decisively

how problematic major change in a subject can be:

While there were some admirable experiments which showed

what might have been done in elementary schools

under the name of new mathematics, «experimental» outcomes

were not generalized to school systems as wholes.

Perhaps the best that can be said about the widespread

introduction of the new mathematics was that its teaching

did not inhibit the traditional teaching of arithmetic too

much.

And what was the result of the implicit, though often tacit

assumption of the period in which new mathematics was

the vogue that the yield of a a “subject” reform could be

secured equally in every culture independent of the degree

to which formal education was institutionalised? There can

be no doubt that Dienes’ efforts to introduce a modern

mathematics project in Papua-New Guinea in the mid-

1960s was as successful as he claimed to be. But 20 or so

years later, after intensive attempts to adapt Dienes’ curriculum

to local conditions, Souviney comments, in his discussion

of mathematics education in Papua New Guinea,

that it is not enough for the educational establishment

«simply to institute a selection procedure which identifies

and promotes children who exhibit ‘high’ e d u c a t i o n a l

potential while failing to address adequately the needs of

the vast majority.» «Increased attention must be paid to the

needs of [the group of children who will return to their villages

after completing community school] who presently

constitute 70% of the community school graduates.»2 By

implication, something much larger than the new

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1 Damerow, P. et al. (1974): Elementarmathematik: Lernen fur die

Praxis? Ein exemplarischer Versuch zur Bestimmung fachuberschreitender

Curriculumziele. Stuttgart: Klett.

2 Souviney, R. J. (1983): Mathematics Achievement Language and

Cognitive Development: Classroom Practices in Papua-N e w

Guinea. In: Educational Studies in Mathematics 14.

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mathematics seems to have surfaced in the context which

rendered the Dienes’ «modern» program moot.

This instance is from the developing world but parallel instances

can be found in the industrialized world. It is cultural

and contextual factors which, as they interact with

mathematics itself (or any subject area), pose the most

serious problem which slogans like «new mathematics and

«mathematics (or science) for all» must face. Do we keep,

for example, the highly selective frameworks and methods

of traditional mathematics education but give up the privileged

position of the subject as part of the core of general

education? Or do we seek to keep mathematics at the core

of the curriculum but find a way of teaching the subject to

all students?

*Two Alternatives*

What might these alternatives mean — for ideology and for

reality? The first possibility would make « mathematics» a

subject of early specialization, with the present role of the

subject being taken over by physics, technical education,

economics, etc. In this way most students would experience

mathematics as a useful tool and concentrate on creative

mastery of and application of the problem-solving techniques

which result from mathematical thinking. The core

of mathematics, its ideas, conceptual structures, methods

of proof and the like, would only be taught to those who

specialize in some way or other in the subject.

This suggestion comes close to the actual situation of

mathematics education in many nations. In the Federal

Republic of Germany, for example, only 11% entering university

students enroll in the subject areas of mathematics

and natural sciences; 21% enter programs in engineering

and the remaining 68% range over all other fields. For most

students, therefore, mathematics is but a potential tool, and

all we are saying is that mathematics programs in school

might reflect this situation. But mathematics in Germany is

not taught in this way and most mathematics teachers

would probably deny the possibility and would instead

emphasize the specialist, pure mathematical aspects of

their work. It is worth, however, mentioning that the alternative

possibility we sketched above once played a substantial

role in curricular thinking. When the influential

«Verein zur Förderung des mathematisch-naturwissenschaftlichen

Unterrichts» (Association for the Support of

Mathematics and Science Education) was mooted in 1890,

the majority of its potential members argued *against* a

continuing role for pure mathematics as a core subject in

the high school curriculum. At the founding of the association

in 1891 a motion was passed against the teaching

of pure mathematics. It was only as a result of the later

influence of the Gottingen Professor of Mathematics, Felix

Klein, that this policy was changed. But Klein promised

that, in the near future, there would be reintegration of

mathematics with its applications in other sciences and

practices and therefore the continuation of the traditional

kind and place of mathematics education was justified as

an interim measure. The possibility of such a new mathematics

became the goal of the association - and this

t u r n-o f-t h e-century anew mathematic» was profoundly

influential both in Germany and internationally. Klein, of

course, did all that he could to promote the development of

such a mathematics with its implied integration with the

domains of practice but he failed and given this it can be

argued that the case for the abandonment of a «pure»

mathematics for all is still as relevant today as it was in

Gemmany in the 1890s.3

The second alternative we mentioned above is keeping

mathematics as a fundamental part of the school curriculum

but finding a way of teaching it effectively to the majority.

What problems must be faced as we contemplate this?

We have first to consider the fact that mathematical education

in the traditional sense had its origins in a specific

cultural tradition. The canonical curriculum of Traditional

mathematics» was created in the l9th century as a study

for an elite and this pattern persists. In Germany, for

example, «advanced» school mathematics (i.e. analytical

geometry and calculus) are only offered in Gymnasium

which in 1981 enrolled only 10.9% of the 18-year-old age

cohort. And as enrolments in Gymnasium have increased it

has seemed necessary to relax the once-fixed expectation

that all students in the Gymnasium would complete a full

program in school mathematics in order to maintain «standards.

» In 1977—78 only about 70% of German students in

grade 12 (the second last year in the Gymnasium) were

taking the traditional sequence in mathematics, i.e. about

10% of the relevant age cohort.4 While the particular curricular

patterns of different societies vary, mathematics is

constructed in most places in ways that lead to few of the

students who begin mathematics in the early years of the

secondary school continuing to take it in their last secondary

years. The separation of students into groups who

are tagged as «able-mathematically» and, «less abler is

endemic. The heart of mathematics teaching is, moreover,

widely seen as being centered on this curriculum for the

able - although all students *begin* the study of mathematics.

There are some important differences between countries in

their retention rates but in the main we see the patterns

which were created in the l9th century still holding; advanced

mathematics is a study for a few.

The transfer of the European mathematics curriculum to

developing countries was, of course, closely associated

with the creation of schools for elites by colonial administrations.

Under these circumstances it seemed natural to

simply copy

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3 Lorey, W. (1938): Der Deutsche Verein zur Förderung mathematischen

und naturwissenschaftlichen Unterrichts, E. V. 1891-1938.

Ein Rückblick zugleich auch auf die mathematische und natur vissenschaftliche

Erziehung und Bildung in den letzten fünfzig

Jahren. Frankfurt a. M.: Salle.

4 S t e i n e r, H.-G. (1983): Mathematical and Experimental Sciences in the

FRG - Upper Secondary Schools. Arbeiten aus dem Institut fur

Didaktik der Mathematik. Universität Bielefeld, Occasional Paper 40.

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European patterns — but, as Souviney makes clear, it is

quite another problem to build a system of mass education

in the Third World and embed mathematics education in

the specific cultural contexts of that world.5 How is this to

be done? Is a mathematics curriculum desirable if it causes

students in these countries to develop the antipathies

against mathematics which are commonly found in

Europe—but in social contexts which lack the

culturally-based consensus found in Europe that abstract

mathematical activity is as good as such and must therefore

be supported even if it seems on its surface to be useless.

Even if this argument is inappropriate, it does raise

the question of the relation between mathematics and culture

which may be the first problem which arises when the

idea of mathematics for all is raised as a platform for a program

of action.

We must consider, second, the problem of conceiving,

even for industrialized societies, a mathematics which is

appropriate for those who will not have contact with pure

mathematics after their school days. Most current attempts

to face the problem of a basic, «minimal-competency» curriculum

reduce the traditional curriculum by pushing out

every mathematical idea and every possible difficulty to

make it feasible to teach the remaining skeleton to the

majority. But there is only a limited basis for an appeal to

«utility» as an argument or a rationale for curriculum building

to support this approach. *Students who* win *not have to*

*deal with an explicit pure mathematics in their adult lives but*

*will face instead only the exploitation of the developed pro -*

*ducts of mathematical thinking {e. g program packages) will*

*only be enabled by mathematics instruction in school if they*

*can translate the mathematical knowledge they have acquired*

*into the terms of real-life situations which are only implicitly*

*structured mathematically.* Very little explicit mathematics is

required in such situations and it is possible to survive

without any substantial mathematical attainments whatsoever.

6

But is this kind of argument a way of making the case for

the first alternative we considered earlier? And is that

alternative the only one we might be left with? It may be,

but if this is true it would seem to deny the significance of

the topic we are concerned with. Thus we might observe

that to draw this kind of conclusion is to look backwards

in order to determine educational aims for a future. T h e

facts we have cited suggest that a program of mathematics

for all implies the need for a *higher* level of attainment

that has been typically produced under the conditions of

traditional school mathematics—and this is especially

true for mathematics education at the level of general

education. To put this another ways we might claim

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5 Souviney, R. J. (1983): See Footnote 2.

6 See, for example, Bailey, D. (1981): Mathematics in Employment

(16-18) (University of Bath); Sewell, B.: Use of Mathematics by

Adults in Daily Life: Enquiry Officer’s Report (Advisory Council for

Adult and Continuing Education, London); ADVISORY COUNCIL

FOR ADULT AND CONTINUING EDUCATION (1982): Adult’s

Mathematical Ability and Performance. London: ACACE.

that *mathematics for all has to be considered as a program to*

*overcome the subordination of elementary mathematics to*

*higher mathematics, to overcome its preliminary character,*

*and to overcome its irrelevance to real-life situations.*

*The Mathematics Classroom*

A number of recent research studies have suggested that

it is very likely that the structure of classroom interaction

itself creates ability differences among students which

grow during the school years. What might cause these growing

differences in mathematical aptitude? The simplest

explanation rests on the assumption that these differences

are due to predispositions for mathematical thinking - with

the implication that nothing can really be done to change

the situation. But this explanation is too simple to be the

whole truth.

The understanding of elementary mathematics in the first

classes of primary school is, we know, based on preconditions

like the acquisition of notions like conservation of

quantity which are, in their turn, embedded in exploratory

activity outside the school. As long as the genesis of general

mathematical abilities is as little understood as it is, the

possibility that extra-school experience with mathematical

or premathematical ideas influences school learning cannot

be excluded. Furthermore, we know from classroom

interaction studies that the differences between intended

mathematical understandings and the understanding which

is embedded in normal classroom work is vast. We cannot

exclude the possibility (aggressively suggested by

Lundgren) that classroom interaction itself in fact produces

growing differences in mathematical aptitude and achievement

by a system of positive feedback mechanisms which

increase high achievement and decrease further low achievement.

7

Such classroom level phenomena also interact in profound

ways with curricular factors. The English Cockcroft

Committee on teaching of mathematics pointed to the

significance of the notion of *curricular pace* as a critical

variable affecting school achievement. If a pace necessary

to cover an overall curriculum (i. e. to reach the levels of

understanding necessary for, say, English sixth-form work)

is to be sustained, a given rate of coverage is required of

teachers. The Cockoroft Report claims that in England

there has been little change in this implicit rate since the

pre-war years despite the fact that the cohorts of children

ostensibly learning mathematics are now drawn from the

second and third quartiles of the general ability distributions

(as a result of increased access to secondary schooling).

The result, the Cockcroft Committee has suggested,

is an overall rate of coverage and pace of instruction which

is far too fast for many if not most pupils. For such pupils

math-

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7 Lundgren, U. P. (1977): Model Analysis of Pedagogical Processes.

Stockholm Institute of Education, Studies in Curriculum Theory

and Cultural Reproduction 2. Lund: CWK Gleerup.

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matics as a subject is abstract, mechanical, and procedure-

based, and success is hard to come by.8

*Implications*

Our discussions to this point make it clear that to talk of

mathematics for all entails an intention to change general

attitudes towards mathematics as a subject, to slow the

pace of teaching, to eliminate divisions between those who

are friends of mathematics and those who are not, to diminish

variance in the achievement outcomes of mathematics

teaching. This, in its turn, involves us in an analysis of

the forces found in social contexts, curricula, and teaching

inasmuch as it is these forces together which create a set

of frames which *create* situations in which mathematics becomes

one of the subjects in the secondary school in which

*selection* of students into aptitude and ability groups is an

omnipresent reality from almost the earliest days of secondary

schooling.9

As we ponder what such notions might mean we have to

address three very different levels of analysis of the mathematics

curriculum.

*1. The distribution of knowledge.* With the implication that we

reject assumptions that mathematical knowledge is the

prerogative of some cultural communities and not others

and instead see mathematics as something potentially

appropriate to all people. At this level the idea of mathematics

for all involves

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8 Cockcroft Committee (1982): Mathematics Counts. Report of the

Committee on Inquiry into the Teaching of Mathematics in Schools

under the Chairmanship of Dr. W. H. Cockcroft. London: HMSO.

9 For an English study see Ball, S. J. (1981): Beachside

Comprehensive: A Case Study of Secondary Schooling.

Cambridge: Cambridge University Press.

issues of cultural exchange and intercultural understanding

- within and between social groups and geopolitical communities.

*2. The school system and its integration into the society.* The

idea of mathematics for all poses an issue of general education

rather than elite education. At this level the idea of

mathematics for all involves us in a rethinking of the traditional

concerns of mathematics education—away from the

«needs» of elites and towards the needs of both elites and

average students; our sense of crowning achievements

would come not from the achievements of the few but from

the achievements of the many. Our index of accomplishment

would be the overall *yield* of the school system (i. e.

the percentage of a cohort mastering given bodies of

content and skill) rather than content and skill achievement

of the most able.10

*3. Classroom interaction.* Mathematics for all is a problem of

opportunities to learn and their relationship to the dynamics

of the learning process. This level of concern must include

an analysis of the assumptions, patterns, and practices of

within-school division of students into ability groups, sets

and streams — for setting/streaming is ubiquitous in

mathematics education from the early secondary years.

It is quite clear, of course, that these levels are very closely

linked together and that they serve to do no more than

*define* the dimensions of the complex but coherent problem

labelled by the slogan «mathematics for all.»

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10 For a conceptualization of «yield» see Postlethwaite T. N. (1967):

School Organization and Student Achievement. Stockholm

Studies in Educational Psychology 15. Stockholm: Almquist &

Wiksell.

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**Implications of Some Results of the**

**Second International Mathematics Study**

Howard Russell

Peter Damerow has introduced some important suggestions

which must be considered carefully if there is to be

any progress made towards the goal of «mathematics for

all» in the classrooms of our various countries.l His suggestions

regarding pace and coverage are grounded in discussions

and presentations which have been summarized

in Mathematics Counts2 and they have emerged as debatable

issues in the period of the mid-eighties. The present

paper is offered as a contribution to the debate and the

data presented constitute an important part of a recently

established data set.

The Second International Mathematics Study (SIMS3)

data constitute this data set and these data show substantial

variations among countries in the extent to which

mathematics education is provided for all students. The

data which reveal these variations are coverage data and

retention data and the product of these two, i. e., coverage

x retention, are presented as «yield.» While it is true that

there is not much new in the concepts involved, the data

are sometimes revealing when presented in the new form

used in SIMS. These data may prove to be helpful as we

proceed to consider a shift in mathematics education from

content for the elite, to a more marketable,

mathematics-for-all. The SIMS data may be helpful because

they foreshadow relationships among key variables

which need to be manipulated if the suggested shift in

mathematics education is to take place in the mathematics

classrooms of the world, as opposed to taking place only in

the minds and the writings of educational leaders.

In this paper I propose to consider the SIMS pop A

data first, i. e., from the 13-y e a r-olds, and to use

these data to suggest that already we have mathem

a t i c s-f o r-all at the elementary level in many countries.

There are variations in what mathematics is, at

this level, but whatever it is, all youngsters get it. T h e

SIMS pop B data, i.e., from 18-y e a r-olds more or less,

show signs that mathematics for an elite is still the

prevailing plan in most countries. It is at this age

1 Damerow, P. (1984): Mathematics for All - Ideas, Problems,

Implications. Paper presented to the ICMI Symposium at the

International Congress of Mathematics. Warsaw, August, 1983.

Zentralblatt fur Didaktik der Mathematik, 16, pp. 81-85.

2 Mathematics Counts (1982): Report of the Committee on Inquiry

into the Teaching of Mathematics in Schools under the Chairmanship

of Dr. W. H. Cockcroft. London.

3 Travers, K. et al.: The Second International Mathematics Study,

Vol. 1, (for publication 1985) Pergamon Press.

level that some countries have tried to move towards

mathematics-for-all and comparisons of these countries

with the others which have not moved so far gives us good

information with which to plan future strategies for implementing

a mathematics-for-all curriculum throughout the

secondary school years. But first let us look at pop A

where, although it is true that virtually all youngsters are

retained in school, and in mathematics, it is nevertheless

clearly evident that different amounts of mathematics and

different parts of mathematics are taught.

The pop A coverage data are shown in Table 1. These are

topic by topic coverage data which are presented in country

rows (see Figure 1). Thus, the coverage index for arithmetic

for Country D is .74 and the standard deviation is .12.

This means that the average teacher with a pop A class in

Country D claims his/her students have been taught the

material involved in 74% of the items under consideration.

The standard deviation is the measure of variation in coverage

C among teachers in Country D on that particular set

of arithmetic items.

Since it is true that virtually all youngsters in a nation

eventually make it through the pop A level, the variation in

coverage is the main source of vacation in the mathematics

which is offered to pop A students. The data in Table 1

show substantial variation from country to country, and as

well the standard deviations show sizeable variations

among classes within countries. It is true then that there

are many youngsters who miss out on instruction in a substantial

amount of the content defined by the SIMS Pool,

even in such popular topics as arithmetic. Since this is a

phenomenon which affects all countries, it may be one of

the features of the present mathematics curricula that is difficult

to change. This would be especially interesting if it

can be shown that the youngsters who miss out on coverage

are the ones who cluster in classes which spend time

on unlearned content of earlier grades and/or require more

than the usual amount of time to cover most topics. If this

is true then it would appear that the teachers of slow

classes have adjusted the pace to the needs of their students.

Although retention is uniformly high through the pop A

years, it is nevertheless true that there is considerable

variation in the amount of time taken by students to get

through to the pop A level. This suggests variation in pace.

Table 2 shows the mean age in months for both the pop B

and the pop A students. The first message which emerges

from this table is that some countries appear to take much

less time to get through to the end of the pop A year than

others. Country A requires only 162 months. Country K

requires 166 months, and the other countries spread themselves

over a range of many months. What interpretation

such data have for deliberations about mathematics-for-all

may be clearer when the associated p-values for student

performance are presented at some future point in time. In

their absence it appears that mathematics-for-all can be

pursued as effectively by following the lead of countries

with a low mean age as those with high mean age.

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The above argument seems to be in conflict with the

hypothesis proposed by Cockoroft4 that the pace of mathematics

education must be slowed down if we are to keep

enough students in the mathematics courses to accurately

label it mathematics-for-all. Table 2 showed us there is

wide variation in pace because there is wide variation in

age among counties. If pace were related to retention at

pop A or retention later then the age variation should be

somehow related to retention. We have made the supposition

that pace, for our purposes, is really coverage divided

by time (age in months) and we have found that pace at

pop A is unrelated to retention or any other variable of central

interest. What seems to have happened then is that

most contries have maintained a «promotion-

by-performance» standard policy and this, in turn,

has lead to retardation or failing of significant numbers of

students. The failing of students, or the forced repetition of

grades by students thus shows up a slowing of pace, but

this type of slowing the pace seems not to have provided

any positive outcomes. Another way of stating it is that no

violence is done to the concept of mathematics-for-all

when age promotion rather than the slower paced, promotion-

byperformance standards is adopted at the elementary

grade levels. Indeed, it can be postulated that violence

may be done to the concept of mathematics-for-all if annual

p r o m o t i o n-b y-performance standards has the effect of

retarding student progress to the point that many students

give up schooling and mathematics the minute that they

attain the legal leaving age. Clearly, more data and more

study are required before such generalisations can be

widely accepted.

The story is different at the pop B level. Table 3 shows the

coverage data means and standard deviations. Again,

there is wide variation both among contries and within. But

these coverage data cannot speak to the issue of mathematics-

for-all until they are augmented by the retention

data. Table 4 shows coverage, retention and yield data for

each country which possessed the necessary data. Now

the variation among countries is even more evident, and

the possibility of finding relationships seems promising.

In order to get one central relationship quickly we

can look back at Table 2 which shows the mean age

for the students in pop B. Along with these mean

ages I have introduced the difference in ages between

pop A and pop B. Now it seems evident that

the countries which are attaining the highest yields

are those which provided their students with the longest

time interval between pop A and pop B. namely

Countries E, O. A and K. What emerges then is a

tentative conclusion which supports the Cockcroft

hypothesis and as well reinforces the Damerow position

elaborated in his paper on mathematics-f o r-a l l .

It must be conceded that Countries C and F possess

high yield, but a more thorough analysis of these

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4 Cf. Footnote 2.

countries in the Volume I of SIMS indicates that Country C

has the lowest level of attainment by far and hence what is

taught is no indication at all of what is learned. Country F

also pays a price of low student attainment, and for purposes

of the present argument that country as well as

Country C is discounted. The SIMS data not only offer

assistance in our debate by providing empirical support for

the Cockcroft and the Damerow positions but also they

help us to identify at least tentatively some of the boundaries

beyond which this central pace hypothesis may be falsified.

Perhaps the first boundary worthy of attention is the one

which separates pop A from pop B. On the surface it

appears that the «slower pace produced higher yield»

hypothesis breaks down between pop A and pop B. An

alternative explanation of the data may be that the type of

slowing down which is produced by annual performance

promotions, i.e., failing the lowest students, has its own

negative effects which in turn counteracts any benefits of

the apparent slowdown. Another point arises here which

requires attention. It is the possibility, indeed probability,

that pace is a variable which has an optimum level such

that either an increase or a decrease from it is likely to produce

a reduction in learning. It is also likely true that optimum

pace is unique to individuals, and hence a general

increase in pace will be beneficial to some students and

detrimental to others. If we return to the data now, and

focus on the fact that the two countries with the lowest

mean age at pop A are also high in yield at pop B. we may

suggest that the pace in the elementary grades could be

too slow in many countries. If this were not true then why

would the apparently fast-paced nations do so well? When

we move to the pace issue at the secondary level the situation

is quite clearly reversed and the nations with the slowest

pace through the pop B years are the ones which

seem to be benefitting the most. Thus, the

Cockcroft-Damerow hypothesis is most likely to be helpful

in our analysis of the secondary school program.

The SIMS data provide another way of looking at the

pace issue. The «opportunity-t o-learn» data have been

aggregated in a matrix form which reveals item-by-i t e m

and class-b y-class coverage. In the case of Country E,

for instance, there are five matrix displays shown in

Figure 4, one for each of the item clusters arithmetic,

algebra, geometry, probability and statistics and measurement.

Each row corresponds to a test item in the

SIMS Pool and each column corresponds to a class

(Country E had a sample of 85 classes), and hence

there are 85 columns in each of the matrix displays.

The reason that this form of display takes on meaning

in the pace debate is that the ordered columns clearly

distinguish between classes with higher and lower

coverage. This same distinguishing among classes on

coverage is, in fact, a distinguishing among classes on

the variable we have called pace. The fast-p a c e d

classes are on the left; the slow-paced on the right. My

discussions with classroom teachers about these

matrices has reinforced my own view that we have at

present wide variations in pace among our

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classes and that these variations are likely the result of the

classroom teachers’ attempt to accommodate the pace to

the capabilities of the students. The fact that the matrix displays

for virtually all countries reveal similar variations in

pace among classes suggests that there is some common

reality tied to such data. What that reality is, and how it can

be utilized in moving towards a viable mathematics-for-all

is the present problem. My feeling is that the reality here is

precisely the one we are looking for, namely that pace can

be slowed, and in fact is already slowed, for the advantage

of the below average student.

The potential for benefits from manipulating pace is suggested

by a consideration of some matrix displays at pop

B. Figure 5 shows Country E and Country F. and the feature

of these displays which is most evident at first glance

is the low coverage in «analysis» for Country F compared

to high coverage for Country E. If Country F should prefer

to cover the same amount of analysis as Country E it could

consider the possibility that the extra time in secondary

school (55 months versus 47 months) could do it. Also, the

possibility that lower retention could be a factor must be

considered. My own speculation is that the slower pace in

secondary school in Country E accounts for both higher

coverage (than Country F) and for higher retention (than

other countries except Country F).

There are insufficient data at the present time to warrant

definitive claims, but the debate is just beginning now, and

I believe the SIMS data and their various new forms of

aggregation will eventually lead us to general relationships

among truly fundamental educational variables. If we can

indeed manipulate learning through the manipulation of

pace then we may begin the process of moving towards

mathematics-for-all even before we know what the nature

of the content should be.

Table 1: Implemented Coverage Indices C\* (see Figure I

below)

I wish to close my contribution to the debate on mathematics-

for-all with a brief and simple analysis of the issue

of content selection. I wish to suggest that a rationale for

mathematics which appeals to the «new» clients of mathematics,

i.e., the middle level students who constitute the

backbone of society, must be carefully constructed. l believe

that a market oriented rationale is quite appropriate.

Such an orientation is likely to be widely accepted if it is

true, and if it can be shown to be true, that the students in

the middle and below the middle on our mathematics competence

scale will be required to use mathematics, or

«mathematics-for-all» in their chosen work in the marketplace.

Some observers suggest that

mathematical-skill-based sophistication in the marketplace

will increase dramatically as hi tech moves into a dominant

market position. Under such circumstances it is true that

mathematical-skill-based sophistication should be introduced

into the core of the mathematics curriculum which

Damerow is proposing. How to identify the precise ingredients

in this new core is not known, but there should be

mechanisms available for arriving at a best guess.

While it is true that many observers see an increasing

need for mathematical sophistication on the part of the average

worker, there are other observers who suggest that

the «user friendliness» of computers yet to be introduced in

the marketplace will place fewer demands of a mathematical

nature on the typical citizen in the workplace. I have not

yet seen a clear answer for this issue. My intuition suggests

dramatic increases in sophistication but I have no

data to support my intuition. Perhaps mathematics educators

should be prepared to collaborate with representatives

of government and business in an effort to identify the

generic skills needed in our new core mathematics-for-all.

Then we can hope to make significant progress in the

quest for mathematics-for-all.

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Table 2: Mean Age in Months

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Country Pop A Pop B Diff.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

A 162 217 55

B 171 218 47

C 171 217 46

D 170 217 47

E 167 222 55

F 168 215 47

G 170 214 44

K 166 223 57

L 170 217 47

N 168 214 46

O 167 228 61

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Table 3: Implemented Coverage Indices C

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Population B

POP B ALG ANAL NUM SETS FIN P&S GEO WGTD

(25) (45) (19) (7) (4) (7) (29) MEAN (SD)

MN (SD) MN (SD) MN (SD) MN (SD) MN (SD) MN (SD) MN (SD)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

CountryA 100 (1) 94 (7) 82 (32) 95 (9) 99 (0) 83 (26) 85 (28) 91

E 91 (10) 92 (10) 81 (15) 83 (19) 93 (6) 72 (16) 74 (23) 85

J 92 (10) 94 (11) 88 (12) 85 (22) 52 (10) 86 (11) 68 (28) 85

K 92 (19) 88 (13) 87 (13) 87 (16) 83 (6) 85 (17) 70(37) 84

L 91 (10) 87 (13) 76 (21) 89 (11) 63 (3) 44 (16) 76 (24) 82

D 85 (12) 86 (13) 73 (18) 55 (19) 60 (6) 67 (20) 62 (30) 75

O 84 (18) 83 (18) 80 (21) 56 (28) 83 (4) 73 (16) 55 (37) 75

C 62 (12) 73 (16) 68 (14) 75 (8) 70(22) 77 (20) 62 (19) 68

H 87 (16) 57 (24) 80 (19) 81 (19) 55 (17) 46 (30) 52 (37) 65

B 70 (14) 60 (27) 69 (13) 75 (12) 85 (6) 81 (7) 53 (28) 64

N 56 (20) 73 (20) 50 (21) 28 (17) 43 (4) 17 (6) 35 (35) 52

F 81 (22) 35 (33) 74 (28) 66 (27) 9 (12) 28 (34) 43 (38) 51

MEAN 82 (13) 77 (18) 76 (10) 73 (19) 66 (25) 63 (24) 61 (14) 73 (13)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\* (Not collecting OTL data were: Belgium (FR), Hong Kong, Nigeria and Scotland.)

Table 4: Coverage Retention and Yield

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Population B

Country Ct R Y

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

A 89 12 107

B 59

C 65 50 325

D 71 6 43

E 84 19 160

F 49 30 147

H 65 12 78

J 84 11 72

K 84 15 126

L 84 10 84

N 46 6 28

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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Figure 1 : Item Classification Figure 5 : Country E

Figure 4 : Country E Figure 6 : Country F

**Implications of the Cockcroft Report**

Afzal Ahmed

1. Mathematics permeates the whole society and its use

seems to assume ever increasing importance as our societies

become more technological and complex.

Mathematical methods and thinking are not the prerogative

of scientists, engineers and technologists only, they are

used by people in making everyday decisions. Their use in

analysing individual behaviour, to study opinions and attitudes

is also increasing. The place of mathematics in both

primary and secondary school curricula for all pupils is also

evidence of general agreement that the study of mathematics,

along with language is regarded by most as essential.

Is it not ironic that the subject which has assumed such

prominence in society is also that which is most closely

related to failure? Low attainment in mathematics has certainly

been at the centre of the education debate in Britain

throughout this century.

2. The Committee of Inquiry into the Teaching of

Mathematics in Schools in England and Wales, of which I

was a member, was set up in 1978 as a result of concern

about the mathematical attainment of pupils.

I must be careful not even to attempt to summarise the

report of this committee «Mathematics Counts», which was

published in January 1982.1 I would, however, use relevant

evidence from this report to discuss the aims and focus of

the Curriculum Development Project for Low Attaining

Pupils in Secondary Schools Mathematics which I am

directing. This is a three-year project which commenced in

September 1983 and is one of three projects commissioned

by the Department of Education and Science as

a result of the concern raised in the Cockcroft Report (The

other two projects concern the assessment procedures for

low attaining pupils.) I shall not confine this paper to the

work of this project nor does the paper contain official

views of the project. I merely use the findings of the

Cockcroft Report and my own project to support the views

expressed in this paper.

Paragraph 334 of the Cockcroft Report begins with the following

sentence:

Low attainment in mathematics can occur in children whose general

ability is not low. «

This, of course, is true of adults too, and this fact is illustrated

quite vividly in Brigid Sewell’s report on

\_\_\_\_\_\_\_\_\_\_\_\_\_

1 Great Britain Department of Education and Science (1982):

Mathematics Counts Report of the committee of Enquiry into the

Teaching of Mathematics in Schools the Cockcroft Reporter

HMSO.

the use of mathematics by adults in daily life.2 This enquiry

was undertaken in association with the Cockcroft inquiry,

Section 6(ii) of this report states:

«Many of the people interviewed during this enquiry were inhibited

about using mathematics, this led them to avoid it as much as possible

and in some instances it has affected their careers. The inhibition

was most marked among women who had specialised in arts

subjects. The more educated were affected to a much greater extent

than the less educated.»

3. The solution to the problems of low attainers in mathematics

is not simply dividing pupils into the following three

groups and then providing separate curricula for each:

a. those who are good at mathematics;

b. those whose general ability is not low but are failing at

mathematics;

c. those whose general ability is low and are failing at

mathematics.

The problem is much more complex since we do not know

enough about the way children learn, we are not agreed on

the nature of mathematics and there is little known about

effective teaching methods. Moreover, the differentiates at

which pupils learn and a wide variation in attainment at a

particular age, make it impossible to categorise pupils in

the above groupings with any degree of permanence.

Another main factor is that teachers want to keep all

options open to enable pupils to enter public examinations

at the highest level possible. There are further factors such

as previous school experience, environment, attitude and

motivation which influence the attainment of pupils in

mathematics, so the idea of separate maths curricula for

separate groups does not offer much promise.

4. Past attempts at making suitable curriculum provision for

mathematics for all pupils have focused on change of

content, groupings of pupils and management of

resources. The impact of these changes on mathematics

education has not been significant Mathematics for the

M a j o r i t y.3 The Schools Council Project in Secondary

School Mathematics was set up in 1967 to help teachers

construct for pupils of average and above ability. The

courses relate mathematics to pupils’ experience and provide

them with some insight into the processes that lie

behind the use of mathematics as the language of science

and a source of interest in everyday life.

This was an admirable aim and the project was inspired

by the «Newsom Report» published in 1963 under the title

Half our Future.4

The following two quotes would, I hope, indicate the inspiring

nature of this report:

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2 Sewell, B. (1982): Use of Mathematics by Adults in Daily Life.

Advisory Council for Adult and Continuing Education.

3 Mathematics for the Majority (1970): Chatto & Windus for the

Schools Council.

4 Ministry of Education (1963): Half our Future. HMSO.

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«Our aim in the teaching of mathematics to all pupils, to those with

average and below average ability no less than to those with marked

academic talent, should be to bring them to an interest in the content

of mathematics itself at however modest a level.» (Paragraph 459)

«Few, if any, of our pupils are ever likely to become mathematicians,

but some may well come to find satisfaction in mathematical work if

its purpose has first been clearly seen and confidence established

through the successful use of mathematics as a tool.» (Paragraph

422)

Note: *Our pupils* refers to half the pupils of secondary schools, i. e

those for whom public examinations were not originally designed.

This project contained some very exciting, attractive and

relevant material for teachers and pupils but the evidence

of its lack of impact on schools is provided by the following

comments from recent reports on mathematics teaching in

secondary schools.

«The work was predominantly teacher controlled: teachers explained,

illustrated, demonstrated, and perhaps gave notes on procedures

and examples (...) A common pattern particularly with lower

ability pupils was to show a few examples on the board at the start

of the lesson and then set similar exercises for the pupils to work on

their own (. ) At the worst it became direct ‘telling how’by the teacher,

followed by incomprehension on the part of the pupils (...)» (H M I

Secondary Survey, Chapter 7, Section 6.35)

«(. . .) and in the majority of the classrooms the teaching did not aspire

to do more than prepare the pupils for examination (...)» (H. M. 1.

Secondary Survey, Chapter 7, Section 6.25)

From the Cockcroft Report6

«(...) Mathematics lessons in secondary schools are very often not

about anything. You collect like terms, or learn the law of indices, with

no perception of why anyone needs to do such things. There is

excessive preoccupation with a sequence of skills and quite inadequate

opportunity to see the skills emerging from the solution of problems.

As a consequence of this approach, school mathematics

contains very little incidental information. A French lesson might well

contain incidental information about France - so on across the

curriculum; but in mathematics the incidental information which one

might expect (current exchange and interest rates, general knowledge

of climate, communications and geography, the rules and scoring

systems of games; social statistics) is rarely there, because most

teachers in no way see this as part of their responsibility when teaching

mathematics We believe that this points out in a very succinct

way the need -which is by no means confined only to courses for

low-attaining pupils - to relate the content of the mathematics course

to pupils experience of everyday life.» (Paragraph 462)

5. It is interesting to note that although recent reports point

out that teachers seem to concentrate on teaching rigidly to

examination syllabuses and spend a vast amount of time

on teaching routine skills they are not successful in increasing

the proportions of pupils who Can perform these skills.

Interesting evidence of this is provided by Dr. Margaret

Brown in her article Rules Without Reasons? «7 According

to her, in some cases the proportion actually decreases!

5 Great Britain, Department of Education and Science (1979):

Aspects of Secondary Education in England. HMSO.

6 See Footnote 1.

7 Brown, M. (1982): Rules without Reasons? In: International

Journal of Mathematics Education, Science and Technology. Vol.

13, No. 4, pp. 449-461.

Further serious and disturbing evidence has been quoted

in Paragraph 444 of the Cockcroft Report. It points out that

according to examination board regulations, the

16-year-old pupil of average ability who has applied himself

to a course of study regarded by teachers of the subject as

appropriate to his age, ability and aptitude may reasonably

expect to secure grade 4 in the certificate of secondary

education. The mark required to achieve grade 4 in mathematics

is often little more than 30% ! This implies that

pupils of average ability can only obtain one-third of the

possible marks, and it can only damage pupils’ confidence

since the examinations are normally set on the syllabuses

which teachers say they need to spend most of their time

on.

6. So what are the reasons for teachers continuing to teach

in largely ineffective methods? The reasons are complex

but not unsusceptible to some analysis. In many cases teachers

are not unaware of the failure of the system they are

operating but their perception of the constraints which force

them to operate in a restricted way is often misleading and

mixed up. If one were to ask them the reasons for not changing

their teaching approaches, as I often do, one is likely

to receive a fairly standard catalogue of reasons such as

resources, time, class size, disruptive pupils, rigid examination

system, lack of pupil motivation, demands from

employers and universities, pressure from parents, political

pressure, lack of suitably qualified mathematics teachers,

lack of technical support in mathematics departments and

so on. Some of these pressures are real and others’ perceived

ones only but they do keep teachers locked up in

operating an ineffective system.

7. The Cockcroft Report has pointed out that the mathematical

education which many pupils are receiving is not

satisfactory and that major changes are essential .

The major changes as far as the teaching approach is

concerned are outlined in Paragraph 243, the most quoted

paragraph of the report—

" Mathematics teaching at all levels should include

opportunities for

- exposition by the teacher;

- discussion between teacher and pupils and between

pupils themselves;

- appropriate practical work;

- consolidation and practice of fundamental skills and routines;

- problem solving, including the application of mathematics

to everyday situations;

- investigational work "

Unlike previous reports which were mainly aimed at teachers,

the Cockcroft Report in Chapter 17 has recommended

active co-operation of the six main agencies for effective

change. Teachers at the front local education authorities

providing support, examining boards, central government,

teachers’ training institutions and those funding curriculum

development and educational research. Co-operation

is also sought from employers and the public at large.

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8. In this climate of support from the above agencies, with

a nation-wide program of «post Cockcroft» activities supporting

us, we have found that the following aims of our

project for low attaining pupils in secondary schools can be

realistic and help focus attention on aspects crucial in bringing

about change in the teaching of mathematics:

- to encourage teachers to change their attitudes about

ways in which low attainers learn mathematics;

- to help teachers to interpret the Cockcroft Committee’s

«Foundation Lists» (Paragraph 458) in the spirit of

Paragraph 455 to 457 and 460 to 466, i.e., suggest activities

which should involve low attaining pupils in a wider

range of mathematics than the usual restrictive diet of

«basics»;

- to provide teachers with ideas and strategies which

should enable pupils to change their perceptions of

mathematics, encourage them not to view the subject

just as a body of knowledge which has to be «passed

on» fact by fact;

- to suggest ways in which teachers can continually gain

insight into pupils’ mathematics without having to rely on

formal tests;

- to suggest ways in which pupils can arrive at conventional

methods and terminology through their participation

in problem-solving activities and investigatory

mathematics;

- to suggest ways of working which should enable pupils

to see links between mathematics and other subject

areas;

- to suggest ways of working which should help teachers

to develop pupils’ confidence and independence in

handling mathematics;

- to suggest approaches which should help teachers cope

with different rates of learning amongst low attainers.

9. We have begun with a viewpoint that there is already a

large amount of material available for pupils, and the problem

lies in its incorporation into the classroom. Transfer of

material from one teacher to another, or one classroom to

another is not straightforward and often causes sufficient

difficulties for the second teacher to reject the material as

unsuitable or justify its failure by cataloguing external pressures.

We have concentrated in the first instance on developing

«good practice» in twelve chosen schools from six local

counties. One teacher/researcher has been released from

timetable commitment from each case study school on a

fixed day every week. We find it essential that

teacher/researchers are not released full-time and for the

rest of their working week they are in the real environment

of schools where changes are intended to take place. The

value of this work will lie in the opportunity to find common

and distinctive features, to follow these up and probe further

in order to provide a basis for making decisions about

any contributory reasons for success and failure of material

and methods.

These case studies should enable other teachers to identify

with situations and experience and to predict the likely

measures of success in their own classrooms. We are also

convinced that the growth in implementation of changes

will not come about by written publication only. We are

using a «cellular-growth» model for development and dissemination

of ideas. In all the six counties we have developed

a growing network of teachers, through full and

part-time inservice courses, who are participating in trials

and feedback. Naturally, the teacher/researchers are

increasingly involving all the other teachers of mathematics

in their own school, and we have considered it most important

that teachers of other subjects appreciate the changes

taking place and support them.

We are hoping that inservice packages (including video

tapes) related to case studies will be produced for general

dissemination to advisory staff, heads of departments in

schools and teacher-training institutions. It is even now

apparent that this will serve to outline the development process

outlined above. We think it very unlikely that there is

any short cut.

10. It is not my intention to discuss in detail all the areas of

exploration we have undertaken so far, but only to provide

a glimpse of some significant issues relevant to the theme

of this paper.

In considering the courses for 11- to 16-year-old pupils the

Cockcroft Report in Paragraph 451 states:

«we believe it should be a fundamental principle that no topic should

be included unless it can be developed sufficiently for it to be applied

in ways which the pupils can understand (...)»

The chief reason offered by teachers for not using methods

which enable pupils to apply their mathematics is the lack

of time. I believe that the issues are more complex than this

and are associated with confidence and the scale of perceptual

leap required in changing their beliefs about how

children learn mathematics.

The Cockcroft Report points out:

«In order to present mathematics to pupils in the ways we

have described it will be necessary for many teachers to

make very great changes in the ways in which they work at

present (...) (Paragraph 465)

Enabling these changes to take place so that teachers

implement them from the position of conviction and confidence

is at the centre of our project.

11. One major obstacle for teachers in changing their

methods of teaching is their anxiety about «covering» the

syllabus. This is further complicated by the fact that very little

effort has been made to disentangle the teaching of those

aspects which almost all pupils need to come across such as

reading charts, diagrams and tables, interpreting simple statistical

data, simple ideas of probability, ideas of inference

and logical deductions, developing a feel for simple measurements,

visualising simple mechanical movements and

many other areas outlined in the Cockcroft Foundation List

(Paragraph 458) from the teaching of those more sophisticated

parts of mathematics which the able and interested

pupils might study, e. g. deductive geometry, calculus, algebraic

manipulation etc. The able minority often misses out

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on the aspects of mathematics for all outlined earlier since

these are glossed over by their teachers who regard these

topics not as an area of experience but as «bits» of knowledge

pupils need to possess.

Teachers of mathematics, who themselves were mainly

an able minority, as well as pupils tend to resort to the way

they were taught mathematics. There is little change in the

methods used to teach the aspects of mathematics which

the able minority will learn compared with those aspects

covered by most pupils (including the able). The main

change consists in diluting, or what is referred to as «watering

down» the content and presenting topics in small easy

steps with plenty of practice examples. The result is trivialising

and makes mathematics meaningless for most

pupils. A large number of pupils, even those are able, tend

to lose interest and find little meaning in this activity and

hence lose confidence in the subject. There is also a tendency

for teachers to teach topics on a syllabus, systematically,

item by item and believe that if all the topics have

not been taught their pupils will suffer.

In order to ensure that topics are covered, it is possible to

rush these through in a narrow, restricted manner rather

than embedding them in a wider context allowing pupils to

reflect upon the use of the mathematics presented to them

and discussing the appropriateness of methods used by

them. Algorithms are often taught to pupils too soon, and

there seems to be an assumption that once taught they are

remembered. The Chelsea Report, Understanding Mathematics

11 - 168 points out:

«The teaching of algorithms when the child does not understand may

be positively harmful in that what the child sees the teacher doing is

‘magic’ and entirely ‘divorced’ from problem solving.» (Chapter 14)

Mathematics, in daily life, is not encountered in small packages

as taught by fragmenting a syllabus or in the form of

the straight-forward command of the textbooks. It appears

in context in a variety of spoken or written language and

social situations. The Cockcroft Report proposes that its

Foundation List of Mathematical Topics (Paragraph 455)

should form a part of the mathematics syllabus for all

pupils. This list reflects situations in which people meet

mathematics in life, and the emphasis of the report is on

presenting mathematics in a context in which it will be

applied to solving problems.

12. «At all stages pupils should be encouraged to discuss

and justify the methods which they use.» (Cockcroft

Report, Paragraph 458)

The implications of the suggested changes in teaching

styles for teachers are great for those who have not worked

in this manner. It would certainly call in question the

conventional method of designing curriculum in terms of

topics, concepts and skills. For example, it could mean that

a teacher would need to have a collection of «situations»,

problems and inves-

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8 Hart, K. M. et al. (1981): Children’s Understanding of Mathematics

11-16. John Murray.

tigations to offer to pupils and the consideration of mathematical

outcome of these in terms of content and processes

for individual pupils would be a retrospective activity

by the teacher. For example, consider a situation

where a pupil is presented with 5 pence and 7 pence

stamps and asked:

a. What totals can he make if he has as many of each as

he wishes?

b. Are there more?

c. Can he convince others?

d. What happens with other stamps,

3 pence and 5 pence?

5 pence and 9 pence?

Some of the outcomes of this activity, depending on the

levels at which individuals tackle this problem, could be

listed as follows:

Generalisation and Factors

Organisation of work Multiples

Pattern spotting Addition and

multiplication

Hypothesising and Algebra

testing Permutation

Formalising Combinations

Exclusion Primes and composites

Checking etc. Triangles

Modular arithmetic etc.

For a confident and experienced teacher, this way of working

could enable him to overcome the major problem of

not limiting a child’s attainment since the choice of rich starters

would enable all pupils to become engaged in the activity

and offer an opportunity for follow-up work in depth for

those who were interested and able to do so: This would

certainly help to overcome the problem of offering motivating,

relevant and challenging mathematics without restricting

pupils’ opportunities for taking any external examinations.

Although the change in emphasis may appear

small, the solution is not as simple as it appears. Teachers

who are used to assuming the major control of their pupils’

learning find it extremely difficult to change their focus.

Pupils also become used to the idea that teachers will

always have the right answers and the right method for all

problems, and if they wait long enough these answers

would be provided by the teacher directly or through the

«bright» pupils in their group. Under these circumstances,

pupils do not find it easy to change their role and assume

responsibility for their own learning.

These changed perceptions of mathematics learning and

teaching need to develop in a climate of mutual trust and

confidence.

13. In her publication «Generating Mathematical Activity in

the Classroom»9 Marion Bird has used written records of a

class of 11-year-old pupils to demonstrate that it is possible

to teach in a way which encourages pupils to begin to ask

their own

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9 Bird, M. (February 1983): Generating Mathematical Activity in the

classroom. west Sussex Institute of Higher Education.

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questions, to control the direction of their investigations, to

make conjectures and think how to test them to make and

agree on definitions and equivalences, to search for patterns,

to make generalisations and to seek for reasons for

what seems to be happening.

One of our project activities has been to examine a large

number of case studies developed by teachers who have

been trying out these teaching approaches in the classroom

and identifying general features which facilitate the

work and the mathematical activities and those features

which inhibit them. These «facilitators» and «inhibitors»

have been found very helpful by teachers attempting such

an approach in the classroom. We have been exploring

methods of involving teachers of other subjects in supporting

these developments since their attitude can have a significant

influence on pupils. We have also been exploring

simple but effective methods of sustaining networks of teachers

who can offer each other support, encouragement

and stimulus - all important ingredients of bringing about

effective change. A collection of ideas and strategies which

would enable teachers to initiate a greater change of focus

in the control of learning in the classroom has been compiled.

This has also provided some initial starting strategies

which have been tried out in several classrooms.

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10»The Mathematical Association Diploma in the Teaching of

Mathematics to Low Attaining Pupils in Secondary Schools» was

piloted at the West Sussex Institute of Higher Education, the

Mathematics Education Centre and at Bishop Grosseteste

College Lincoln.

We are also observing and developing case studies on

effectiveness of various strategies for inservice support,

e.g. teachers visiting each others schools, teachers released

to work with other teachers in their own schools etc.

Finally, it seems clear to us that the most effective change

can come about by starting from the teachers’ o w n

strengths and building from there. This, in the time of great

pressure on inservice finance, can be an extremely slow

process. It would be counterproductive and a retrograde

step if we allowed ourselves to be seduced by short cuts

which entail offering crutches to support teachers.

The development of the Mathematical Association Diploma

in the Teaching of Mathematics to Low Attaining Pupils 10,

for which some teachers can be released for one day a

week over a year under a central government scheme is

already proving an effective agent for change. We need to

think seriously about the continuation of these developments

undertaken by teachers who have completed these

courses and widening the means of supporting those who

are not able to have such an opportunity. The extreme importance

and enormous benefits of teachers released from

schools to work with other teachers and the effective programs

for such activities can be a theme for another paper!

Details available from:

The Secretary of the Diploma

Board

The Mathematical Association

259 London Road

Leicester LE2 3BE

England

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**Universal Mathematics Education and its**

**Conditions in Social Interaction**

Achmad Arifin

As far as I understand, universal mathematics education is

mathematics education having everyone in a society as its

target. Its objective is to educate the society to be more

intelligent in utilising the available resources or opportunities

to improve their welfare and prosperity. What we need

is a mechanism to bring mathematics education to everyone

in the society. This mechanism should have the ability

to disseminate the intended changes or improvements at

any time. School system should be in the mechanism. But

the school system alone cannot be made responsible for

bringing mathematics education to everyone or to carry out

improvements in mathematics education.

Improvement followed by another improvement, or change

followed by another change, will be the feature of mathematics

education in particular as a consequence of the

rapid development of mathematics as well as that of science

and technology.

The mechanism should be able to channel the needed

changes and improvements to the school system without

any disturbances, that is to say, as smooth as possible. For

a country with a large population the need in such mechanism

can be very urgent.

In the implementation of universal mathematics education

the active participation of the community can contribute a

great deal in reaching the target.

In this paper I will try to describe how we should raise

community participation in carrying out universal mathematics

education through looking at the aspect of interaction.

In developing the mechanism the mathematics itself

and its process of development should be brought as close

as possible to the place where mathematics education is

going to be developed. Because the role of the mathematicians

and their activities is also significant in initiating the

development of universal mathematics education.

*1. Social Structure*

Setting the ultimate objective of education as to develop

the society toward attaining a better standard of living

means that education should be the concern of everyone in

the society. To enable the society or the community to participate

in the process of education a certain interrelationship

has to be developed between the society and the

school system. Since interaction is a component which

basically supports the educational process, we may examine

the interrelationship mentioned above from the

aspect of interaction. To pave our way to this purpose, we

generalise our examination by looking at the interactions

that happen in a society.

Social structure in a society is understood to be the totality

of interactions among people or groups of people.

Depending on its quality, social structure can influence the

survival and the development of the society. Through interactions

the society improves its ability for continuing its survival

by utilising the resources and opportunities that are

available in its disposal. These interactions are social interactions.

Let us direct our attention to the social interactions which

contribute to the improvement of people’s ability and refer

to this particular interaction as a positive interaction. Since

interaction can happen between people and their environment,

social as well as natural one, we generalise the meaning

of positive interaction as to include not just the social

one.

*2. Positive Interaction*

Looking at a particular society we may always ask whether

interactions happen among individuals in that society.

Assuming that interactions happen among them we may

further ask, what are the kinds of interactions and how

intensive they are. What we should identify are those interactions

which have the effect on increasing individual’s

knowledge or skill, or individual’s ability in general.

On the other hand, someone may gain additional knowledge

or skill through reading books or observing natural

phenomena. Can we say that someone gains additional

knowledge and skill through interaction between him or her

and the books he or she is reading or between him or her

and the natural phenomena he or she is observing? The

answer is yes, but this will depend on the way he or she

reads the books and observes the phenomena. Certainly

someone needs a certain ability in order to undergo these

kinds of interactions.

We generalise the meaning of positive interaction to include

any interactions which contribute to the development of

abilities of individuals or groups of individuals. It can happen

in various kinds and patterns between individuals and

their surroundings. This positive interaction is actually an

important aspect in the educational process. It provides

opportunities to individuals to improve their ability. With

their continually improving ability the people will have

opportunities to contribute to the development of their

social structure.

Positive interaction is expected to happen life-long for

each individual. Someone needs some knowledge and skill

to initiate positive interaction with his or her surroundings.

In particular, someone should be able to utilise information

to get some additional knowledge and skill, or to improve

his or her ability in general. Facilities like libraries or

museums which exhibit new developments, for instance in

science and technology, form resources of information.

These can provide stimuli to motivate positive interaction to

happen continuously with increasing intensity and quality

as to fulfill the needed ability.

Someone’s continual efforts for improving abilities, or

developing new abilities can be considered as the consequences

of the rapid development in various sectors. The

developments in science and technology in particular crea-

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te challenges in their utilisation. To cope with these challenges,

particular abilities have to be made to exist.

Leaving the development of such needed abilities to individuals’

motivation and initiation, through their involvement in

positive interaction, might take a long time. Therefore, an

organised effort is needed to design positive interaction to

happen in a certain time period and space.

*3. School Interaction*

Someone needs to have certain abilities in order to be able

to get involved in a positive interaction. Someone needs to

know the language of the book he is reading, or perhaps he

needs some knowledge about the topics discussed in the

book in order to get some additional knowledge about the

topics. In general, someone needs some basic knowledge

and skill to draw additional knowledge from the book he is

reading, from the discussion with someone else, from the

observation of a natural phenomenon, etc.

To organise positive interaction to happen in a certain time

period and space is particularly aimed at providing individuals

with some basic knowledge and skill. Positive interaction

which is designed to happen in a certain time period

and space is an important aspect pertinent to what we

understand as school. I refer to this positive interaction as

school interaction or classroom interaction. The ability of

individuals in carrying out further development of their own

ability as well as that of their society independently is usually

set as the main objective of school interaction. This further

development of ability can be carried out through positive

interaction.

A school is a place where people who have different backgrounds

and come from different environments come together

with the intention to learn. The people who come to

learn at a certain school constitute a society of students

with a certain characteristic, that is a certain level of readiness

to get involved in positive interactions.

On the other hand, the school is equipped with a curriculum

containing a series of programs of teaching and learning

process as a means to achieve the objective as set in

the curriculum. In this junction where the interface between

student background and school curriculum take place we

expect the teachers to play their role; that is to manage the

school interaction to happen as to give an optimal result.

In managing school interaction we should pay attention to

the individual’s background as differences related to value

systems or behaviour patterns might occur. Referring to

those differences the teachers or someone else who acts

as a facilitator should manage the interaction so as to happen

without disturbances.

Each of the three components: social structure, positive

interaction and school interaction should have the ability to

absorb some developments in mathematics that are relevant

to the society development and to utilise them in educating

the people in accordance with their respective role.

The parts of each component include:

- Social structure: To appreciate and support necessary

changes in school mathematics and create conducive

environment outside school for learning mathematics.

- Positive interaction: To set up facilities other than

schools that motivate and create opportunities for

mathematics learning.

- School interaction: To develop the environment in

school and the teaching methodology in mathematics

class as well as that for individual approach so to enable

them to inspire, to stimulate and to direct learning activities.

Referring to the roles of the components as described

above, we are not looking at them with their passive meaning;

but with their active participation in providing opportunities

and stimulation for mathematics learning. This justifies

the purpose of the three-component functional relationship:

social structure - positive interaction - school interaction,

that is to provide opportunities and stimulation for

mathematics learning through interaction, aimed at everyone

in the society.

*4. MathematicsforALL*

Once again, universal mathematics education is mathematics

education for everyone in a society, so it is mathematics

education for all. The school system is one of the

places where universal mathematics education can be

channeled to reach a part of the members of a society. In

other words, school interaction is one of the components

for reaching individuals in a society and to manage them to

get involved in learning mathematics. Since learning

mathematics at school is related to the development of a

society as a whole we also have to examine the components

which can contribute in paving the way to make the

relationship really happen. These components are social

structure and positive interaction.

What parts of mathematics that a person learns at school

should be accepted and appreciated by the society as a

part of the society development process. On the other

hand, the school should be knowledgeable and be aware

of the needs of the society, particularly those concerning

the society development and those related to increasing

the individual’s ability.

From now on, we need to attach more operational meaning

to positive interaction. That is, some facilities need to

be created or to be established in order to stimulate and

provide opportunities and means for positive interaction to

happen. In this connection, the targets are the society and

the school system.

From what has been explained above, we can see that

the interrelation among social structure, positive interaction

and school interaction enable the development of the

school system as well as the quality and the intensity of

social interaction continually. This will result in increasing

the ability of individuals. Furthermore, referring to the role

of individuals in increasing the quality and the intensity of

interaction the continual development of school mathematics

teaching will, after all, result in increasing the ability of

the society itself. Therefore, taking also into consideration

the individual’s background during teaching-learning processes

we can describe a functional relationship among

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social structure, positive interaction and school interaction as

follows.

This three-component functional relationship forms a

mechanism for developing the needed ability of a society in

coping with challenges. Therefore, it can also be utilised as

a means of transferring mathematics and its development

to everyone in the society, through various kinds and patterns

of interaction.

This is aimed at achieving certain mathematical abilities.

This gives an idea as to how to carry out universal mathematics

education. To initiate universal mathematics education

it is therefore desirable that in any society the

three-component functional relationship can be identified.

In this era of science and technology development, the

intelligence of the people will contribute a great deal and

meaningfully to the society’s ability. Mathematics education

can play a role in developing the intelligence of the people.

But this will depend on the persons who have to make

opportunities to enable mathematics to play its role fully;

that is to create a conducive environment where mathematics

learning can happen through interaction. Referring to

this, we are dealing with the following questions.

1. Which part of mathematics that can function as a developer

of an individual’s intelligence?

2. Those parts of mathematics that have been chosen,

how should they be presented?

To learn mathematics through interaction can take place

everywhere and any time, not necessarily in a classroom

during a mathematics class. Therefore, those two questions

are relevant to the school interaction as well as to the

other two components: social structure and positive interaction.

Referring to the purpose of developing a society,

we should apply the two questions to the three components:

social structure, positive interaction and

school interaction equally balanced. The answer will

depend much on the existing cultural condition, for

example, the average mathematical knowledge possessed

by the people in the society, the intensity and the quality of

interaction that constitute the social structure. Furthermore,

the existence of mathematicians and their activities related

to mathematics should provide some assets in seeking

answers to the questions.

In developing countries mathematics for all can be earmarked

as universal mathematics education, that is mathematics

education for everybody. The mechanism for carrying

out universal mathematics education is the

three-component functional relationship. In developing universal

mathematics education it is always necessary that

we examine the mechanism whether it is an appropriate

condition to carry out the new changes. Otherwise, we

have to develop the mechanism itself, that is to develop the

components or the interrelation among the components.

In developing countries the problems related to the development

of universal mathematics education always are

concerned with the development of the mechanism

besides those of mathematics and its teaching. In developing

countries with a large population, the problem will

mainly concern the development of the mechanism, to

enable it to function accordingly. This means that in carrying

out universal mathematics education we have to pay

attention to the three-component functional relationship:

social structure — positive interaction —school interaction.

Its development includes the development of the components

and their interrelationship.

The role of the mathematicians from the country concerned

is to form a pool of expertise for the development of the

mathematical content and the way it should be presented.

This includes the development of the teaching methodologies

as well as that of the mechanism, that is the

three-component functional relationship. Therefore, the

development of universal mathematics education will never

be free from the need to develop mathematical activities in

the developing country concerned and the willingness of

the mathematicians to take part in the efforts to improve the

intelligence of their people.

*5. The Role of Mathematicians*

The two questions related to what and how as mentioned

previously are: Which part of mathematics that can function

as a developer of an individual’s intelligence and how

should they be presented, always occur during the development

of universal mathematics education. The development

of mathematics provides us with many choices in fulfilling

the mathematical content, while the development in

technology provides us with many alternatives for presenting

certain mathematical topics.

The questions related to what and how will always

be relevant from time to time. Therefore, what is more

important here is the availability of a mechanism

which is able to provide answers to the questions of

what and how in accordance with the needs and

conditions of the developing society or nation. T h e

mechanism should include the three-component functional

relationship which carries out the universal

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mathematics education and the local mathematicians who

provide information and expertise for the development of

the mathematical content.

On the other hand, the local mathematicians should be

actively involved in stimulating and upgrading the three

components that are aimed at developing the three-component

functional relationship.

To fulfil this task the mathematicians need to keep themselves

informed on the development of mathematics and

actively maintain their communication with mathematical

activities as wide as possible.

*6. Conclusion*

Referring to the role of mathematics education in developing

individuals’ intelligence that is aimed at developing

the ability of a society or nation, the role of mathematicians

in providing information and expertise, the

expected active involvement of mathematicians in the

development of mathematics education, the fact that

mathematics and also science and technology are developing

rapidly, the fact that the growth of education depends

on the cultural condition of the society, we may close this

paper with the following remarks:

1. Universal mathematics education can function as a

means for increasing the ability of a society or a nation

through developing individuals’ intelligence.

2. Universal mathematics education needs to undergo

continual development to maintain its function in a developing

society or nation.

3. The three-component functional relationship can function

as a mechanism for universal mathematics education

to reach everyone in a society through the school

system as well as outside the school system.

4. The local mathematicians and their mathematical activities

could be supportive to the continual development of

universal mathematics education including the

three-component functional relationship as its mechanism.

5. The development of the three-component functional

relationship as a mechanism and the supportive attitude

and activities of the local mathematician reflect the

effectiveness of the community participation.

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**Alternative Mathematics Programs**

Andrew J. C. Begg

**Introduction**

Whenever a group of high school teachers discuss their

mathematics programs they make certain assumptions.

They usually take for granted the way the programs have

developed over recent years, the available resources, and

the purpose of the programs. In this paper I want to suggest

a number of questions that may cause us to question

our syllabi and teaching methods and consider alternative

programs. Before looking at these questions we need to

consider some aims, assumptions, and constraints that we

probably all share.

*Aims*

In mathematics education the three most common aims of

our programs are summed up as:

Personal — to help students solve the everyday problems

of adult life;

Vocational — to give a foundation upon which a range of

specialised skills can be built;

Humanistic — to show mathematics as part of our cultural

heritage.

These three aims imply that basics are necessary but not

sufficient, that we must present a broad-based course as

the future needs of our students will vary tremendously,

that historical topics and career information should be

included, and mathematics should be given a warm and

human flavour rather than a formal or logical one.

*Assumptions*

In my country we assume that universal secondary education,

including mathematics for at least two years, and

usually for three or four years, is the right of all students.

This assumption varies in other countries and may make

some considerations less relevant.

The other assumption I would make is that for many students

mathematics is not inherently interesting — indeed

they may have been turned off the subject. Usefulness is

not enough, we want motivation and fun, and our mathematics

programs must build in this motivation so that students

look forward to our classes.

*Other Constraints*

A huge variety of schools exist: large/small, urban/ rural,

traditional/alternative, wealthy/poor, academic/ technical

etc. In spite of this variety many places, including New

Zealand, have a national system of education that has one

syllabus for each year of schooling.

Our yearly programs are based on 3 or 4 hours week of

mathematics. In New Zealand this used to be taught in 5 x

40 minute periods but is now usually 3 x 60 minute periods.

This change should cause significant movements in the

teaching of our mathematics program.

The third constraint is caused by the pressures on our

school syllabi. We are asked to cover numerous new items.

Computer education, careers education, outdoor education,

health education, and multicultural education are

examples of areas that either require time from the total

school program or need to be integrated across the curricula

and this includes the mathematics curricula.

*General or Special Purpose Mathematics*

It is usually possible to look at a group of students and see

some aim they have in mind, e. g. to graduate from high

school, to pass an external exam, or to prepare for employment

or unemployment. When we look more closely we

can usually isolate a number of subsets of students with

differing needs. In the same class we find students who are

terminating their mathematics education, others who hope

to pass an exam and others who expect to take the subject

on to a higher stage.

In large schools it may be possible to separate these students

into different classes which cope with their specific

needs, but in small schools this is not possible. Further, the

students may not be certain about their future plans and

needs.

In New Zealand I find that many classes contain students

requiring enrichment while others need remedial assistance,

some are sitting one exam while others are sitting another,

some expect to leave school for a job, others for

unemployment; yet generally students are not given alternative

programs to cater for these needs. At the most

senior level in New Zealand we are looking at alternative

mathematics programs with a statistics or calculus bias

according to the student’s needs, but for the majority of

school leavers no basic course exists with significant elements

of budgeting, tax, insurance, rents and the other

skills needed by school leavers. I believe alternative programs

are needed where content decisions are based on

the needs of the participating students.

*Teaching: Mathematics or Students*

A mathematics program is part of a total educational package.

As teachers and program designers we must consider

the aims of this whole package and then adjust our programs

to suit. These general aims would include the development

of

- self-respect,

- concern for others,

- urge to enquire

and we would want to develop the skills of - communication,

- responsibility,

- criticism,

- cooperation.

If we wish to achieve some of these aims we must stress

cooperative learning, encourage project work

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and displays of work by individuals and groups. We must

use discovery approaches and not try to shortcut this

time-consuming process by presenting results too quickly.

We must give all students success in their eyes, in their

peers’ eyes and in their teachers’ eyes. Here alternative

programs are needed as our present ones are not achieving

these goals, and we must consider other teaching

methods as an important part of these programs.

*Monocultural or Multicultural*

Traditionally, our programs, both contents and methods,

reflect the traditions of European education. Many of us

have assumed this European background and the associated

attitudes to learning. Now we find that mathematics

programs are required for monocultural groups that are not

European and for multicultural groups that reflect a broad

range of cultures.

In New Zealand the two main non-European groups are

Maoris and Pacific Islanders. Practically no research has

been done on their different views and attitudes to mathematics,

nor to the way their language reflects different

views of the subject. What we are aware of is that many

Polynesians prefer group work and do not enjoy being ranked

apart from their peers. This fact has obvious implications

in designing a program that stresses group rather

than individual success.

Other problems experienced by groups from other cultures

and in particular by new immigrants are the problems

associated with language. If teachers are aware of these

difficulties and modify their programs these difficulties can

be reduced, but it is difficult for a teacher to cope when a

very broad range of backgrounds occur at the same time.

In building self-esteem we must at least build respect and

understanding for the differences that our students reflect.

We can at least use names and subjects from other cultures

in our examples, but we must be careful not to offend.

Assuming this cultural sensitivity, opportunities exist in

numerous areas to incorporate ideas from other cultures

and help students of all races appreciate the differences

between members of their communities. This two-way

appreciation helps our students «stand tall».

*Streamed or Mixed Ability*

Disregarding all other differences and factors affecting the

achievement of our students we all accept that students of

a particular age group vary from very talented to those of

low ability. One way of handling this variation is streaming.

The difficulty with streaming (or even broad-banding) is

that it is practically impossible to stream exactly, and when

one considers the factors affecting achievement (illness,

schoolchange, bad teaching etc.) one realises that streaming

can never be perfect.

Many teachers believe that because of social factors it is

desirable to keep mixed-ability forms. Certainly mixed-ability

forms mean we must offer alternative programs within

our classroom where with streamed groups it is easy to

think one program is suitable for all students when in fact it

is only suitable for the majority of them.

In smaller schools and in schools where option structures

affect mathematics we are forced to cope with mixed-ability

classes and to design alternatives within the program.

These alternatives include enrichment and extension for

the more able students and more practice in basic skills for

the less able. These alternative programs need to consider

the appropriateness of topics according to whether or not

students are ready for the subject and whether or not they

expect to continue their mathematics education into the

future.

*Class, Individual, or Group Programs*

Traditionally, most of us taught our classes as one group.

More recently, with the advent of mixedability classes,

some of us have tried individual programs. Having tried

both methods, I would believe that neither a class approach

nor an individual program is satisfactory if used all the

time, so again I look for alternatives.

Analysing the possibilities we have six main teaching

modes:

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Teaching Mode Work Rate

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Class teaching Same Same

Individual program Same Different

Individual program Same Different

Group teaching Same Different (between groups)

Group teaching Different Different (between groups)

Group teaching Different Different (within groups)

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No approach is necessarily correct and variation of style

throughout the program is probably the most desirable answer

but within this varied program I think we need to use

groups to a much greater extent than at present.

Sometimes groups will be doing the same work at different

speeds, sometimes the same work at different speeds but

achieving the work at different levels, and sometimes different

work. All these modes need consideration in our programs

and obviously our decisions will affect the resources

we need for these modes of teaching.

*Cooperative or Competitive Learning*

I have mentioned the way some cultural groups prefer working

in groups and how using groups may help overcome

some problems associated with aspects of whole-class

and individual programs. I want to suggest that a group

approach is the real-life approach to many problem-solving

situations and that our programs should reflect this cooperative

and realistic approach.

I know that we all value excellence and feel that the student

who ranks first in our class deserves praise and

reward, but I believe that this should not be achieved using

a competitive strategy which recognises one at the expense

of ranking others in lower positions. It can be done with

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comments rather than with numbers. At the same time,

other students also do well in presentation or in attitude

while others improve in some facet of their work, and these

aspects need positive encouragement and praise too.

When a cooperative group is working together (whether it

is mixed ability, streamed, special needs or whatever) then

the whole group should share the praise and encouragement,

and there is no need to separate out individual skills

to achieve our objectives.

*Traditional or Alternative Teaching Styles*

The teaching style of many of us still reflects the style of

many years ago, the way we were taught and even the lecture/

tutorial system of traditional universities. Meanwhile,

our clients have changed, they represent a broader range

of backgrounds and abilities, their interest in mathematics

is different, and they are more sophisticated. Some teachers

have altered their teaching styles and some have

tried discovery approaches. The new maths revolution had

some teachers trying more formal approaches and others

more intuitive approaches.

I believe that, in terms of our general aims, we need to

encourage open-ended approaches whenever appropriate,

through discovery learning and through project activities.

The important aspect here is that a variety of approaches

is needed, and it is often useful to have one group doing

projects while another group requires more teacher attention.

We sometimes talk of integrating mathematics with other

subjects in the curriculum. Science, economics, geography,

technical drawing and home economics are examples

of subjects where obvious overlaps exist. From our knowledge

of transfer of training we must assume that subject

overlap must be made explicit and that when we talk of

applications in mathematics, surely the most relevant applications

are those ones which the students are at present

involved in with their other class work. This change must be

reflected in our programs and will vary according to the

other option choices of our students.

A final aspect of teaching style that worries me in New

Zealand is a result of the change in school structures from

7 x 40 minute periods each day to 5 x 60 minute periods.

In mathematics this has usually meant 3 x 60 minute periods

replacing 5 x 40 minute periods. At the same time,

many teachers have not significantly adjusted their teaching

style. I believe this 60 minutes should be split into at

least 4 shorter periods with a greater variety of activities

occurring during the hour. Maintenance should be built in,

the range of activity should include not only oral and written

work, but also physical activity and a greater use of

visuals both in teaching and summarising. I believe this

variation should be built into our programs in recognition of

the short attention span of many young students. The need

to provide varied and interesting stimulation over an extended

period is the only way we can avoid having our students

bored.

*Text-oriented or Multi-media*

Textbooks are the most commonly used resource in our

classrooms. They have usually been written for a particular

course and are the cheapest available resource. With the

advent of alternative programs we see the need to build up

numerous supplementary resources. With groups working

on different topics in the same classroom and with students

with reading difficulties, we see other reasons for more resources.

Mathcards and worksheets are needed to direct

students into alternative activities such as games for maintenance

work, project starters, and enrichment. Much of

the work on these cards could be self- m o t i v a t i n g .

Computer-assisted learning has obvious applications to

remedial, revision and enrichment work. Films, slides and

videos all have their place in providing variation. Our

mathematics programs must «build in» these resources so

that within a school all the students are getting these advantages

and not merely those in the classes of one or two

keen teachers.

Imaginative texts are still necessary, and we must remember

that every student should learn to learn from a book

without assistance.

*Logical or Humanistic Approach*

Mathematics was taught very formally, then with the «new

math» we saw logical approaches, mathematical

approaches and psychological approaches. Some people

have tried more intuitive approaches, and I understand one

or two have used an historical approach.

What I would prefer is a humanistic approach, I mean an

approach that is student-centred and develops from the

students’ particular interests and needs. An approach that

links their work to real life and to applications that are relevant

to them.

I want to see a warm approach that treats every student

as someone special, that works positively to avoid sex or

race stereotyping, and that builds self-esteem in our students.

I am sure that once this self-esteem is present, teachers

will be amazed at the progress students can make.

**Conclusion**

I know that schools have limited resources, that teachers

have limited time, and that numerous other constraints are

put on us by our schools, but I believe we can all introduce

more alternative elements into our programs. I know most

of us like to have a class start together, but we can still produce

various endpoints, we can use group work, and we

can encourage more cooperative problem solving.

I am sure we must give students the opportunity to make

decisions that are relevant to their education and each of

us should be «the guide on the side not the sage on the

stage».

Program development will keep happening, it is our responsibility

to make sure it helps our students achieve the

aims of education in general as well as the aims of mathematics

education.

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**Part II:**

**Problems and Developments**

**in Industrialised Countries**

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**Arithmetic Pedagogy at the Beginning of the**

**School System of Japan**

Genichi Matsubara and Zennosuke Kusumoto

In Japan the Emperor had held the reins of government,

although the Samurai held it for some time in Japanese

history. In the Meiji revolution in 1868, the Emperor took the

reins of government again and the new government in the

Meiji Era was born. To make the nation modernise, it made

great efforts to learn a great deal from foreign countries

and to adopt new educational policies.

About 110 years have passed since the elementary school

system was established in Japan. Here I’d like to tell something

about arithmetic pedagogy at the beginning of the

Meiji Era in Japan.

Today everyone thinks it is the turning point in education in

Japan, so that to reflect on arithmetic pedagogy in the Meiji

Era will provide some suggestions on how to introduce a

new arithmetic pedagogy.

**I. Arithmetic Pedagogy at the Beginning of the**

**Meiji Era**

First, I’d like to discuss the «Terakoya» (private schools) in

the age of the Samurai, since they played an important role

in forming the basis of arithmetic pedagogy at the beginning

of the Meiji Era.

*A. Arithmetic in «Terakoya»*

At the end of the Edo Era, the Samurai took the reins of

government in Japan. In order to educate the Samurai’s

children, each feudal clan had a school of its own. But they

did not take into consideration the education for ordinary

people. So they established their own schools to get a

minimum knowledge to go into the world—these were called

«Terakoya». There were many «Terakoyas» but there

was no communication in terms of teaching methodologies

among them. Arithmetic in «Terakoya» was mostly concerned

with how to handle abacuses. Many kinds of textbooks

used at «Terakoya» were published in the Edo Era.

Among these publications, about 1,126 books were mathematics

books such as UJinkoki-, «Sanpo zukai». These

were used as textbooks for children and teachers at

«Terakoya» or other private schools. Next I’ll introduce to

you something about «Jinkoki» to let you know the

contents of arithmetic pedagogy in those days.

*B. «Jinkoki»*

This is the book on abacuses written by the mathematician

Mitsuyoshi Yoshida in 1627. This was modelled after

«Sampotoshu» which was a manual for abacuses in China.

But the contents mainly consisted of how to use abacuses

for business transactions in daily life. It is noticeable that

what was dealt with in it was just the same as what we

dealt with in problem solving, which was discussed all over

the world in the early 20th century. But teaching methods

were not dealt with in it. After many teachers of «Terakoya»

used it, the contents were revised and published several

times.

*C. «Terakoyosho «*

The content of arithmetic at that time was to learn the four

operations by using the abacus. The teaching materials

connected with daily life transactions were arranged in the

same way as in problem solving. There were many children

who were able to do division of two-digit numbers.

Graduating from «Terakoya», they went to another private

school where they studied the latter half of «Jinkoki».

Teachers of «Terakoya» taught them according to their own

philosophy. So, when the government introduced school

systems in education and build elementary schools,

«Terakoya» was one of the models in mathematical education.

In December 1873, Director David Murray reported

to the Ministry of Education that the average standard of

education was very high.

II. **Arithmetic Pedagogy After the Proclamation of**

**School Systems**

After the revolution of the Meiji Era, the Samurai reign was

replaced by the Tenno (Emperor) reign. Then the unified

Meiji government was born. One of the policies of the

government was to build elementary schools all over Japan

and to put an emphasis on the 3R’s. Prior to this, there

were already a few private elementary schools established.

But feudal clan «Terakoya» and other private schools exerted

a great influence to make the new educational policies

possible.

In March 1869, the government ordered the building of elementary

schools in every prefecture. The government promoted

the new educational policy, so people were eager to

equalise ordinary education. After the Samurai Era passed

on to the Tenno reign, each local government took over the

policies. The curriculum of each school, similar to that of

«Terakoya», was as follows: The official age of enrollment

was five, but it was usually six. Almost all children could

understand basic addition and subtraction. In To k y o

«Terakoya» was not admitted as a school, but later was

admitted as a private elementary school. In those days the

government authorised two kinds of schools.

(1) One was a school for people going into business after

graduation, which was established in each prefecture. By

establishing these schools, the government aimed at the

decentralisation of education.

(2) The other was a school for people going on to college and

the university after graduation. Then there were three kinds of

elementary schools considered from other perspectives.

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a. the national school — established by the Ministry of

Education; the attached school - established as the

preparatory course for colleges and universities; b. b.

elementary schools of each prefecture — established by

private funds;

c. the elementary school — established by feudal clans.

**III. Proclamation of School Systems**

*A. Establishment of the Ministry of Education and*

*Proclamation of School Systems*

When the Meiji government started, much thought was

given to educational policies. It established a kind of school

administration section in the government of Kyoto. Then

the capital was moved to Tokyo and the same section was

established in Tokyo. So, it was thought necessary to establish

the Ministry of Education, which deals with educational

matters such as making rules, building schools and

assisting financing. In September 1871, the government

decided to abolish schools for the Samurai and to establish

the Ministry of Education.

The Ministry of Education had the task of making rules

about elementary schools and junior high schools and did

everything for schools. It made efforts to equalise the standard

of education of every school in Japan. This made it

reinforce school systems in Japan.

In Japan the law on school systems was the first one that

was established for education. It controlled the systems

and the curricula of schools from elementary level to university

level in Japan. To make the law, the government not

only gathered the materials from other countries, but also

established pilot schools to get data from them.

In August 1872, the outline of school systems was

decreed as follows:

a. the necessity of schools in terms of man’s development;

b. to study regardless of positions and sex;

c. to explain the mistakes of traditional learning;

d. to give everyone opportunities of learning;

e. to make parents responsible for children’s education;

f. to make parents pay money for children’s education;

The contents which children had to study were arithmetic

or abacuses. We can find in «Gakumon no susume»

*(Encouragement of Study* written by Yukichi Fukuzawa),

how to learn the angular style of the Japanese, how to write

Chinese characters and to drill using abacuses, to deal

with the balance etc. were indicated.

Now I stall summarise the school systems.

*1) The large, middle and small districts*

The Ministry of Education, which was responsible for

controlling and managing schools in Japan, divided all

areas as follows:

*The large districts —* the country was divided into eight. A

university was established in each large district.

*The middle districts —* each large district was divided into

32. A junior high school was established in each middle district.

*The small districts —* each middle district was divided into

210. A elementary school was established in each small

district.

So, there were 6,720 elementary schools in one large district

and 53,760 elementary schools in all.

*2) Schools*

There were three kinds of schools - universities, junior high

schools and elementary schools.

The curricula were prepared in a special book.

*3) Elementary schools*

Elementary schools were considered to be the primary

stage of school education, so all the people had to go to

school by all means. Schools were generally called elementary

schools, but there were several kinds.

a. Infant schools - preschools for children under six years

of age.

b. Private elementary schools - the man who had a licence

taught in his private home.

c. Schools for the poor— schools for the children of poor

people who could not support themselves. The rich

contributed money to these schools.

d. Village elementary schools — schools in the remote

areas. The teacher omitted a part of curriculum or attended

evening school.

e. Girls’ elementary schools — schools for girls in which

they were taught handicrafts and ordinary subjects.

f. Ordinary elementary schools — It was divided into lower

and upper schools.

*4) The subjects of elementary schools*

At first there were about 20 subjects. They were revised

soon and only a few of them remained. I’ll introduce something

about arithmetic here.

*Arithmetic —* order and the four operations; they were

explained in the western style.

*Geometry—*the subject for upper schools;

*Arithmetic* was taught using a western-style method and

avoided teaching how to use the abacus.

In the lower elementary school systems, it was prescribed

that elementary education was compulsory. Since all the

laws had not yet been put in order, I think that the school

systems remained just a model which could not be followed

as an ideal at that time. Not all the children of school age

could go to school.

In 1883 the percentage of school attendance was about

50%. It was still a very low rate at that time.

In 1870 compulsory education started in England for the

first time in the world. Two years later the school systems

started also in Japan, so it may be interesting to note that

compulsory education started in Japan next to England,

but elementary school education was already spreading in

advanced countries.

*B. The Elementary School Syllabus*

Following the proclamation of the school system laws, the

elementary school syllabus was proclaimed.

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It was a little similar to the course of study which was introduced

from the USA after World War II. But it was just the

syllabus before World War II, or the one which was used in

countries except the USA.

The syllabus is as follows:

The elementary school was divided into two stages — an

upper and a lower one. The lower one was from the age of

6 to 9. The upper one was from the age of 10 to 12. So the

children studied for eight years. The course of the lower

elementary school was divided into eight grades. The term

of each grade was six months. The contents of each grade

were given in this period. But as it was only a model, it was

recom mended to revise and to use it in each prefecture.

Next, I shall introduce arithmetic only. I can only find in the

eighth grade the same course of study of arithmetic as

today’s. But I cannot find it in the first grade to the seventh

grade.

- the 8th grade: 6 months, 5 hours a day, 30hours a week,

except Sunday; Western-style arithmetic: 6 hours a

week; Using textbooks, the teacher wrote the Arabic

numbers, order and the fundamental calculations of

addition and subtraction on the blackboard, and the children

wrote them on their paper. Children practised the

calculation of figures and mental arithmetic every other

day. When a teacher made the children do mental arithmetic,

children could not use paper, and only answered

questions on the blackboard. The questions which were

answered in exercises the day before remained unerased

on the blackboard, and the next day, all children

were made to do the exercise again.

- the 7th grade: 6 months; arithmetic: 6 hours a week; to

teach multiplication and division as in the 8th grade, and

to do exercises on calculation of figures and mental

arithmetic every other day;

- the 6th grade: 6 months; arithmetic: 6 hours a week; to

teach multiplication and division;

- the 5th grade: 6 months; arithmetic: 6 hours a week; to

study the application of the four operations, and to do

the exercises on the calculation of figures and mental

arithmetic every day;

- the 4th grade: 6 months; arithmetic: 6 hours a week; to

teach the four operations with compound numbers;

- the 3rd and 2nd grades: 6 months; arithmetic: 6 hours

a week; to teach fractions;

- the 1st grade: 6 months; arithmetic: 6 hours a week; to

teach fractions and proportions; review of each subject:

2 hours a week; to review all subjects studied before.

After passing the test, children went to upper ele- v o l u -

me mentary schools. Children who could not pass the test

studied for six months more in the 1st grade.

The lower elementary school was more modern ised compared

with the schools of the clans and «Terakoya». But

the Ministry of Education did not consider the unification of

teaching methods all over Japan soon.

In the syllabus of the lower elementary school, arithmetic

consisted of teaching western-style arithmetic and not teaching

how to use the abacus. But at that time there were

few who could teach westernstyle arithmetic, so the

government took the measure of making plans to train teachers.

There was a big objection against only teaching

western-style arithmetic and not teaching to use the abacus.

There were few books about western-style arithmetic till

«Shogaku sanzitsuyo» was published by the Ministry of

Education. So it introduced «Hitssan kunmo», «Yozan

sogaku» as textbooks, and explained a little about the teaching

methods. I do not think all the teachers could teach

the children. In the Ministry of Education there occurred

discussions about it. In 1874 it gave a notice, «We do not

intend to use only western-style arithmetic, but we will let

the children study the abacus, too.»

Arithmetic in the syllabus of elementary schools did not

limit the size of numbers for each grade. Today it is said

that each teacher determines the limit of numbers according

to the children’s ability. It is not necessary to show the

details. At that time, not everyone knew about the modern

school, so it could not be helped. It might be unavoidable

when the development of children was unknown.

Geometry was the subject for upper elementary schools.

Measurement was only thought as the adaptation of calculation.

Now I shall introduce to you «Hitssan kunmo» to

show how it was taught at that time.

It was published in September 1869. The writer was Meiki

Tsukamoto — a geographer. He was a talented man who

played an active part in the navy and bureaucracy. He was

the first writer who offered arithmetic in a systematic and

modernised textbook,» said Kinnosuke Ogura. It was

because he studied western-style arithmetic from the

beginning — he was not a man of the abacus. This book

was published as a primer, but it was of high level.

This book was the first that showed the style of the subjects

as it is common now.

- The first stage is a general explanation.

- The second stage is a detailed explanation of the

methods with examples.

- The third stage are exercises of calculations.

- The fourth stage are exercises of applications and problems.

The book contained four volumes. For each volume

existed another book which contained formulas and answers.

- the first volume: number, four operations; the second

volume: fractions;

- the third volume: proportions;

Now I shall introduce to you a part of the first

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*1) Number*

The number in Chinese character

The large number (larger than 10)

The decimal number (smaller than 1)

The cardinal number

The notation

the number placing and notation of the decimal

system

*2) The four operations*

Addition: the method of calculation of figures, the

addition of numbers of 4 or 5 figures; the

number divides 4 figures; applied problems

Subtraction: the same way as addition

Multiplication: explained as addition; the calculation

Division: with 10 to 12 is added to the fundamental

multiplication table; to explain multiplication

of the number of I or 2 figures to explain the

method of division when the divisor

consists of I or 2 figures

*C. Upper Elementary School Syllabus*

- the 8th grade: *arithmetic:* 6 hours a week

- the 7th grade: *arithmetic:* 6 hours a week

- the 6th grade: *arithmetic:* 6 hours a week

to teach proportionate distribution

*drawing:* 2 hours a week

to draw point, line and regular

polygon

- the 5th grade: *arithmetic:* 6 hours a week

to teach proportionate distribution

*geometry:* 4 hours a week

to teach regular polygon

*drawing:* 2 hours a week

the same as the 6th grade

- the 4th grade: *arithmetic: 6* hours a week

to teach proportionate distribution

*geometry:* 2 hours a week

to teach line, angle and triangle

*drawing:* 2 hours a week

to draw plain, straight line and

volume

- the 3rd grade: *arithmetic: 6* hours a week

to teach square and square root

*geometry:* 4 hours a week

to teach circle and polygon

*drawing:* 2 hours a week

to draw plain, straight line and

volume without shadow

- the 2nd grade: *arithmetic: 6* hours a week

to teach the calculation of interest

*geometry:* 4 hours a week

to teach comparison with each figure

*drawing:* 3 hours a week

to draw arc, line and volume

- the 1st grade: *arithmetic: 6* hours a week

to teach series and logarithm

*geometry:* 4 hours a week

to teach practical use

*drawing:* 4 hours a week

to draw a map and others

«Sokuchi ryaku» was the appointed textbook for geometry.

It was written by Tora Uryu in 1872. It was for the measurement

of land. So part of the book is used for the text. The

contents of it were almost definitions and their explanation.

**IV. TeacherTraining**

Positive educational policies of the Ministry of Education

were to build schools, to arrange syllabuses and to train

teachers.

To get rid of defects of traditional education, it was necessary

to adopt western-style training and to train teachers by

foreign teachers.

In September 1872, the normal school was built in Tokyo

and lectures began. The following April, the attached elementary

school was established. This was the first attached

school for the pupils of the normal school. It was used

not only to practice teaching but also to investigate teaching

methods. Then it developed into the research center

of the normal school and the model school for all prefectures.

At that time, the contents of teaching in normal

schools were mainly concerning teaching methods. In June

1873, it adopted academic subjects of study. Until then the

term of the school year was not determined, so pupils

could graduate according to the results at any time. They

were dispatched to each prefecture as teachers. The term

of the school year for upper and lower schools was determined

to be two years and each grade was divided into two

stages.

As schools were built in each prefecture, there arose a

problem of shortage of teachers.

In general, people thought that the contents of elementary

schools were three subjects: reading, writing and abacus

just like in «Terakoya» and other private schools. So teachers

of punctuation, writing and abacus were employed

for special subjects.

At that time it was easy to get teachers from «Terakoya»

and other private schools. The Ministry of Education made

efforts to train prospective teachers. In 1874 each prefecture

established teachers’ training schools without permission

of the Ministry of Education. About 46 of this kind were

built all over the country.

Many books on education were published by the Ministry

of Education. It thought that it was of no use only to teach

teaching methods, but it was neces-

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sary to explain to newly appointed teachers in detail the

book of «The Primer of Elementary Schools».

Then the principal of Normal School, Nobuzumi Morokuzu

published «The Required Manual for Elementary School

Teachers», and many other books of this kind were published.

On the other hand, M. M. Scott taught carefully the teaching

method of modernised schools, not through the

books. The students who graduated from the schools went

to provincial normal schools to teach the teaching

methods. But as the percentage of school attendance

increased, the number of schools increased. So there was

a shortage of teachers for a long time.

It was necessary not only for teachers but also for the

people to have knowledge about modernised schools —

«What is education? Explanation of the mission of the

school, the system and management of modernised school

and teaching method in details.» I introduce one of the

popular editions of the book, «The Required Manual for

Elementary Teachers» written by Nobuzumi Morokuzu in

1873.

In this book he explained the important duties of teachers

in three items - «The teachers arrange the seats of pupils

according to their results. They move to upper classes after

the test. Where does the teacher stand in the classroom?

How are the desks arranged? etc.»

This description was easy to understand for teachers. It

may have been difficult to teach the style of modernised

school lessons and to get rid of the «Terakoya’s» teaching

concept.

Now I shall introduce a part of arithmetic education.

*The 8th grade: arithmetic*

To show the picture of numbers; it was necessary first to

know how to read the numbers and to teach Arabic numbers.

Teachers wrote both numbers on the blackboard and

asked the pupils to read them or teachers read them themselves.

Then children wrote on their slate board.

Afterwards, some of them wrote on the blackboard.

Teachers checked them and said to the children, «Raise

your right hand if it is right.»

*The 6th grade: question and answer*

Using pictures about shape, volume, line and angle, teachers

taught shapes and surfaces of material or the kinds

and names of lines etc.

**V. «The Book of Elementary Arithmetic»**

The Book of Elementary Arithmetic was chosen as the textbook

for the elementary school syllabus of the normal

school and the elementary school syllabus of many prefectures.

It was written by the Tokyo Normal School and published

by the Ministry of Education. It was a progressive syllabus

compared with the textbooks of other subjects at that

time. It was considered to be one of the best books on

arithmetic education in the country.

The textbook was much influenced by W. Colburn — his

thought was based on the «intuition» of J. H. Pestalozzi.

In this edition the main editors were M. M. Scott and C.

Davis. It was used in many prefectures. It was excellent,

but it was doubted whether it was actually used by teachers.

In March 1873 the first volume was published; in April

1873 the second volume was published;in May 1873 the

third and fourth volumes were published; in September

1876 the fifth volume was published.

I introduce a part of it.

*The first volume*

«The Elementary School Syllabus» of the normal school is

taught in the 6th grade. In the 7th and 8th grades, the primer

of elementary school — numbers, the memorisation of

the calculation tables of addition, subtraction and multiplication

were already taught.

Japanese numbered

Arabic number

Explained in two pages, the first edition of the book written

by W. Colburn was published in 1821; it was used for 60

years with several revised editions. The last version of ‘84

was published after his death. The Book of Elementary

Arithmetic was, although it was published in 1873, similar

to the hook of ‘84. Material selection and explanation of the

problems were like that of ‘84. Considering the fact of M. M.

Scott’s coming to Japan, it may have been modelled after

the book of ‘63. In the first volume, children study addition

- example: add 1 to 1-10 and 2 to 1-10. Question and answer

were just as in the book of ‘63.

So, how did M. M. Scott see the Japanese? Recently, the

following letter of M. M. Scott was found at the Griffis collection

in Rutgers University. A part of his letter to Griffis is

cited below.

«You ask me what I think of the Japanese after thirty-five years’

experience of them. I may say that I always had a very high opinion

of their intellectual qualities but had some misgiving as to the practical

application of what they could so easily learn.

Those misgivings have been completely brushed away from my

mind. They have proven themselves great in nearly every department

of human effort and I predict for them even in the near future

greater achievement still. When I left Japan it was with much regret

at what I thought then to be disserverance of a ten years’ acquaintance

much appreciated by me, but I have had now twenty years’

experience with a large number of Japanese in Hawaii, with a different

class indeed, but still with their able and amiable official here.

You are quite right in thinking this country a very interesting one. I call

it ‘a museum of ethnology’. It would pay you to take a vacation and

come here in the near future, and with your powers of observation

and your slashing pen you could show us to ourselves as others see

us. Pray do come. l will give you a welcome.»

**Vl. The Situation of Arithmetic Education at the**

**Beginning of the School Systems**

A superintendent visited schools throughout Japan.

How did he feel?

*A. The report of the superintendent D. Murrey*

He said, «There remains traditional style of education.» But

he thought that it was better to change

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only slowly. I think he understood that making gradual progress

was better for Japan. He held the same opinion

concerning the abacus.

*B. The situation in the country*

Over a short period of time, each prefecture established

training centers and normal schools to lessen the shortage

of teachers. Each prefecture devised the method of a circuit

teaching independent of the school administration.

Developing school systems in the Meiji Era meant fighting

against poverty. They were hard pressed to pay money for

education, but realised the value of education gradually

and became interested in it.

All the heads of the Ministry of Education made a tour of

inspection. It was because people complained of the

school systems, and also because there arose a tendency

of revising the syllabus of schools.

The school buildings were temples or people’s houses.

The shortage of teachers was a crucial problem. So the

excellent children who left elementary schools with an

excellent score were adopted as teachers. The children

aged more than 15 years were adopted as assistants. The

children aged more than 13 years were adopted «leaders

in the lesson». Their ability was just the same as the 6th

graders of today.

As the subsidy of government and prefecture was very

small, the money for school was expanded for the elementary

school district. Farmers and fishermen in those days

were so poor that they could not afford it.

There was a time when there were no blackboards and

notebooks in the classrooms. Children wrote the numbers

and words on trays which were filled with sand or rice bran.

Some of the teachers were not competent to handle the

four operations with calculation figures and did not understand

why the product of the decimal was less than the multiplicand.

When a child learned the Arabic numbers, he

asked, «Is number 6 just like the shape of the nose?»

There were a few regular teachers who began to study

themselves in each prefecture.

*C. Promotion and examination*

In elementary schools, pupils had to pass the examination

to go up to an upper grade. They took the examination for

each grade, and one who succeeded in the examination

could go up to the higher grade.

When the pupils left the elementary school, the middle

school and the university, they had to take the examination

to leave school.

Each prefecture established «the test of the elementary

school». There were many books on teaching methods

which were published by a private company. I found the

background and the method of pedagogy in them.

*D. Conclusion*

The leaders of the Meiji government were overwhelmed

with civilisations of foreign countries. They thought that

there was not a moment to lose to equalise the standard of

education to catch up with the advanced countries.

The government made efforts especially to establish elementary

schools compulsorily, and also to let children go to

school, but they did not know people’s opinion on education.

The government intended to change the existing system

to develop modernised schools in Japan.

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**On the Value of Mathematical Education**

**Retained by Japanese Society as a Whole**

Takashi Izushi and Akira Yamashita

We would like to talk about «On the Value of Mathematical

Education Retained by Japanese Society as a Whole».

This is a research on how school mathematics, which students

learned at junior and senior high schools, has been

retained by them when they grew up into society. The purpose

of the study is to try to get a fundamental viewpoint

for the organisation of curriculum for the department of

mathematics in the future.

**1. The History of Mathematical Education in Japan**

We think that the history of mathematical education in

Japan may be divided into the following six stages:

*The first stage*

This stage was the period of about 40 years of the Meiji

Period (1868 — 1911) through the Taisho (1912—1925) to

the beginning of the Showa Period (from 1926). In this

stage, the main objectives in mathematical education were

to get skills in calculating, to train thinking power and to

gain practical knowledge.

*The second stage*

This stage was the period of about ten years in which

mathematical education was seriously affected by the

Perry Movement. The main objectives were to develop the

concept of function and to foster one’s power of space

observation. World War II took place during this stage.

*The third stage*

This stage was the period of about ten years after World

War II and was influenced by the USA. The method in

mathematics teaching was focused on daily life experience.

In this stage, the six-year term of compulsory education

was extended to nine years.

*The fourth stage*

This stage was the period of about ten years which was a

period of review of the former stage. Mathematics teaching

was taken as the matter of mathematics systems seriously.

And Japan’s economic growth began in this stage.

*The fifth stage*

This stage was the period of about ten years in which

mathematical education was seriously affected by modernisation.

The percentage of the number of students going

on to high school was over 90%.

*The sixth stage*

We are now in this stage, the period of about ten years

which is a period of review of the modernisation efforts.

And efforts are made in mathematics curricula to meet the

differences among the students.

**2. Purposes**

The purposes are classified into the following three.

The *first purpose is* to examine the following: Do they

remember or understand mathematics contents which

were learned at junior and senior high school?

The *second purpose is* to examine the following: What

kind of contents of mathematics are useful to their work?

The *third purpose* is to examine what may be called formal

discipline. In the first stage of the history of mathematical

education in Japan, formal discipline was emphasised

and Euclidean geometry took a great part as formal discipline

for a great guiding principle.

So, we examined what impression was made on them.

From the examinations mentioned above, we try to get the

mathematics contents which are retained by Japanese

society, and we would like to use their knowledge to organise

the curriculum for the department of mathematics.

**3. The Way of Examination**

The examinations were carried out twice.

*(1) The first examination*

This examination was carried out in 1955. The time was

during the fourth stage of the history of mathematical education

in Japan.

*a) Participants*

The participants are members of society who learned

mathematics before the third stage. They have contributed

to economic growth in Japan, more or less. The number of

participants was 976 and they were sampled from the

whole society according to their occupations: technologist,

teacher, specialist, administrator, office worker, farmer,

fisher, seller, etc.

*(b) by Content*

They were asked to solve problems connected to the following

contents:

1 Calculation (including positive and negative

number, literal expression)

2 Round number, percentage

3 Proportion, reciprocal proportion

4 Fundamental figure

5 Solid of revolution

6 Scale

7 Projective figure

8 Word problem (equation of the first degree)

9 Congruence and similarity of triangles

10 Statistics (graph)

11 Pythagoras’ theorem

12 Trigonometric function

13 Coordinates

14 Word problem (simultaneous equation)

Furthermore, they were given the following questionnaires

corresponding to the problem: «If you understand the

content related to this problem, is it useful to your work?»

*(2) The second examination*

This examination was carried out in 1982. A part of the

result of this examination was presented at the

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I C M I-JSME Regional Conference in Mathematical

Education (1983, Japan).

*a) Participants*

The participants were graduated from a senior high school

belonging to a national university. This senior high school

is an eminent school for the graduates going on to college.

The 62 persons of the object were chosen corresponding

to the years they were in senior high school, 19 persons

from the years of the third stage, 18 persons from the years

of the fourth stage and 25 persons from the years of the

fifth stage. Their occupations were technologist, scholar,

doctor, etc. and they contributed to the improvement of

technology and science in Japan, more or less.

*b) Content*

They were asked to solve problems which were mainly

related to elementary geometry, because we wanted to

make a study of formal discipline.

**4. The Outline of the Examinations**

*(I ) The result of the first examination*

The problems were connected to the following

contents:

I questionnaires of elementary geometry

II 1 perpendicular of line and plane

2 parallel

3 sum of internal angles of triangle

4 projective figure

5 equivalent transformation

6 condition of triangle construction

7 congruence and isosceles triangle

8 measurement

III logic

IV axiomatic method

V non-geometric model of the axioms of the affine

geometry

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Table 1 : Correct Answer (%)

a specialist and administrator (218 persons)

b office worker (219 persons)

Table 2 : Answer That is Useful to Your Work (%)

a specialist and administrator (218 persons)

b office worker (219 persons)

*(I ) The result of the second examination*

Table 3 : Correct Answer (%)

a specialist and administrator (218 persons)

*(3) The outline of consideration*

*a) Knowledge*

Simple mathematics knowledge which was learned in

junior and senior high school is well remembered by every

person, though many years have passed since they learned

it.

In these tables (Table 1, Table 3) only a, b are displaced,

but the persons of other occupations answered also more

than 40% of the problems correctly.

It is not suprising that a gets higher percentage of correct

answers than b. But the difference is not so much.

The problems in which a relatively differs from b in percentage,

are connected to proportion and reciprocal proportion.

As for the problems of calculation, b is better than

a.

The response about the problems of calculation, round

number, fundamental figure, and congruence and similarity

of triangles, shows high percentages for each person.

On the other hand, the problems of projective figures and

trigonometric functions, show low percentages. This is a

consequence of the times in which they learned. We cannot

find any differences between the results of the first

examination and that of the second examination.

In the second examination, the objects were chosen

concerning the years of graduation from senior high

school. We find that knowledge, such as of perpendicular

line and plane, which may be observed in daily life are forgotten

as the time passes after they learned them.

*b) Usefulness*

The number of persons who agreed to the questionnaire «If

you understand the content related to this problem, is it

useful to your work?» is smaller than the number of those

who answered the problem correctly .

The contents judged relatively useful for the persons of

both a and b are scales and statistics. The contents of proportion,

reciprocal proportion and fundamental figure are

thought to be more useful by the persons of a.

Many persons answered correctly the problems of calculation

and solids of revolution, but few persons thought that

these contents were useful. And many persons judged that

content of coordinates not useful to their work.

The persons of a more than b think that the contents of

proportion, reciprocal proportion and fundamental figure

are useful.

*c) The way of thinking*

It is examined here whether the attitude of deductive thinking

which was obtained by learning elementary geometry

is still in their mind or not.

IV and V of the second examination are the problems

which need deductive thought. V is a problem of a

non-geometric model of the axioms of affine geometry.

The result of examinations is satisfactory. It has been a

long time since they learned, but they have not forgotten

the attitude of thinking deductively.

The result of examinations about equivalent transformation

is satisfactory too. The result of examinations about

logic is satisfactory.

Y ounger persons replied that they used mathematics

knowledge.

But older persons replied that they judged by common

sense, though they might use mathematics knowledge

unconsciously.

Many members of Japanese society judge that thinking

and reasoning powers are developed by learning elementary

geometry. And they judge that knowledge of elementary

geometry is useful to their daily life, but not to their

work.

From these findings, we may conclude that formal discipline

is supported by Japanese society as a whole.

What was mentioned above is the results of our examinations,

but it can show a viewpoint on what school

mathematics should be.

**5. Examples of Problems**

*(I ) Problems of First Examination*

1. Proportion and reciprocal proportion

Mark an x if the following statement is false: A train can

travel 100 km/h. The distance traveled

is proportional to the time traveled. ( )

2. Calculation

a) (+12) - (-7) + (-15)

b) (0.15 - 3 : 4) -0.3

c) 4a2b x 2a2b4

3. Fundamental figures

Mark an x if the following statement is false: For three

lines 1, m, n in a plane, if 1^m and 1^n, then m//n. ( )

4. Congruence and similarity

Which of the following are congruent or similar triangles?

a) ( ) and ( ) are congruent triangles.

b) ( ) and ( ) are similar triangles.

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5. Projective figures

Mark an x on a projective figure which expresses

true length.

(a) (b) (c)

( ) ( ) ( )

6. Scale

Find the actual distance using the scale:1 :50000

a) 3 cm b) 4 cm c) 14 cm

7. Solids of revolution

What are the following solids of revolution?

a) b)

8. Coordinates

a) Give the coordinate of point P.

b) Find the equation of the line PQ.

*(2) Problems of Second Examination*

Problems of perpendicular line and plane

Describe the definition of the following word.

a) A line is perpendicular to a plane (in a space).

b) A plane is perpendicular to another plane (in a space).

2. Problems which need deductive thought IV Assume the

following premise about the organisation of roads and bus

stops.

- There are at least two bus stops.

- For every two bus stops, there must be one and only

one road which is through them.

- There is one and only one bus stop on the intersection

where two roads cross.

- All bus stops don’t go along one road.

- For each road and a bus stop (not along this road), there

is one and only one other road which doesn’t intersect

the first road and goes through the bus stop.

According to these assumptions, show in the following

order that there are at least four bus stops.

Ql. There are at least two bus stops.

Why? Let these bus stops be named A and B.

Q2. There is only one road which goes through A and B.

Why?

Q3. There is at least one bus stop (not along the road in

Q2). Why? Let it be named C.

Q4. There is only one road which goes through A and C.

Why? Let the road be named b.

There is only one road which goes through B and C.

Why? Let the road be named a.

Q5. There is only one other road which goes through A

and does not intersect the road a in Q4. Why?

There is only one other road which goes through B and

does not intersect the road b in Q4. Why?

Q6. Two roads in Q5 certainly intersect. Why?

Q7. There is only one bus stop at the intersection in Q6.

Why?

VA’s family go on a journey. Assume the premise about the

organisation of cars and passengers.

- There are at least two cars.

- For every car, there are at least two passengers.

- For any two passengers there is only one car containing

both passengers.

For every car show in the following order that

there is a passenger who is not in the car.

1) There is another car which differs from car or. Why?

Let the car be named b.

2) A is in car b. Then there is a member other than A who

is in b. Let the passenger be named B.

3) Both A and B are not in car a. Why? So there is a passenger

B who is not in car a.

In addition to these, the following assumption holds good.

- For each car and a passenger (not in the car), there is

one and only one other car containing this passenger

but not containing any passenger in the first car.

For each car, there are at least two passengers who are

not in this car. Why?

For each passenger, there are at least two cars which do

not contain this passenger. Why?

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**An Evolution Towards Mathematics for All in Upper**

**Secondary Education in Denmark**

Ulla Kürstein Jensen

**1. Upper Secondary Education in Denmark**

A description of the evolution in the teaching of upper

secondary mathematics in Denmark would be built on sand

without a few introductory remarks about upper secondary

education in Denmark.

The traditional Danish upper secondary school called

gymnasium is a three-year education based on nine years

of compulsory education that contains mathematics throughout.

Nevertheless, many students choose to postpone

their start in the gymnasium until after the optional tenth

year offered by the schools where they receive their compulsory

education and originally meant for those who want

to increase their qualifications without going to the gymnasium.

The gymnasium consists of two lines, the languages line

and the mathematics line, each of which is divided into

several branches. Mathematics is compulsory on all

branches and is taught at three levels in the gymnasium.

The lowest is the one for the languages branches, and

there are two for the mathematics branches.

The branching doesn’t take place until after the first year,

and the maximum number of mathematics lessons per

week on a branch is six whereas the minimum is three. The

standard timetable on each branch comprises about ten

subjects all of which are compulsory.

In 1983 about 20,000 students finished the gymnasium

and about 12,000 of these came from the mathematics

line. Nearly 4,000 students finished another academically

oriented education, the so-called HF-education. Under certain

conditions the HF-examination offers the same opportunities

for advanced studies as does the «studentereksamen

» the final examination of the gymnasium.

Mathematics is a compulsory subject in the first year of the

HF-education.

In 1983 a total of about 24,000 students, which is approximately

40% of a year of Danish students, completed an

upper secondary education with some mathematics.

**2. The Evolution that Started in 1961 and a Step**

**Towards an** «**Upper Secondary Mathematics for**

**All» Course**

When describing the evolution in the teaching of upper

secondary mathematics it is natural to start in 1961 when

new curriculum regulations for the upper secondary school,

the gymnasium, were signed. The regulation for mathematics

was penetrated by the new-maths-wave and intended

for a small proportion of the students, but it was to be

applied by a rapidly increasing number of students during

the next 20 years. It is important to notice that it restored

mathematics as a compulsory subject for the languages

line students after a pause of a decade. This turned out to

be a small step towards mathematics for all.

The purpose of mathematics for the language students

was to give the pupils an impression of mathematical way

of thinking and method and to provide them with mathematical

tools that could be useful in other subjects at

school and during their future activities, so it wasn’t only

aiming at university studies.

The topics to be taught were: the concept of a function,

elementary functions, infinitesimal calculus, computation of

compound interest, combinatorics and probability theory.

The first textbooks were very theoretic and the whole

course nearly failed completely, but it was rescued by a

new textbook at a suitable level.

When the HF-education, that is a type of further education

meant as an offer to everybody who wishes to qualify

for more advanced theoretic education, came into existence,

it was but natural that the purpose of the mathematics

regulation 1967 was nearly the same as the one for the languages

line students. The course was to be more elementary

though, for instance, it shouldn’t comprise infinitesimal

calculus, but the textbooks included chapters from the textbooks

for language students. After a short time, it became

evident that fundamental changes were necessary in order

to change this compulsory one-year, five lessons per week

course to a success.

In the revised mathematics regulation for HF it was mentioned

as the first goal to provide the students with mathematical

knowledge that could be useful in other subjects

and in their daily life, and as the second, to give them an

impression of mathematical method and way of thinking.

Theoretical algebra and group theory were removed from

the curriculum and statistics, probability theory, and binomial

test were entered. The so-called free lessons meant

for cooperation with other subjects or for elaboration of previously

treated material appeared in the curriculum. A textbook

for the course was written on the basis of an experiment.

This textbook comprised many examples connected

to everyday life and it encouraged the pupils to find others.

The style of it made it easy to understand also for many

who had earlier given up mathematics as too difficult or as

uninteresting. It contained few proofs, but it was very good

at helping the students to create relevant concept images.

The text encouraged the teacher to let the students spend

much of the lessons working in groups of three to five persons

solving and discussing problems. In this course,

mathematics was no longer a terrifying subject.

In retrospect, the new HF-curriculum together with the new textbook

and the applied methods of teaching seem to have served as a catalyst

for the succeeding evolution. The course was the first course

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that might be called an upper secondary mathematics

course for all.

The HF-mathematics that at the beginning had been heavily

influenced by the mathematics curriculum of the languages

line now in turn influenced this curriculum little by

little. Among the reasons for this evolution should probably

be mentioned that the increasing number of students not

aiming at university studies or similar advanced studies

created difficulties for the mathematics teachers, especially

when they were teaching the languages line students,

and the fact that teachers often feel more free to experiment

when teaching these students than when teaching

those on the mathematics line because the former have

only an oral exam to pass whereas the latter have to pass

an oral as well as a written exam, the latter being centrally

set by the Ministry of Education. So the teachers tried

consciously or subconsciously to remove some of their difficulties

in the mathematics lessons for the languages line

students by using ideas or methods from the teaching of

the HF-students, and during the last half of the seventies

an increasing number of teachers chose an optional

mathematics syllabus for their languages line students.

The above mentioned optional curriculum had for some

years been used by 90% of the classes when in 1981 it

became regulation. Let me state a few remarks in order to

characterise it. The objectives are:

The students should acquire:

- some mathematical knowledge which can be of use in

other subjects and in their daily life,

- a knowledge of the framing and application of some

mathematical models,

- an impression of mathematical methodology and reasoning.

It is noteworthy that it is now explicitly mentioned in the

objectives that the students should acquire some mathematical

knowledge that can be of use in their daily life and

a knowledge of the framing and application of some mathematical

models. It is also remarkable that differential calculus

in this syllabus may be substituted by another coherent

material of the same extent and value if the teacher and the

students so wish. Another interesting feature is the so-called

free lessons. These approximately 25 lessons can be

used for going deeper into the compulsory topics, for working

with new topics, for instance some that are connected

to other subjects or for providing an introduction to electronic

data processing and its role in society. The teacher and

the students choose how to use these lessons.

**3**. **Drafts for Courses in Advanced Mathematics that**

**Are at the Same Time Worthwhile Courses for**

**Students Not Intending to Go to University**

The evolution on the mathematics line accelerated later.

During the last ten years, the number of students choosing

this line has increased from about 7,000 to about 12,000,

but at the same time the percentage going to university or

similar advanced education has dropped considerably. As

a result several booklets written by teachers as most textbooks

are and offering alternative and often more intuitive

and less formal approaches to many topics have appeared

during the last five years. The majority is intended to be

used either the first year in the gymnasium or at the lower

of the two levels. The booklets represent a new interpretation

of the regulations. The evolution in HF-mathematics

and mathematics on the languages line have inspired the

authors of the booklets.

A new interpretation of the old regulation isn’t enough to

take into account the change in the students’ qualifications

from their preceding schooling as well as the growing

influence of computers and the fact that for the majority the

mathematics course in the gymnasium isn’t just a kind of

an introduction course but the students’ final mathematics

education. Therefore, the Ministry has just issued a draft

for a new curriculum for mathematics for each of the

branches of the mathematics line.

The intentions leading to the construction of the draft for

the curriculum common to all branches but the one on

which mathematics is taught at the highest level, the

mathematics-physics branch, were to create a curriculum

that:

- within some central mathematical fields shows mathematics

as a subject with its own essential values,

- permits an all-round elucidation of the interaction between

mathematics and other subjects,

- allows time for absorption in major concepts and correlations,

- allows time to meet special wishes from the class or the

school,

- encourages that the content of the lessons should be

influenced by the teacher and the class to a greater

extent than before,

- guarantees that the student has received an all-round

mathematical education with perspective and width,

- prepares the students for a wide range of types of further

training for which a foundation in mathematics is

required.

Both drafts live up to these intentions by comprising not

only a list of topics to be taught but also a list of aspects

that are to be brought into the teaching, and various comments

on ways and means of teaching. The following

objectives, topics, aspects and comments are included in

both drafts.

Objectives:

The students should acquire insight into mathematics as a

form of cognition and as a means of description.

Topics:

1. Integers, rational and real numbers

2. Plane geometry

3. Functions

4. Infinitesimal calculus

5. Statistics and probability theory.

Aspects:

i) Through suitable examples the students should experience

how an algorithmic approach throws new light on

the mathematics they work with, and they

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should acquire a knowledge of the practical application of

electronic data processing.

ii) The students should acquire knowledge of parts of the

history of mathematics and of mathematics in cultural,

philosophical, and social context.

iii) The students should obtain knowledge of formulation

of mathematical models as idealised representations of

reality and get an impression of the possible applications

of mathematical models and of the limitations in

the applications.

iv) The students should learn about mathematics, they

should be aware of mathematics as a form of cognition

and as a language.

Comments:

As to the comments, I have chosen to include just the following:

- The choice of methods of work is to be adapted to the

students as well as the mathematical content, and the

students should be acquainted with several methods of

work so that they can take part in the choice.

- As to the use of textbooks and texts, it is desirable that

the students, apart from reading ordinary textbooks,

become acquainted with texts about mathematics. It is

also desirable that the students try to read a mathematical

text in a foreign language.

- Topics should be approached from different angles.

There has got to be deductive sequences as well as

intuitive ones. Also, the students should become increasingly

familiar with the language of mathematics including

symbols and concepts from set theory and logic.

- When planning the lessons, respect should be paid to

subjects where mathematics is applied.

The draft for the curriculum for the branch with the highest

level of mathematics, the mathematics-physics branch, differs

from the previously mentioned by including, for instance,

some numerical analysis, induction, mathematics from

an algorithmic standpoint and recursion that should provide

the students with a general theoretical background for

future work demanding and involving the use of computers.

It also comprises a considerable amount of free lessons,

the content of which has to be chosen by the teacher and

the students, and it requires a more elaborate treatment of

some topics. Among the additional demands on this branch

I should also like to draw attention to the fact that the

students should learn to express themselves precisely and

clearly orally as well as in writing, and to the fact that the

students should acquire an understanding of the deductive

nature of the subject by working with proofs in connection

with which they should also get the opportunity to construct

proofs of their own.

Teachers who want to cooperate and increase experience

by testing the drafts, thereby helping to formulate other

concepts of them, are going to use them now. Drafts for

textbooks are also being published. On the basis of these

efforts, curriculum regulations beneficial to many, not only

to future mathematicians, should appear in a couple of

years.

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**Fight Against Academic Failure in Mathematics**

Josette Adda

In France in recent years, studies on the rate of academic

failure have revealed the fact that the education system

functions by a process of successive elimination of pupils

from the normal streams at each level of orientation.

In the appended diagram, an extract from (12), it can be

seen that among children born in 1962 in France there

remained at the «theoretically normal level» only 72.2% at

age 7, 59.5% at 9, 44.1% at 11, 34% at 13, 21.9% at 15

and 16.1% at 17 (the remainder having repeated years or

having been put into marginal-type classes).

Moreover, it appears that, on the one hand, these eliminations

concern more selectively children of socio-culturally

deprived families and, on the other hand, that the responsibility

of «the teaching of mathematics» (and not

mathematics themselves) is essential in these orientations,

which have the effect of confirming social inequalities.

In order to evaluate this «inequality of opportunity» as far

as mathematics are concerned, it has been noted that, for

1976—77 (cf. (2)), 52% of the children of upper executives

in the corresponding age group were following the

C-stream (i. e. a course with a predominance of mathematics),

the rate being 6% for the children of workers; their

chances of reaching that particular class being respectively

91% and 23%. Thus, entry to these classes was far from

being equal for all and the «socio-cultural handicap» was

2.2 times more disastrous for the C-section than for all the

classes.

An examination of the socio-professional category of the

family head for students at the Ecole Polytechnique in

1978—79 (cf. (2)) reveals that, out of the 602 students, 422

come from the category of «liberal professions and upper

executives», (or 70%), whereas this category represented

8.3% of the French population for the age group under

consideration.

As for the final year mathematics specialists of the Ecole

Normale Superieure of the same year, one notes that, out

of the total of 21 students, 12 had at least one parent who

was a teacher.

What a lot of «wasted intelligence» (to use the expression

of M. Schiff)!

We shall sum up briefly some research carried out at the

University of Paris 7 which aims to analyse *why* certain

children fail and others succeed and *how* the process of

failure works, so as to find what changes should be made

in teaching to remedy the situation.

In order to analyse the phenomenon, it is first of all necessary

to be aware that children are not normally in direct

contact with mathematics, that be coming familiar with

mathematics is achieved by the intermediary of «mathematics

teaching», which in fact plays the role of a simple

intermediary for only a minority but is rather a filter for the

majority. A study of its workings is thus indispensable.

Classes are essentially carried out, of course, not in

«mathematical language» but in natural language, and so

create numerous external difficulties of linguistic origin (on

the semantic level rather than the lexical or syntactic levels

as is often thought). These difficulties have been analysed

at length by D. Lacombe (cf. his lectures at Paris 7).

However, the purely linguistic explanation is still insufficient,

and it is also in the pragmatic and the rhetoric of the

discourse of the mathematics teacher that we must seek

the sources of faulty comprehension and misunderstandings.

First of all, being abstract, the objects of mathematics that

are treated, the properties and the relations that are studied

can never be seen (in contrast, for example, with the

objects studied by the physical and natural sciences) and

so the distance between the *signified* and the *signifiers*

plays here a role that is more crucial than for any other type

of discourse. Signifiers such as mathematical symbols, diagrams,

graphic representations are necessary and yet are

the source of poor comprehension of the mathematical

objects signified (cf. (1), (4), (6), (8)). By studying the

«misunderstandings» brought about by this confusion between

signifier and signified we have observed the responsibility

they bear not only in a very great number of errors

but also in the impossibility of acquiring the concepts themselves.

For example, for many children (and teachers!)

there are no sets without a string and numerous adolescents

affirm that 2 is a whole number but neither decimal

nor rational, 2.00 is decimal but neither whole nor rational,

and 4/2 is rational but neither decimal nor whole!

Another way of representing mathematical concepts is the

use by teachers and textbooks of metaphors (cf. (14)) that

are supposed to refer to the experience of the student.

Almost always far too simplistic really to conform to any

recognisable reality, they are nevertheless still too complex

not to be burdened likewise with numerous extraneous

meanings that block access to mathematics.

Instead of seeking to show through appropriate exercises

the abstract and generical nature of mathematical

concepts, one type of pedagogy has sought for some years

to «make mathematics concrete» through the method of

teaching: an absurd enterprise and as such inevitably

bound for failure. The initial idea that starting with real-life

situations and anathematising them can be a form of motivation

in early stages was in itself quite reasonable. The

constraints of the school system, however, led to presenting

mathematical questions rigged out in «disguises» that

were extremely artificial, pseudo-concrete and the source

of misunderstandings, becoming thus extramathematical

causes of errors in problems claimed to be «mathematical

». Even problems of the type «Mummy goes shopping,

she buys (...)» are not really natural and the expenditure

calculated is often very different from that of actual

purchases (unrealistic prices, proportions out of line with

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commercial usage .. .). Moreover, this variable «Mummy»

(each pupil supposedly feeling involved) introduces an

emotional factor that is not necessarily positive: for

example, when the mother has financial difficulties, has little

time to do the shopping, is sick, far away or deceased .

. . (cf. (3)). Reactions in the face of these *academic*

«situationproblems» are very different from those of students

to whom one can give the opportunity to «mathematise

» a *real-life* «situation» problem: doing the shopping

themselves, for example.

F. Cerquetti has shown that when pupils in an apprentice

class for baker-pastrycook have to do all the calculations

for purchases necessary for making croissants and for selling

them, considerable success may be noted, whereas

the same students react against all the artificial «word»

problems put to them in textbooks and prefer and succeed

better in purely abstract games with numbers (cf. (9)).

Young children have a potential for abstraction which is

not exploited. The fact that primary school teachers are

often recruited from the students who have the least positive

feelings towards mathematics is very worrying in

France. It sets up an interlocking process of failure (cf. (8))

and declining performance is one of the most distressing

phenomena of our educational system (cf. work in progress

by F. Carayol and M. Olvera in particular). For example, the

use of clear symbolism is perfectly well allowed as a simplification

by young children (cf. the well-known experiments

of Davydov in the USSR), but certain ways of introducing

badly understood formalism are rejected by students in the

secondary system: in fact, when one seeks the causes of

rejection in the teaching of mathematics, one almost

always finds that it is a question of notions, the presentation

of which has been carried out in an inconsistent way,

with inner contradictions .

It is important to stress also that class use of questions

«disguising» mathematics, a method fraught with errors

because of the outside influences that are introduced, is

not «socially neutral» and this constitutes a further factor of

selection (cf. (5) and (10)). At the beginning of this century,

exercises referred above all to a rural universe of landowner

adults who exploited their holdings, transmitted inheritances,

invested their savings, and so on. Today, there is

an attempt to involve the child more and so school exercises

refer often to children but these are children living

in towns or cities (often the capital), receiving lots of presents,

making journeys, and so on. Thus, not only can certain

children not be familiar with certain of the elements

necessary to understand the questions but, above all,

these «disguises» contribute to giving many of them the

idea (immigrants or not — some speak of «home-grown

foreigners»!) that they are foreigners in this world, this universe

of schoolroom problems that they believe to be (the

ultimate mistake!) the universe of mathematics. It is striking

to see the archetypes that certain pupils propose when

they are asked to invent the text of a problem (cf. (3) for

example).

Questionings used in *evaluation* do not so much reveal

inequality but create it (cf. (7)). The formulation and the

presentation of mathematical problems present the same

sort of bias as those (denounced for years now) of IQ tests.

Moreover, poor results have an all the more disastrous

influence in that the present school system sets up a *loc -*

*ked-in process of failure* through the «Pygmalion effect»

(self-fulfilling prophecy) and above all by the irreversible

streaming off towards poorly thought of types of classes.

The struggle against academic failure in mathematics is

not a question of change in curriculum. It requires a

concerted attack on the true basic problems, for otherwise

all the sources of difficulty can recur, in a more or less

serious form, on any chapter of mathematics. T h e

«reform» of recent years has been a good example of this,

with the result that all the criticisms expressed at the time

of the survey on teaching carried out by the review *L*

*‘Enseignement scientifique* in 1932 are easily transposed

into the present situation.

Teachers must become aware of those aspects of their

teaching practice which create misunderstandings and lack

of comprehension; they must not only have a solid knowledge

of the mathematics that they have to teach but also

be capable of understanding how these are to be transmitted

in the mathematics class and study the ethnological

features of that universe where, in the interrelationship between

teacher, pupils and mathematics, communication is

threatened by interference which can be called «socio-logical

» (according to P. Bourdieu).

In order to give all pupils access to mathematical culture

with its own specific features, it would be necessary, as for

physical culture, to offer to all the pleasure and the opportunity

to carry out exercises (here intellectual and abstract

ones) (see for example (11)). Above all, let us not forget

that we are talking about an integral part of the cultural

heritage to be transmitted and that theorems are also

works of art: Pythagora’s theorem is a classic on the same

plane as a play by Shakespeare or a painting by Leonardo

da Vinci, and there is in it an aesthetic wealth that we must

try to offer to all. Contrary to those who want to confine

underprivileged children to «useful mathematics» (in the

sense of creating minimal automatic responses with no

practice in deductive reasoning), I think that we must

attempt to allow all children to exercise a right to abstraction

(for which mathematics teaching offers the best opportunity),

the formative element in the loftiest activities of the

human mind.

Certain mathematicians, conscious of the collective responsibility

they bear for the harm done by misuse of their

discipline, recognise that it is their duty to react. Academic

failure at the present time is no longer solely the failure of

pupils, it is the failure of the whole educational system,

and, if teachers fail in their struggle against this failure, the

responsibility will fall on those whose mission it is to train

teachers, in other words, those working in tertiary education.

The mathematical community must conscientiously do

its duty towards the school system through the initial training

of teachers and inservice training (with

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special emphasis on recent research on mathematics teaching)

for practising teachers, as well as by the development

and improvement of the forms of support

necessary for all those for whom it cannot be provided by

the family context.

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**EQUALS: An Inservice Program to Promote the**

**Participation of Underrepresented Students in**

**Mathematics**

Sherry Fraser

Mathematics has been called the queen of the sciences.

She could also be called the gatekeeper to the job market.

Too often, students who might find job satisfaction in a

scientific or technical field are unable to enter that field

because of inadequate preparation in mathematics. Why is

it that students, especially female and minority students,

stop taking mathematics in high school, as soon as it

becomes optional to do so?

Many women and minority students don’t see the relevance

of mathematics to their future lives. This perceived

lack of usefulness of mathematics contributes to the high

dropout rate. If students don’t see the need for math they

do not take the elective mathematics courses and effectively

screen themselves out of many potential careers.

Another factor in students dropping mathematics is their

lack of confidence in their ability to be successful in doing

mathematics. Traditionally, mathematics has been seen as

a male domain. In the United States it is socially acceptable,

especially for girls, not to be good in math. Unless

the student feels competent and confident in doing mathematics

she or he will not continue on when the courses

become optional.

Students need support from their teachers, counselors,

parents, and peers if they are to continue on in their mathematics

education. Intervention programs that develop students’

awareness of the importance of math to their future

work, increase their confidence and competence in doing

mathematics, and encourage their persistence in mathematics

have the best chance of success. Thus, EQUALS

uses these strands - awareness, confidence and competence,

and encouragement in its programs.

EQUALS is a mathematics inservice program at the

Lawrence Hall of Science, University of California,

B e r k e l e y, serving teachers, counselors, administrators,

and others concerned with keeping women and minority

students in mathematics education. It focuses on methods

and materials for the kindergarten through twelfth grade

level that will help attract and retain underrepresented students

in mathematics. Since 1977, 10,000 educators have

participated in EQUALS programs. Six national sites have

been created to disseminate the program throughout the

country.

The EQUALS program includes multiple approaches to

the issues of access and retention. At first, EQUALS

mathematics instructors—all former public school teachers—

set out to sharpen the participants’ awareness that

disproportionate numbers of women and minority students

decide not to continue on in mathematics in high school

and are thus unprepared to enroll in vocational or college

programs requiring quantitative skills. To develop a commitment

to recruiting and retaining students in mathematics,

participants must have an investment in the issues. So

we ask participants to investigate specific conditions indicating

their schools’ performance in bringing women and

minorities into mathematics (such as comparing math course

enrolments of males and females or surveying students’

career aspirations). The participants then become the

experts on the situation in their schools. They begin to define

the problem and are ready to work on solutions with

others.

Secondly, EQUALS provides teachers with learning materials

and methods that engage them in doing mathematics

with competence and enjoyment. EQUALS participants

encounter logic, probability and statistics, estimation, geometry,

and nonroutine applications of arithmetic for problem

solving. These are areas of mathematics that are

relevant for mathbased occupations and in which women

and minority students tend to lag behind males and majority

students in achievement tests. The learning environment

must be one that is cooperative and non-threatening.

EQUALS models the kinds of teaching approaches we

hope to encourage in the classroom, such as providing

time for people to work together on math problems and

other new materials; minimizing the fear of failure and

encouraging risk taking; providing manipulative materials

to use in making abstract math concepts concrete; and

helping people develop a range of problem-solving strategies

that suit their style of teaching and learning.

Recognizing that a number of people must have a stake

in such a program of change, the EQUALS approach has

been to build, where possible, coalitions of administrators,

teachers, and parents who will work cooperatively to

spread EQUALS through their schools. EQUALS participants

are strongly urged to teach fellow teachers and

parents the math activities they’ve learned in the program,

as well as some of the startling statistics about women’s

and minorities’ disadvantage in employment and earnings.

They bring to their classrooms or schools women and

minority men who work in math-based professions or

skilled trades. These people serve as role models and

encourage students to think about their futures in terms of

necessary and realistic work.

These activities help teachers convince themselves and

co-workers at school of the importance of EQUALS goals

and generate support for the often difficult task of inserting

EQUALS into a text-and-test dominated math syllabus.

The activities also reinforce the idea that EQUALS participants

are collaborators in the effort to make mathematics

meaningful and productive for students who otherwise may

be filtered out of a wide range of occupational choices.

During the year-long program, EQUALS participants keep

journals of their experiences using EQUALS in the classroom.

The journals reveal that many EQUALS participants

identify strongly with the math-avoiding students for whom

the program is designed. Again and again the program is

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experienced as a breakthrough for the teachers themselves.

A feeling of personal achievement perhaps contributes

as strongly as the practicality of the curriculum and

the vitality of the workshops to the program’s unusually

high evaluations — mean scores of 4.5 and above on a

scale of 5 in teachers’ ratings of the workshops, and findings

that at least 84% of participants apply EQUALS

immediately and continually in their classrooms. Schools

sending teachers to EQUALS report that in two or three

years they observe increased enrollments of previously

underrepresented students in advanced mathematics

classes and more favorable attitudes about mathematics

among all students. Most recent pre- and posttest data

indicate that EQUALS teachers and their students are

improving in their problem-solving skills as well.

Because of the need expressed by teachers for more

experience with computers, and its usefulness as a tool in

the mathematics curriculum, EQUALS in Computer

Technology was developed and offered for the first time

this year. Whether they have participated in a math or computer

workshop, EQUALS teachers experience astounding

growth, particularly in leadership skills, because they are

encouraged to speak up, make presentations, and deliver

ideas. Small victories are quickly acknowledged. As the

person grows, his or her commitment to the program and

the people who fostered that growth remains. As a result,

we have advocates everywhere, whose support, in turn, is

crucial to us.

What we have learned from the thousands of teachers

with whom we have worked is renewed respect for the difficult

work they do each day without the public support they

so desperately need. Many elementary and secondary

schools are alienating environments where teachers are

placed in adversary roles to students, parents, and administrators.

When they come to a program like EQUALS,

which provides a non-threatening, supportive environment

where they can take risks, make mistakes, and learn new

skills, their response is one of gratitude and enthusiasm.

What this tells us is that there is little opportunity for cooperation

and creativity in their own schools.

Our task then, as math educators, is to remember and be

sensitive to the many difficulties facing teachers as they try

to strengthen their mathematics programs and to provide

them with the respect and resources they need to accomplish

their goals. As teachers grow in their confidence and

competence in mathematics, so will their students.

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**FAMILY MATH**

Virginia Thompson

*Background*

Several years ago, we were asked by teachers in our

EQUALS inservice program to think about ways to provide

parents with ideas and materials to work with their children

in mathematics at home. Many parents had expressed

frustration in not knowing enough about their children’s

math program to help them or in not understanding the

mathematics their children were studying. In 1980, the

EQUALS program received a three-year grant from the

Fund for the Improvement of Postsecondary Education

(FIPSE) of the U. S. Department of Education to develop a

FAMILY MATH program for parents and children to learn

math activities together that would reinforce and complement

the school curriculum. Although the activities are

appropriate for all students, a major focus has been to

ensure that underrepresented students — primarily

females and minorities — are helped to increase their

enjoyment of mathematics.

The 12- to 16-hour FAMILY MATH courses provide

parents and children (kindergarten through 8th grade)

opportunities to develop problem-solving skills and build

understanding of mathematical concepts with «hands-on»

materials. Parents are given overviews of the mathematics

topics at their children’s grade level and explanations of

how these topics relate to each other. Men and women

working in math-based occupations meet with the families

to talk about how math is used in many occupations; other

activities are used to demonstrate the importance of

mathematics to future fields of study and work.

*Course Content*

Materials for each series of FAMILY MATH courses are

based on the school mathematics program for those grade

levels and reinforce fundamental concepts throughout that

curriculum. Topics include: arithmetic, geometry, probability,

statistics, measurement, patterns, relations, calculators,

computers, and logical thinking. The activities included in

this FAMILY MATH sampler illustrate how math topics are

approached. A career activity is also included. In any given

class, four to six activities are presented for parents and

children to do together. They then talk about how they solved

the problems and how these activities help with school

mathematics. Families are then given these and other activities

to continue the help at home. Often, parents will bring

in new books and activities they’ve found to share with

other class members.

*Participants*

Parents learn about FAMILY MATH from their children’s

teachers or principals, at PTA or school site council meetings,

through newspaper articles or church bulletins, or

from radio announcements. Classes are offered in the

afternoon or evening at schools, churches, community centers,

community colleges, or the Lawrence Hall of Science.

*Impact of the Program*

Evaluation shows that families can and do use FAMILY

MATH activities at home and that they have become motivated

to continue their exploration of mathematics.

Teachers and principals find that FAMILY MATH creates a

positive dialogue between home and school and a way to

involve parents in their children’s education.

*The Future*

During 1983 — 84, the FAMILY MATH staff will be offering

workshops to help parents and teachers to learn how to

establish and conduct FAMILY MATH classes; the full curriculum

will be published; and a film will be made of the program

to help disseminate its philosophy and approach to

communities outside of the San Francisco Bay area.

If you would like to be on the FAMILY MATH mailing list to

receive notices of available materials and workshops, please

send your name and address

to :

Virginia Thompson and Ruth Cossey

FAMILY MATH/EQUALS

Lawrence Hall of Science

University of California

Berkeley, CA 94720

We welcome your comments and suggestions for

future FAMILY MATH activities.

*Appendix*

Growth of FAMILY MATH Classes

in San Francisco Bay Area

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Year No. of No. of No. of Total No. of

Offered Classes Sites FamiliesParticipants

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1981 -82 6 3 46 67

1982-83 11 8 136 197

1983-84 16 12 345 654

(to date)

Total 33 23 527 918

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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FAMILY MATH Trainer-of-Trainer Workshops

at Lawrence Hall of Science

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Year No. of % Educators% Parents (w/o

Offered Participants direct educational

responsibilities)

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1983 115 77%

23%

1984 133 85% 15%

Total 248

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Evaluation of the courses, through observations and a follow-

up questionnaire, evidences a high level of math-related

activity undertaken by FAMILY MATH participants.

Over 90% of the 67 parents who attended classes regularly

during the first year have played math games with their

children and helped them with their math homework; over

80% have talked to their children’s teacher about their

mathematics progress. Parents have also taken actions for

themselves, including getting a math puzzle or game book

(50%); a math refresher book (27%); or taking another

math class (18%). These numbers compare favorably with

the implementation levels observed during the FAMILY

MATH sessions. It appears that the math-related activities

that are begun during the class are sustained.

In October 1983 and February 1984, we conducted two

2-day FA M I LY M ATH training sessions for interested

parents and teachers. The response to these workshops

was overwhelming: 140 applied to the October session and

164 to the February one. The logistics of handling that

many people for two days, 6 hours each day, was formidable.

Yet, because we could call on the entire EQUALS

staff of 9 mathematics educators, we were able to organize

people into groups of 30 and take them through the concepts

and activities of the 12-hour course. Evaluations of

these training sessions indicate extreme satisfaction and

high enthusiasm for the organization and conduct of the

program. Further, participants were asked how they would

use the training by means of a FAMILY MATH Planning

Sheet. From the October session, 51% indicated that they

would establish or team teach a FAMILY MATH course this

academic year and, according to our information, 22% of

them have already done so; an additional 16% have firmly

scheduled a class to begin this spring. In the February session,

53% stated they would offer a FAMILY MATH course

either over summer 1984 or during the 1984-85 academic

year. The majority of trainees who did *not* intend to conduct

classes said that they intended to use the materials in their

classrooms or at home with their children, at faculty and

school board meetings, at church, or at community meetings.

Project staff will conduct a follow-up of all trainees in

late spring 1984 to determine the extent of these dissemination

activities.

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**Mathematics for All is No Mathematics at All**

Jan de Lange Jzn.

Under the influence of Prof. Freudenthal’s Institute IOWO

(Institute for the Development of Mathematics Education)

Mathematics for ally has been a much discussed item in

the Netherlands for the last decade. Since 1971 lots of

materials were developed for primary education by the

Wiskobas department of IOWO and since 1974 Wiskovon

developed texts for secondary education.

It is not easy to characterise mathematics the way it was

developed during that period, but some of the much-used

slogans were:

Mathematics as a human activity - Everyday-life mathematics -

Mathematics in the world around you.

During the initial years it was not clear how big the influence

of IOWA was, but recent research (1984) carried out by

Rob de Jong showed that the influence of the

Wiskobas-group is very large as of this moment. As de

Jong stated:

«The lOWO-Wiskobas paradigm for math education can be characterised

as realistic which means among other things: connection with

informal strategies of children, using inspirmg contexts and aiming at

the comprehension of fundamental concepts. «

«The results of the research: Wiskobas characteristics have been

traced to a large extent and in correspondence with the original intentions

in five series of textbooks taking 35% of Dutch primary school,

and increasing.»

«Moreover: about 80% of the materials used in teacher training can

be characterised as IOWO-like.»

«Finally: when teachers are considering a new textbook, 4 out of s

take a ‘realistic’ method.»

This kind of primary mathematics for all may be illustrated

by the following examples:

*Examples*

1. The first example is meant for children of about ten

years of age.

Question: «Which camera presents the picture shown?»

2. Another one, for about the same age-group:

A map of part of the Island of Bermuda is presented:

Question: «Which drawing shows the situation as it is

seen on the Bermuda?»

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3. At a somewhat higher level are the following examples

(11— 13 years) about straight lines. A very simple problem:

«How can you place three cubes on a straight line»

is illustrated in the following way

A ship is finding its way on a river with numerous shallow

spots. To make navigation easy, a number of signs have

been placed on the border. Now you have to sail Jon the

straight line» formed by two of these signs that form a pair.

As soon as the next two signs are collinear, you change

course. This idea is also used when entering harbours.

Here we see a map of a harbour. Ll and L are lights. Ll is

much higher than L2.

The tugboat Constance and its tow are reaching the harbour

at Perry. The captain of the Constance estimates that

they will pull into port in about 15 minutes. He watches the

harbour lights very closely, especially Ll and L2.

During the last minutes he sees them like this:

Assignment: «Draw the route of the last few miles on the

worksheet.»

In a discussion it becomes apparent that the children are

capable of understanding the principle. Some of them are

even able to place themselves in the position at sea and

can translate horizontal information (the L1, L2 line) into

vertical information and make the right conclusions.

For lower secondary education quite a number of experimental

texts were developed by IOWO. Some 20 booklets,

mainly of a geometrical background, were the result of five

years of experimenting, observing and evaluating. Some of

this can be found back in one of the biggest and most

influential series of textbooks in the Netherlands. Some

continuation of the project — that had to stop when IOWO

terminated all activities in 1980 — takes place at the Foundation

of Curriculum Development, especially in the field of

global graphs.

It is interesting to note that the reactions of teachers to

IOWO-like material that is part of an established series of

books is at present more favourable than to the original

material eight years ago. Via those textbooks they sometimes

rediscover the original IOWO-material.

Most reactions are like «Mathematics can be fun» and this

seems to surprise teachers even more than students.

Some examples from the original IOWO-Wiskivon materials:

5. The closer you come, the less you can see. That is also

the problem of the lighthouse man.

The man walks towards the lighthouse. Behind the tower

rabbits are playing in the grass. At home the man tells his

children: «When I approach the light-thouse, I get closer to

the rabbits. Although they don’t run away I see less rabbits

when getting closer. Why?»

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6. Question: «Is the tower higher than the bridge or not?

Explain your answer!» Without proper preparation this is a

very difficult problem. Everybody knows the phenomenon,

but very few people are aware of what really causes it. The

designer hoped that a side view would arise more or less

spontaneously, but this was not the fact. But as soon as a

side view was suggested the pupils were able to say some

sensible things. Although these problems presented to 12-

to 13-year-olds still offer many difficulties.

7. Question: «How do you know the earth is a sphere?»

Answer: «Because when you are at the beach and a ship

is approaching the coast, you first see the upper part, and

only later the whole ship.»

Now this answer may not be completely correct, but the

next one is quite sophisticated: «When you see a picture of

the earth from a satellite, you see a circle.» The teacher:

«But then it can be a flat pie?!» «No, because wherever the

satellite flies, it always is a circle .»

8. Ratio and proportion, as well as the introduction of

angles can be done with shadows, as indicated above. But

there are, of course, other possibilities. One of them is

experimenting with flying model paper planes.

There are numerous plans to build a successful paper

model plane within 15 minutes. This activity alone has, of

course, some interesting geometrical aspects. But the

planes can be used for further experiments. It is necessary

that they fly reasonably, that means more or less in a

straight line.

Interesting is to compare this performance of the planes.

This can be done by observing how far each plane flies in

relation to the height it was thrown from.

It is obvious that for each student the height *h* will be different

(and more or less the same for one student) and that

the distance flown will vary. Let’s compare two planes:

*Plane 1 : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

*h* 90 90 90 90 90

*\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

*d* 450 400 360 500 480

*\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

*Plane 2: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

*h* 120 120 120 120 120

*\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

*d* 600 550 620 550 580

*\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*

It looks like some more experiments with plane 1 are

necessary to make conclusions about *the* distance flown.

Plane 2 behaves very decent. One might say it flies

around 580 cm, when launched at 120 cm.

Some additional flights with plane I make it fair to say that

plane 1 flies 480 cm when launched at 90cm.

The question arises: «Which plane is the best?»

This leads right into numerous aspects of ratio, proportions,

fractions, angles and percentages.

A rather simple way of solving the problem in a geometrical

way is by making scale drawings, cutting them out, placing

them on to each other and comparing the «glide

angles».

The smallest angle gives the best plane! Why?

Finally, some remarks on mathematics for all at upper

secondary level. Since 1981 experiments were carried out

that will lead to a completely new curriculum for mathematics

from August 1985 on. Applications and modelling play

a vital role in this curriculum for Math A, aiming at students

who will need mathematics as a tool. Math B is for students

heading for studies in exact sciences.

Math A seems to be very successful: Applications and

useful mathematics starting in reality seem a fruitful

approach even for students who are poor at «traditional»

mathematics.

More than 90% of the students at present choose mathematics

at upper secondary level which is up from 70% in

previous years.

During the experiment 20 booklets were developed for

use by the students.

Many of the ideas from these books are to be found in the

well-known textbook series in the Netherlands: once more

the influence of OW and OC (the research group that can

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be considered being the successor of IOWO) proved to be

considerable.

As a matter of fact, the experiments with Math A were so

successful that the vice minister of education considered

making math compulsory for upper secondary level.

Some examples of the Math A program are as follows:

*Examples*

One of the subjects that is part of the Math A program is

*periodic functions.* And as a special case: *goniometric*

*functions.* The latter is not a very popular subject in most

courses as we all know. From the experiments we get the

impression that embedding goniometric functions in the

periodic functions, and in real-life situations, makes the

subject much more motivating to the students.

9. The book starts with the electrocardiogram (ECG). An

ECG, taken at 16 different spots on the body looks like this:

From this, one can take the average (over the whole body),

and finally make a mathematical model that look like this :

In an earlier version, it was not explained what exactly caused

the different peaks in the graph. When students asked

questions about the heart functioning, (math) teachers

were unable to explain. So now we explain the relation between

the pumping of the heart and the ECG.

This first periodic phenomenon offers ample possibilities

for further questioning. For instance:

This is a part — one period — of an ECG of someone suffering

from a heart attack. The P-top is identified.

Explain in what way this ECG differs from the ECG of a

healthy person.

10. An interesting phenomenon that is worth mentioning

within the framework of periodic functions is the biological

clock.

The fiddler crab is very active during low tide, and rests

during high tide.

When the crab is taken away from the beach, the graph

changes dramatically:

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In this case, we say the periodic activity of the crab is

caused by an external biological clock.

The same crab has another periodic phenomenon:

It is coloured darker at daytime and lighter at night. Again,

when taken away from its natural environment and placed

in a situation with constant light, the periodic colouring

remains.

The fiddler crab has an internal biological clock as well.

11. The prey-predator model usually is not found in curricula

for secondary education. Certainly not in mathematics.

We tried to introduce this model in a very simple way.

From a story the students have to draw a graph representing

the growth cycle of the predator (lynx) and prey (rabbit)

as well.

From:

and finally, because of the periodicity to:

This — not so realistic — story and the graphs are analysed

and confronted with real-life graphs, which look pretty

much the same and have the same characteristics.

Also questions are posed about a very simple model:

Nl (t) = 200 sin t + 400

N2(t) = 300 sin (t -2 :5p) + 500

Finally, the children are given another way of drawing

graphs of prey-predator models, which emphasises the

periodicity of the model:

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Very important within Math A is the activity mathematising

and modelling. This is a complex and difficult matter and

offers lots of discussion.

13. The yearly average tide graph of a coastal town in the

Netherlands (Vlissingen) is indicated in this graph:

Assignment : «Find a simple (goniometric) model to describe

the tidal movement.»

Initially, three rather different models were found by the students

(17 years of age):

f(x) = 2 sin 1/2x

g(x) = 190 sin 1/2x + 8

h(x) = 190 sin pi /6.2

Of course, a lively discussion was the result :

f(x), that was clear was a very rough model : the amplitude

was «more or less equal to two meters» and the period

was 4p or 12.56 which is not «far away» from 12

hours and 25 minutes.

g(x), as the girl explained, was better in respect to the

amplitude: The amplitude of 190 cm, together with a

vertical translation of 8 cm gave exactly the proper

high and low tides which was very relevant to her.

h(x), was more precise about the period. This boy considered

the period more relevant «because you have to

know when it is high tide». The period proposed by his

model was 12 hours and 24 minutes, which really is

very close.

After a long discussion it was agreed that:

k(x) = 190 sin pi/6.2 + 8

was a nice model although some students still wanted to

make the period more precise.

Many people think that this kind of mathematics is no

mathematics at all. When Math A was introduced some teachers

asked if they really had to teach this. After working

for a year or two with Math A most teachers change their

minds: Math A is full of mathematical activities of a very

high level. On the other hand, we have to consider that the

ultimate mathematics for all is no mathematics — as a

separate discipline — at all: It could be possible that especially

this kind of mathematics disappears when all other

disciplines that use mathematics teach their part of mathematics

integrated in the discipline involved.

And so daily-life mathematics for all may disappear in

daily-life sciences or general education.

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**Organising Ideas in the Focus of**

**Mathematics for All**

Roland J. K. Stowasser To Hans Freudenthal

on his 80th Birthday

*Summary*

The history of mathematics offers outstanding examples of

simple, and at the same time, powerful *ideas which orga -*

*nise their surroundings,* ideas connected with each other in

a transparent network. Rather Pascal’s «esprit de finesse»

rules the process of thinking, and to some extent, by analogy

learning, too.

An impressive example shows that even less able students

can take profit by such an organisation of mathematical

knowledge. There is more hope that a new quality of

mathematics teaching might result from the epistemological

and historical point of view rather than from the currently

flourishing empirical research, categorising and doctoring

merely the symptoms.

*About Organising Ideas*

There is a lot of lip-service and well-intended general advice

for the use of history in math teaching, but very few worked

out examples of the kind I would like to talk about.

In the history of mathematics, I was looking out for *ideas*

- influential in the development of mathematics;

- simple and useful, even powerful which at the same time

could act as

- «centers of gravity» within the curriculum;

- knots in cognitive networks.

In that sense I call them *organising ideas.*

In the course of history new central ideas developed by

reorganisation of the old stocks of knowledge allowing to

draw a better general map from those «higher points of

view».

«La vue synoptique» brings to light associations hitherto

hidden.

This does not work by «longues chaînes de raison»

(Descartes). Not Euclid’s «I’esprit de géométries» rules the

process of cognition and by analogy the process of learning,

but rather *a mode of thinking related to Pascal’s*

*«esprit de finesse» paving short ways from a few central*

*points to many stations.* \*

Pascal himself provides a splendid example. When 16

years old, he reorganised the knowledge about conics handed

over by Desargues unveiling the «mysterium hexagrammicum

», ever since called Pascal’s theorem: the high

point surrounded by a lot of close corollaries.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

«(...) il faut tout d’un coup voir la chose d’un seul regard, et non par

progrès du raisonnement, au moins jusqu’à un certain degré (...)

(Pascal, Pensées, ed. Lafuma 512 : the difference between l’esprit

de géométrie and l’esprit de finesse.)

Having cut organising ideas out of the historical context

one is left with the hard task to process them into more or

less comprehensive teaching modules (or even schoolbook

chapters). The products can be problem fields in

which very few organising ideas instead of dozens of theorems

operate as a means of problem solving.

Concentrating on a few simple, and at the same time,

powerful ideas which organise their surroundings and

which are connected to one another within a simple network,

offers help for the less able student, too. His inability

derives to a large extent from the fact that he is unable to

organise his thought with respect to a complex field in

which the connections are presented in the usual plain logical

systematical way and where teaching is used to administer

only spoonfuls of the subject matter, disconnected

and without depth.

*An Example on the Idea of Congruence*

In German schools, pupils have to learn and apply some

special divisibility rules, the end-digit-rules for 5, 25, 2, 4, 8

and the digit-sum-rule for 9 (not more!).

The organising idea behind the different looking types lies

hidden away as it was before Pascal’s paper about a generalised

digit-sum-rule (see appendix to [1]). He wrote the

paper in a mathematician’s fashion: describing the algorithm

by some simple examples (9-, 7-rule) and giving a

proof by recursion. His approach is not an appropriate proposal

for an interesting lesson on divisibility rules for 11

-year-olds.

Take my approach with the very familiar clock on the face

of which, so to say, Pascal’s idea, and even the more general

idea of congruence comes to light in a very simple way.

I quote from [1].

«In the 11-year-olds’ daily life experience the clock is just the right

thing to start with. I have a big cardboard clock without a minute

hand. The hour hand is on the 12.

A pupil is called to the blackboard. He is told to write down

the number of hours the hour hand should move. He writes

down an unpronounceable number of hours which goes

from left to right across the blackboard. Three seconds

after the last written figure I put the hour hand on the right

hour. For example, the pupil writes:

2045010223053123456789024681357902541403.

I put the hour hand on the 7. The pupils check by dividing

through by 12. One page is filled. Plenty of mistakes. In the

right hand corner is written the only interesting thing: the

remainder.

The pupils know that I am not a magician, especially that I

am not good at mental arithmetic. Of course I do not reveal

the trick. The pupils will work for it, discover it. A prepared

work sheet asking «what time is it», that means asking for

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the remainders regarding 12, shows a pattern: the remainders

of 12 (Zwölferreste) divided into the powers of 10

(Zehnerpotenzen) from 102 on, are all equal, thank God.

R12 (l0n) is constant for n 2.

Every 10th power pushes the hour hand 4 hours ahead.

I assume, otherwise it has to be dealt with, the pupils really

know what the abacus is, that they can see a decimal

number consisting of powers of 10.

Now my mystery trick in arithmetic is solved. No matter

how many digits there are in front, the hour hand simply

jumps to and fro among three positions (beyond the tens).

I prefer to do the rotation of the hand mentally instead of on the

actual clock. In the example R12 (2106437822) my mental arithmetic

looks like this:

The 22 hours at the end, being out of the routine, put the

hour hand in the final position:

The 11-year-olds even understand my enquiry whether the calculations

on the working sheet confirm that R(10000000000) = 4. The

reason why - hidden reasoning by induction because of the recursively

defined powers of 10 - can be found by 11-year-olds with a little

help.

Four hours remain from 102 hours after taking away the half days.

From 103 =10.102 hours remain 10 • 4 hours. From 103 hours remain

therefore again 4 hours after taking away the half days. We proceed

in the same way for 104 = 10.103(...)»

So far, the 11 -year-olds have discovered and fully understood

the quick method for remainders by watching the

familiar (Babylonian) clock. There will be no real difficulties

to transform this method so that it can be applied to arbitrary

non-Babylonian divisions of the day (e.g. 11-, 18-,

37-hour-clocks). Pascal’s idea has been grasped from the

paradigm (12-hour-clock) and transferred, not abstracted

from a lot of examples. As a teacher, I couldn’t but follow

Aristoteles’ advice for teaching, as opposed to research, to

use but one example, *the* paradigm, which embodies the

idea.

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More Examples from TU Berlin — An A n n o t a t e d

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donées a l’école normale en 1 795» teaching about 1300

prospective teachers from all over France, foreseen to form the

staff of local teacher colleges, presents a very simple but universal

method to solve geometrical construction problems. The infinite

halving procedure, similar to the metric system, used already

by Stevin to solve engineering problems, became later with

Bolzano a very simple proof method. One and the same proof

scheme works and can be successfully applied by even high

school students to all the fundamental theorems about real

sequences and continuous or differentiable functions (theorems

about cluster points, ..., intermediate, mean, extremal values, ...).

For the general opinion these facts lie in the deep sea. But actually

they float on the surface, held together and ruled by the simple

idea of repeated halving. The insight, that measuring and proving

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Preparatory Course» concentrates upon the fundamental idea of

recursion as a tool for modelling, especially combinatorial situations,

rather than for proving.

Difference equations treated before differential equations, linear

methods as heuristic means to find say Binet’s formula (1843) for

the Fibonacci sequence (1202), polynomial interpolation etc. offer

a good chance for general practice and preparation of the harder

material to follow: in the Analysis, Linear Algebra, Stochastic

Courses.

The booklet explains in a manner suitable for l.6- to 17-year-olds

a philosophy of math teaching which combines Polya’s problem -

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**CSMP: Realization of a**

**Mathematics Program for All**

Allan Podbelsek

**Introduction**

*Focus and Direction from Damerow ‘s Paper*

In his paper, «Mathematics for All,»\* Damerow raises an

interesting dichotomy relative to mathematics education —

do we continue with a mathematics curriculum designed

essentially for a small, elite group or do we develop a program

designed to bring more of the essence of mathematics

to all students? Stated another way - do we keep the

highly selective framework and methods of traditional

mathematics education but give up the privileged position

of the subject as part of the core of general education; *OR,*

do we seek to keep mathematics at the core of the curriculum

but find a way of teaching the subject to all students?

In spite of 25 years in mathematics education, I still personally

believe in attempting to teach as much of the essence

of mathematics as possible to the general student population.

Therefore, it is issues related to the latter choice that

this paper will address.

*Basis for this Paper*

As a student, mathematics was presented to me in the traditional

manner. Because of some effective teachers and

my own desire to see things related to one another, I developed

an interest in and an appreciation of mathematics as

a unified body of knowledge rather than a set of individual,

isolated topics. Consequently, in the past 10 years, much

energy in my work has been devoted to implement a unified

or integrated mathematics program to a subset of students

in the school system in which I am employed. It is my

hope that someday mathematics programs in the United

States will be unified in nature. Perhaps this will happen by

the turn of the century along with metrication! Through thinking

about mathematics as a unified body of knowledge

and dealing with teachers, administrators and community

members who were often resistent to such ideas, I developed

a strong interest in the very topics that Peter Damerow

addresses in his paper.

In the past five years, I have had quite a lot of involvement

in the implementation. of an innovative,

integrated program at the elementary (K — 6) level.

This program is the CSMP (Comprehensive School

Mathematics Program). CSMP was developed over a

period of several years by mathematicians and mathematics

educators from several countries. I believe

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\* Paper presented to the ICMI Symposium at the International

Congress of Mathematics, Warsaw, August 1983. Zentralblatt fur

Didaktik dot Mathematik, 16 (1984) pp. 81-85.

that one of the motivating factors of its founders was the

work of the mathematicians and mathematics educators

whose ideas were published in the 1963 «Cambridge

Report.» From initial leadership by Burt Kaufman, the federally

funded project moved from Southern Illinois University

at Carbondale, Illinois to its home at the Central Midwest

Regional Educational Laboratory in St. Louis, Missouri

(CEMREL).

When Ian Westbury called me last fall to discuss the

paper written by Peter Damerow, we discussed the

concept of Mathematics for All.» Immediately, CSMP came

to my mind as an excellent example of a program designed

in the spirit that I thought Peter Damerow had in mind. In a

later letter from Peter Damerow I was surprised to learn

that he was surprised to know that CSMP was still in existence!

*Purpose of this Paper*

Because of my belief that CSMP does represent a realization

of a mathematics program for all, I proposed this

paper. *It is the purpose of this paper to discuss/ evaluate*

*CSMP as a possible realizatiorl of a mathematics program*

*for all.* In order to accomplish this task, it seemed reasonable

to enumerate some critical attributes of a program

designed around a mathematics-for-all philosophy. In the

next section of this paper, I will elaborate on the attributes

of such a program based on my study of Peter Damerow’s

paper, and my own reading, understanding of mathematics

and experiences in mathematics program development.

In the third section of this paper I will describe CSMP and

evaluate it with respect to the criteria outlined in the second

section. I will also discuss the implementation of CSMP in

Louisville, Kentucky, as well as perceived sufficient conditions

for successful implementation of this program in

general. The third section will be followed with some

conclusions and recommendations.

**Critical Attributes of a Mathematics-for-AII Program**

What should a mathematics program for all be like? How

should it differ from many of the various, current programs?

Is it the topics and content included that must differ or, is it

the sequencing of the topics that should change? Is the

classroom structure an inhibiting factor in teaching mathematics

for all? Is it the background of the personnel asked

to teach mathematics that serves as a hindrance to teach

mathematics for all? What about the community expectations?

What about the mindset of the teachers of

mathematics — how they have been trained and what their

perceptions of mathematics are?

It would take too much space and time to describe in great

detail the attributes of a mathematics program for all.

However, I believe that using some broad constructs, it is

possible to communicate the essence of what is meant by

such a program. Below is a list of possible goals stated for

the purpose of analyzing CSMP:

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1. Develop the capacity to understand and interpret numerical, spatial,

and logical situations which occur in the world in which one

lives

2. Develop a scientific, questioning, and analytic attitude toward

mathematical problems

3. Develop mathematical knowledge, skills, and understandings relevant

to one’s personal and vocational needs to include:

a. problem solving

b. application of mathematics to everyday situations

c. estimation and approximation

d. using mathematics to predict

e. reading, interpreting, and constructing tables, charts

and graphs

f. computer literacy

g. understanding and application of basic operations

4. Develop an awareness and appreciation of what is mathematics

by recognising and using the following features of the subject:

a. content dealt with in mathematics

b. types of thinking used within the discipline

c. methods of proof

d. orderliness of mathematics

e. beauty of patterns and structures of mathematics

f. power of mathematical processes, patterns, and structures

g. interaction of mathematics with other areas of human activity

h. every spiraling development of mathematics through the history

of pcople

i. balance between inductive and deductive reasoning

Most of the above stated goals are self-explanatory. For

any program which proposes to teach mathematics for all,

it is important to examine how the program would teacher

present each of the goals or Subgoals included above.

**Evaluation of CSMP as a Mathematics-for-AII**

**Program**

*Brief Description of CSMP*

In its recently published report on CSMP, entitled

«Conclusions and Recommendations of the Evaluation

Review Panel,» the Evaluation Review Panel be

gins with a rather concise statement of what CSMP is

all about.

The Comprehensive School Mathematics Program (CSMP) is a dramatic

curricular innovation in elementary school mathematics. During

its development, conscious decisions were made about how mathematics

should be taught. The most important of these were the follo -

wing:

- Mathematically important ideas should be introduced to children

early and often, in ways that are appropriate to their interests and

level of sophistication. The concepts (but not the terminology) of

set, relation and function should have pre-eminent place in the

curriculum. Certain content areas, such as probability, combinatorics,

and geometry should be introduced into the curriculum in a

practical, integrated manner.

- The development of rich problem-solving activities should have a

prominent place in the curriculum. These activities should generate

topics, guide the sequencing of content, and provide the

vehicle for the development of computation skills.

- The curriculum should be organized into a spiral form which would

combine brief exposures to a topic (separated by several days

before the topic appears again) with a thorough integration of

topics from day to day.

- Whole group lessons should occupy a larger and more important

role in mathematics class and teachers should be provided with

highly detailed lesson plans which lay out both the content and

pedagogical development of lessons. Furthermore, training in

both the content and pedagogy of the program should be made

available to the teachers.

These beliefs about the teaching of mathematics were translated

with remarkable integrity into the eventual curriculum materials.

CSMP is a model of one very distinctive way of teaching mathematics

and is one of the few that can be studied in detail by mathematics

education researchers and teachers. Its implementation and evaluation

in schools is, in a sense, an experimental test of these distinctive

features.

In this K— 6 program, the objects of mathematical study

are: numbers of various kinds, operations on and relationships

between numbers, geometrical figures and their properties,

relations and functions, and operations on functions.

Growth in the ability to reason is seen to play an

important role in the study of mathematics. CSMP developers

argue that in mathematics the development of the art

of reasoning goes hand in hand with the growth of imagination,

ingenuity and intuition. In this perspective, elementary

arithmetic takes on the form of «adventures in the

world of numbers. « Individual numbers can assume a personality

of their own in this world. Teaching arithmetic shifts

its emphasis from an obstacle race for mastery of basic

skills to the stimulation of exploring the world of numbers.

Skills such as balancing a checkbook have their place in

real life but are of little interest to a fifth or sixth grader and

are not realistic activities for goals of an elementary school

mathematics program. More genuinely rewarding are the

stretching of the powers of imagination and the challenging

of the mind.

CSMP believes that its program requires a special pedagogy

which they call «a pedagogy of situations.» A pedagogy

of situations is described as Zone which is based on

the belief that learning occurs in reaction to the experience

of confronting a situation (real, simulated, or imagined) that

is rich in consequences, is worthy of confrontation, and has

genuine intellectual «content.» As such, a pedagogy of

situations has the following properties:

1. involves children at a personal level,

2. presents an intellectual challenge that is accessible to a broad

range of abilities,

3. provides opportunities for creativity,

4. supports experiences that have mathematical content.

The CSMP curriculum presents a large number of varied,

yet interrelated situations that provide the experiences out

of which mathematics grows. The philosophy of the program

is based on the idea that there is no reason why very

young children should not have the pleasure of mathematical

thinking at an elementary level and of exploring

mathematical ideas.

CSMP uses three languages to express mathematical ideas at a

young child’s level. These languages are: (1) the language of strings,

(2) the language of arrows, and (3) the language of the Papy

Minicomputer. The language of strings is used to help children think

about classification of objects. Its structure is much like that of the

Venn diagram. The language of arrows helps the children think and

describe relations between objects. Its structure underlies the

concepts of function and vector to be studied at a later stage in the

mathematical development of the child. The third language is based

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on a simple abacus and is called the Papy Minicomputer. With this

concrete model, children explore and learn number concepts such as

place value and develop computation algorithms.

Four major content strands comprise the CSMP curriculum. These

are:

The world of numbers

The languages of strings and arrows

Geometry and measurement

Probability and statistics

Attached in the Appendix is a «Summary of the

Mathematical Content of the K— 6 Curriculum.»

*Some Conclusions of the Evaluation Panel*

Below is a summary of the comments made by the evaluation panel.

1. The most important conclusion about CSMP is that it does teach

problem-solving skills better than the standard textbook curricula.

2. The original CSMP belief that merely doing computations as part

of the problem activities will develop computational skills as well

as the traditional program does is not justified by test data.

(Modest supplementation removes differences.)

3. CSMP belief that emphasizing problems in a group setting and

posing problems directly in the CSMPlanguages will develop adequate

skills in word problems is justified by test data.

4. CSMP student effects should be appreciatively larger when more

experienced teachers use the revised program. (It was found that

often the teacher did not receive sufficient training.)

5. CSMP students probably know more mathematics than the evaluation

results indicate. (Tests given do not measure all of the

mathematics learned.)

6. CSMP has positive effects on students at all ability levels. (This is

important in a mathematics-for-all situation.)

7. The spiral feature of CSMP may be one of the most widely applicable

of all features of the program. (More research is needed to

determine how the mechanics of the spiral curriculum affect student

learning at different points in time.)

To embark upon the implementation of a program as innovative

as CSMP is a complex and difficult task. In the

United States, conditions are not usually conducive to easily

making the kind of changes in teaching mathematics

required by CSMP. The Evaluation Review Panel for CSMP

summarizes these conditions succinctly as follows:

The status quo of mathematics education makes curricular innovation

almost impossible. Content and sequencing of topics have

always been heavily influenced by the very traditional, computationally

oriented view of mathematics held by many school administrators,

principals, and teachers. Recent increased use of commercial

standardized tests, and state and locally mandated competency

tests, together with public dissemination of the results of these tests,

have narrowed the traditional focus further so that, to a large extent,

these tests effectively control the curriculum. (...)

This accountability movement has placed increased pressure on teachers

to have students achieve these goals, even to the exclusion of

other less well measured goals such as problem solving, or less well

understood content such as probability. In the future, successful curricular

innovations are likely to be limited to those which can provide

advance proof of those positive student effects which are valued by

the public as represented by school boards and administrators.

*Analysis of CSMP Using Criteria for Mathematics-for-AII*

*Program*

In the second portion of this paper (page 4), several goals

that attempt to characterize the essence of the content of a

mathematics program-for-all were listed. In this section,

CSMP is analyzed relative to these goals. Each of the

goals is restated below with comments relative to the

CSMP status with respect to the stated goal.

1. Develop the capacity to *understand* and *interpret* numerical, spatial,

and logical situations which occur in the world in which one lives.

CSMP: CSMP is rich in situations where students must develop this

capacity. However, the program’s authors are not so concerned that

these situations be in the «real» world from the adult point of view.

Their goal is to create situations for this goal which are in the «real»

world of the learner which depends on the age and experiences of

the learner. CSMP does this very well.

2. Develop a scientific, questioning, and analytic attitude towards

mathematical problems.

CSMP: The variety of pedagogical situations through which the

content of the program is developed does an excellent job of focusing

on this goal. The detailed dialogue provided for the teacher often

follows a discovery approach in which the students are pushed or

guided to question, analyze, predict or conjecture.

3. Develop mathematical knowledge, skills, and understandings relevant

to one’s personal and vocational needs includmg:

- problem solving,

- applications to everyday situations,

- estimation and approximation,

- using mathematics to predict,

- reading, interpreting, and constructing tables and graphs,

- computer literacy,

- understanding and applying basic operations.

CSMP: CSMP emphasizes problem solving, estimation and approximation,

using mathematics to predict, charts and graphs, and

understanding and application of basic operations. Again it takes a

different point of view relative to applications to everyday situations

because everyday situations are seen from the perspective of the

learner. Computer literacy is not included in the written program.

Emphasis on understanding the basic operations is strong. Less

emphasis on mastery of pencil/paper algorithms is predominant in

the philosophy of the program. It is worth noting, however, that great

stress is placed on mental arithmetic.

4. Develop an awareness and appreciation of what is mathematics

by recognizing and using the following features of the subject:

- content dealt with in mathematics,

- types of thinking used within the discipline,

- methods of proof,

- orderliness of mathematics,

- beauty of patterns and structure of mathematics,

- power of mathematical processes, patterns, and structures,

- interaction of mathematics with other areas of human activity,

- spiraling development of mathematics through history,

- balance between inductive and deductive reasoning.

CSMP: CSMP presents to the learner a broad picture of what is

mathematical content. Through teacher led discussions, student activities

and written work, the types of thinking associated with mathematics

is illustrated and practiced. Through string activities intuitive

arguments are practised. However, formal proof is not dealt with in

this program which terminates at the end of grade six. The stage is

set for pursuit of proof in grades seven and above. Through explora-

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tions and problem-solving situations, the orderliness of mathematics

is frequently acted out. In CSMPproblem-solving situations, patterns

and structures are prominent. Students are frequently asked to utilize

a pattern in order to determine a function rule. In addition, the

power of mathematics is experienced frequently as students use

various structures and models to understand relationships and solve

problems. Interaction of mathematics with other areas of human activity

is seen in some of the special workbooks. Again, these activities

are often those more realistic to the world of the child. The program

is spiraling but no emphasis is made relative to history of people.

There is a nice balance between intuitive/inductive reasoning and

checking guesses in a semi-formal manner.

As is evident in the analysis, CSMP fares very well in

terms of the goals stated in a mathematics-for-all program.

The greatest lack seems to be in the areas of computation

(particularly from a drill and practice/mastery viewpoint),

computer literacy, and history of mathematics.

*Implementation of CSMP*

In the past six or eight years, CSMP has had fairly extensive

implementation in the United States with considerable

success. To successfully implement CSMP at the

local level requires a lot of coordination and special attention

to several areas. These are outlined and discussed

below.

1. *Teacher Training.* CSMP indicates that 30 hours of training is

recommended to help teachers learn about the program and how to

teach it Much time is required to bring teachers to understand and

appreciate the philosophy of CSMP.

2. *Materials.* CSMP materials are mostly consumable; therefore,

there is considerable recurring cost which makes the program more

costly to maintain than a traditional one.

3. *Community Awareness.* Because their children will he bringing

home materials so different from what is brought home in the traditional

programs, there must be a well-articulated plan to acquaint the

parents with the program.

4. *Administrative Stagy Awareness*. Local school principals and central

office staff must know and understand some of the aspects of this

program because they are in positions where they often must explain

a program to community members.

In my own school system, Jefferson County Public

Schools, Louisville, Kentucky, initial implementation began

in 1979 for second and third year students. The initial

implementation at the sixth year was in 1982. Overall, the

teachers and parents are quite pleased with the program.

As was expected, teachers were concerned about students

developing a mastery of certain pencil/paper algorithms

especially those that are tested on the achievement tests.

It is very important in our community to show improvement

on achievement test scores and a great deal of the school

system’s image in the community is determined by performance

on these tests. Therefore, some instruction related

to computation was provided to students from traditional

texts.

The school system’s elementary specialist received over

30 hours of training in St. Louis before assuming responsibility

for training teachers for grades one through five. In

1982, I spent three days in St. Louis preparing to train the

sixth grade teachers.

Early months of implementation for teachers of grades

one through five were hampered by the fact that the teachers

had at most two days of training. A few teachers

received no training. Getting them out of the traditional textbook

was not an easy task. Because of these problems,

special care was taken to provide sixth grade teachers with

more training. Thus, I provided 60 hours of intensive training

to nearly all sixth grade teachers involved in teaching

the program. These teachers were much more confident in

their first year of implementation than were the elementary

teachers.

It was very helpful to have an expert from CSMP come to

our system to provide information to a large group of community

members. Generally, as parents learn and see what

the program can do, they are very supportive .

Getting and maintaining materials and supplies has been

a challenge in our system. The annual cost of consumable

materials threatens the continued use of the program in our

school system.

Recently, a traditional textbook which utilizes some of the

philosophy of CSMP was chosen as a supplement to the

CSMP materials. Perhaps in another five or ten years,

other traditional texts will incorporate some of the CSMP

philosophy.

*Sufficient Conditions for lmplementation of CSMP*

Based on my experiences over the past eight years, the

four areas cited above under Implementation of CSMP»

are absolutely essential. The Evaluation Review Panel for

CSMP strongly supports this claim. In addition, they indicate

that the role of the local coordinator in implementing

and managing the program in school districts is vital to the

success of CSMP. Without a skilled and influential person

at the helm, a solid implementation of CSMP is almost

impossible. Someone has to interpret the program and

help teachers, administrators, and parents realize the need

for and importance of the program.

***Summary***

*Some Conclusions*

The purpose of this paper was to provide evidence that

CSMP is a realization of a mathematics program for all

based on implications of the Damerow paper. According to

the criteria developed in this paper to characterise a

mathematics program for all, CSMP rates very high. The

program is effective with all ability groups and it does much

to foster problem solving through mathematizations.

Shortcomings seem to be in the areas of computer literacy

and history. In addition, CSMP places less emphasis on

computation with paper and pencil than is required by

school systems in order for students to perform well on the

achievement tests.

On the other hand, CSMP is compatible with some recent

trends in mathematics and mathematics education. Some

of these trends are listed below:

1. increased emphasis on problem solving,

2. increased mathematics requirements for highschool

graduation,

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3. need to provide teachers with more mathematics

training,

4. increased use of computers in schools,

5. increased interest in discrete mathematics and algorithmic

thinking in mathematics.

Some Recommendations

CSMP has much to offer as a mathematics program

for all. However, it must be scrutinized and updated

in several ways. Suggestions of areas in which change

should be considered are listed below.

1. The use of computers must be brought into the

program

2. Logo should be used in the text materials

3. More use of history of mathematics should be included

4. Programs should be developed at higher grade

levels to sense as an extension to the present

program which terminates at the end of sixth

grade

5. Ways should be explored to make the program

more cost effective

6. The ICME 5 Theme Group on Mathematics for All should

explore ways that CSMP and other similarIV. Geometry

programs can be supported A. Networks

7. Research should be planned to help develop CSMP. As

this theme group considers issues related to the transformation

of mathematics education from the training of

experts into an essential part of general education, I

hope that the contributions made by CSMP toward this

end will be examined and valued. I believe CSMP is

one of a few, if not the only, ex-emplification of a

mathematics program which is built Kin the spirit» of

what should be a mathematics program for all.

*Appendix*

**A Summary of the Mathematical Content in CSMP**

*Kindergarten*

I. The World of Numbers

A. Counting

1. Count dots in pictures.

2. Draw a given number of dots.

3. Find the dot picture that corresponds to a given numeral.

4. Play counting games.

B. Numeration

Recognize and write numerals for whole

numbers.

C. Order

1. Play a game in which whole numbers are located on

the number line.

2. Compare sets to determine which has more elements.

D. Addition and Subtraction

Interpret and draw dot pictures for

simple addition and subtraction stories.

II. Probability, Statistics, and Graphing Data

Collect and record data in bar graphs.

III. Problem Solving and Logical Thinking

A. Reasoning

1. Use clues to identify unknown numbers.

2. Recognize and represent the intersection of two sets.

B. Relations and Functions

Interpret and draw arrow pictures for

simple relationships.

C. Sorting and Classifying

1. Sort and classify objects such as attribute blocks and

centimeter rods.

2. Place dots for objects in Venn diagrams.

D. Patterns

Determine a rule for a sequence of objects.

IV. Geometry

A. Networks

Follow one-way roads from one point to another.

B. Taxi-Geometry

Draw taxi-paths from one point to another.

C. Measurement

1. Compare lengths of centimeter rods and

lengths of paths on a grid.

2. Use attribute blocks to compare areas.

D. Transformational Concepts

Work with mirror and «cut-out» activities

that involve reflective symmetry.

E. Euclidean Concepts

1. Recognize circles, triangles, squares, and

rectangles.

2. Do activities that involve spatial relationships and

perspective.

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**Mathematics for Translators Specialized in**

**Scientific Texts — On the Teaching of**

**Mathematics to Non-Mathematicians**

Manfred Klika

Mathematics is the unique art and

science that enables us to cope with the

complexity of economic, social and

technical problems in a rational, quantitative

way. The education and training of

students in this field is an international

concern. (Proceedings of ACME 4)

Comprehending that which can be comprehended

is a basic human right. (M.

Wagenschein)

*Introduction*

Perhaps you are surprised that I want to present a contribution

on a topic which seems to be very specific. Yet in

fact I deal directly with the main questions presented in the

paper of the Organizing Committee of this theme group [5]:

\* What kind of mathematics curriculum is adequate

to the needs of the majority, what modifications to

the curriculum are needed for special groups of learners?

In this paper, the extent to which experience gained in our

project could be transferred to the teaching of mathematics

to the majority will be discussed. Starting with specific

objectives, I will develop my argument to reflect on mathematical

education in the future.

*The Structure of the Course «Fachübersetzen «*

Because of a large amount of international co-operation,

the need for high quality translations has increased in

recent years, especially in the physical sciences and engineering.

A trend-setting degree program has been set up at

the Hochschule Hildesheim for training

«Fachübersetzer»—technical translators in specific technical

fields (at present limited to mechanical and electrical

engineering). The program is aimed at providing the students

with both a practically oriented and a theoretically

based foundation in linguistic attainments closely connected

with knowledge in technical fields.

The students should acquire the capability of seeing specific

interrelations within their future fields of activity, of working

independently, of working with problems, between disciplines,

with a scientific approach. The close integration of

linguistic and technological studies is achieved by using

the subjects covered in the technical courses of one year

as the basis for exercises in translating technical texts in

the following year of the program (see the table «Structure

of Coursework ‘Fachubersetzen’»).

In order for the students to gain the solid foundation

necessary for studying technical fields, compulsory lectures

are offered on the «fundamentals of technology»

during their first two semesters in which mathematics plays

an important role.

The program is addressed to students who have a special

inclination and ability for languages and who are also interested

in physical sciences and engineering. Employment

openings for the graduates include: translators in trade and

industry, national and international agencies, and publishing

companies, and linguists in related fields like terminology,

lexicography and documentation.

But there may be a problem in the future here. Far-reaching

changes are now also occurring in the field of linguistics

as a result of advances in electronics; computers

are being used more and more in the work of translation.

Already, future technical translators are being introduced to

the methods and problems of mechanical translation

during their course of study. In the next few years, this

component of their study will be undertaken in conjunction

with a study of computer science. Both «Fachübersetzen»

and computer science could meaningfully work together in

linguistic data processing.

Up to now research has provided some impressive

results, in particular in the translation of very specialized,

greatly standardized texts. Computers will be able to relieve

translators to an increasing extent of routine work, so

that they will be able to devote all of their time to more

demanding linguistic tasks. The linguistic and technical

demands on translators will therefore certainly increase in

the next few years. But I venture to make the following prediction:

computers will not replace translators.

*Experiences with «Fundamentals of Technology» and*

*Conclusions Relating to the Overall Aims of*

*«Mathematics for All»*

Because our lessons on technology and mathematics were

closely related, our major aims had to be these: team-work,

co-operation, and the introduction of alternative forms of

instruction.

The customary courses in university mathematics were

either not suitable or not available, so I had to construct a

new mathematics curriculum myself.

Some details about the framework:

- The mathematical qualifications (abilities, knowledge,

skills etc.) of our students are extremely varied. Tests

which have been carried out show that:

\* knowledge that can be examined is learnt only for examinations

(overemphasising the calculation component);

\* the abilities of imagination, estimation, and visualisation

are developed to a relatively low level;

\* graphs of functions and related equations of functions

are poorly associated with each other;

\* even many students who are very interested in technical

subjects lack confidence in their mathematical ability

to solve a problem in the technical field.

- It was not appropriate for me to teach pure mathematics,

for, as experience shows, linguistically

oriented students frequently have a weak mathe-

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matical background, and, correspondingly, have little

interest in (purely theoretical) questions in this field. (By

the way, the majority of our students are female. This

has given rise to very interesting new insights on the

topic of women and mathematics, e. g. [10]; our experience

up to now has not shown the existence of the attitudes

that are usually attributed to women concerning

technical things.)

The trend to modern technologies (calculators, computers)

appears to be reducing the amount of calculation;

therefore it makes no sense to perform calculations

without calculators and computers in our curriculum.

Furthermore, we cannot exclude the possibility that in

the future the relationship between linguistics and computer

science as described above may even change

attitudes towards mathematics and the learning of

mathematics.

There are a lot of important inquiries into the teaching of

mathematics and many critical papers have been written

which show clearly that the present secondary school

mathematics curriculum doesn’t achieve either its own

goals or those expected by others (e.g. [1, 6, 7]). The

remarks made in the section «Selectivity of the School

System» by the Organizing Committee of our theme group

[5] set us thinking, too.

Our own experiences with the first mathematical courses

for technical translators show the same results: the curriculum

of traditional school mathematics (that is primarily

aimed at formal education and computational skills)

doesn’t prepare the students for questions requiring comprehension

and sense, particularly those involving central

mathematical terms which are used in technology as

«hand tools.» Students have no idea how to use the disconnected

details, the significance of which they do not

understand.

A frequent question, Couldn’t we do some practical exercises

on this subject?» refers clearly to the problem touched

on above. Through a «retreat into calculation» perhaps

some students believe they come to understand the

facts. No wonder this is the case because at school they

have been led to make exactly the same assumption.

Bauer [2] shows that because only aspects such as

«memory,» «cognition,» and occasionally «production,»

are tested in school-leaving examinations, these are therefore

the very things that are expected in school mathematics.

(By the way, in our course formulae are used occasionally

and calculations made, naturally, but only to further consolidate

comprehension of formulae and concepts.)

But it would be worse to Renounce mathematics as a

substantial part of the core curriculum of general education.

»[5] The consequence of this would be that the comprehension

of mathematical interrelations would deteriorate

further, even more than appears to be the case now.

There is no reason to deny the significance of mathematics

for all in the future [6]. On the contrary, it is a very

important objective of mathematical education in the future

to raise the level of attainment to a higher degree.

How could this be done?

In a recent paper, R. Fischer [4] has argued that «one of

the functions of mathematics education within the official

educational system should be to contribute to a liberation

from mathematics.» This means that mathematics has

become independent in view of its richness and wealth of

material, which he calls its «second nature.» Men are running

the risk that mathematics will control them, and it will

be necessary for them to take steps to avoid this possible

state of affairs. «Mathematics education can fulfil this new

function mainly by emphasizing questions of sense in the

classroom and, thus, questions of the relation between

men and knowledge.»

I feel this is a very important aspect, as it improves the

mathematics curriculum for the teaching of all students.

With regard to the suggestions mentioned above and the

fact that the content of the curriculum has to be structured

in a new way, I see as particularly important the role of fundamental

ideas.

*Fundamental Ideas*

A possible answer to some of our main problems in this

theme group «mathematics for all» has to be to point out

«fundamental ideas. « The problems of general mathematical

education stem from courses which are too full and

which contain mathematical topics which are too much atomized.

Interpretations of the concept of «fundamental

ideas» are varied. My colleagues and I have looked into

the role of fundamental ideas in recent years [9].

What is my understanding of fundamental ideas?

I am going to offer you the following approach: to look at

fundamental ideas from different points of view, to clarify

why and in which way ideas are fundamental. For the purpose

of explanation and demonstration, the following complex

of questions will be used:

- Is the establishment of fundamental ideas (including the

mathematical topics) related to their position within the

mathematical context? (e.g., relation to theory, consideration

of relevant logical structure and the topic-specific

systematical hierarchy, taking into account the training

of mathematicians at a university level, «recruitment

problems» of up-and-coming university teachers.)

- How applicable, or even better, how usable are the fundamental

ideas and what is their significance in practice?

(e. g. relation to the real-life situation, to experience,

to the-description of the environment, to the demands of

school and work.)

- Which interrelations exist between fundamental ideas

and educational objectives such as mathematical

modelling, argumentation, acquisition of the ability to

use heuristic methods?

In the light of these questions, we divided the concepts

of fundamental ideas into the following aspects:

\* central ideas («guidelines») embodied in mathematical

terms and theorems which have importance within the

implied framework of a given theory by being the common

basis of numerous postulates based on that theory

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or through which a hierarchical development can be

achieved. These central ideas relate primarily to the

theoretical nature of mathematics. They have only a little

significance within our teaching of fundamentals of

technology. (Central ideas in the sense of didactics are

elementary forms of components in mathematical theories.)

\* major mathematizing models. These are mathematical

ideas (concepts, theorems, methods, etc.) which are

useful for explaining important facts of real life or which

are suitable to serne as the terminological framework for

the mathematical approach to a multiplicity of situations

outside mathematics.

\* field-specific strategies are central strategies for problem

solving, especially for establishing proofs, finding

relationships and concept formation in specific fields of

mathematics. These strategies can be characterized as

being suitable for a variety of different problems in a

field.

*Examples for major mathematizing models:* functions, differential

and difference quotients, integrals, differential and difference equations,

graphs, Cartesian and polar coordinates, systems of equations

and inequalities, vectors, matrices, events, distributions, stochastic

variables, chains, boxes, algorithms, . . .

*Examples for field -specific strategies:* approximation, linearization,

analogy between algebra and geometry, geometrization, estimation,

special algorithms (e. g. of Gaulb), analogy between plane and spatial

facts, transformation, simulation, principle of counting, testing,

special theorems (e.g. fundamental theorem of differential and integral

calculus), ...

My thesis is that major mathematizing models and

field-specific strategies are apt to structure the process of

mathematical learning with a lasting effect. They have to

involve:

- comprehensive relevance

- sense-creating significance

This is the «higher level» I talked about above before pointing

out my views concerning fundamental ideas. R.

Fischer [4] suggests that «it is not necessary and furthermore

not meaningful for the next generation to learn all

those things we have learned.» Indeed, there are a lot of

common outcomes which are taught in the present curriculum

which are not fun and (I suppose) make no sense any

more.

The majority of our future technical translators have not

continued taking mathematics in their last secondary

years, and there is only a little time to teach some mathematics

successfully (within a limit of about 50 hours!). But

there are contexts enough in order to teach mathematizing;

nearly all facts in our

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1 Original German in [4], translated by M. K.

coursework are concerned with fundamentals of technology

and technical subjects which include the teaching of

mathematizing models.2 We have seen that, apparently

because students were working in a context, they actually

became interested in mathematical concepts because they

became relevant to their studies.

There are a lot of difficulties when students have to interpret

graphs and especially when they have to use the

extensive symbolism in mathematics. Interpreting this

notation is necessary for grasping the underlying concepts;

there is no possibility of avoiding these difficulties because

notation occurs in scientific texts. In this respect, the teacher

has the task of helping over and over again, and for

the very reason that this may take a long time, it is necessary

to start at an early age. And, furthermore, it is necessary

to demand questions of sense in the mathematical

curriculum at all times and in all places.

To teach and to learn the process of concept formation is

not easy, but I am convinced that this way is better than the

one the present curriculum provides. This is my last message.

Perhaps this is a chance to help students lose their

widespread fear of mathematics and mathematical education.

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2 Examples are given in [9].

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**Part III:**

**Problems and Developments**

**in Developing Countries**

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**Having a Feel for Calculations\***

David W. Carraher, Terezinha N. Carraher, and Analucia D.

Schliemann

In studying the arithmetic of Liberian tailors Reed and Lave

(1981) proposed that there were two qualitatively different

modes of doing arithmetic. The unschooled tailors used a

«manipulation of quantities» approach — an oral,

context-based way of working with numbers — in contrast

to the «manipulation of symbols» approach employed by

their schooled counterparts. It is possible that such different

modes of doing arithmetic may be found within the

same individuals, especially if they use maths in everyday

work settings. If so, it could be instructive to look at and

compare these modes and to investigate possible relationships

between them.

The present report looks at these two ways of understanding

and doing maths. We will draw primarily upon

a study (Carraher/Carraher/Schliemann, 1982, 1985)

which we conducted among young street vendors in northeast

Brazil; but we should recognise, if only in passing,

the relevance, to our analysis, of cross-cultural studies,

particularly those of Gay and Cole (1967), Lave (forthcoming),

Lave, Murtaugh, & de La Rocha (1984), Scribner

(1984), and Saxe and Posner (1983).

The present study investigated the uses of mathematics

by young schooled vendors who use maths in their jobs in

the informal sector of the economy (Cavalcanti, 1978.) and

who belonged to social classes which characteristically fail

in grade school, often for problems in maths. The study

proposed to compare and contrast the quality of maths performance

among the same children in the market place —

the informal setting — and in a formal setting.

In the informal setting, interviewers were customers who

made purchases of fruits, vegetables, or popcorn from the

vendors. In the course of the transaction they posed questions

about real or possible purchases, such as «How

much would 6 oranges cost (at 15 cruzeiros each)?» or

«How much change will I receive if I pay for the oranges

with a 200 cruzeiro bill? « .

The vendors often worked out their calculations spontaneously

in an outloud fashion, as in the case below:

Customer: «How much is one coconut?»

Vendor (12 years old,3rd grade): «35.»

Customer: «I’d like 10. How much is that?»

Vendor: «Three will be 105, with 3 more, that will be 210

... I need 4 more ... that is ... 315 ... I think it is 350.»

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\* Support for the present research was received from the

Conselho Nacional de Desenvolvimento Cientifico e Tecnologico,

Brasilia.

In cases where the reasoning was not clear, minimal questioning

by the customer was sufficient for the vendor to

describe his steps.

In the above case, the question posed by the interviewer

may be formally represented as 35 x 10. The child’s elaborate

procedure consisted in the use of repeated chunked

additions for multiplying. The response of the child could be

formally represented in the following manner: (3 x 35) + (3

x 35) + (3 x 35) + 35 = 350, where the «chunking» is reflected

in the parentheses. Notice also that the vendor must

keep track of successive subtotals («I need four more») so

he knows when to stop.

We gave five vendors (mean age 11.2 years) who had

diverse levels of schooling (from 1 to 8 years) a total of 63

items in the market place. *They answered correctly, without*

*using paper and pencil, in 98.2% of the cases.*

Similar or formally identical problems were devised for

testing later in the child’s home under conditions which

were «formal» in the sense that testing was done with the

experimenter and child seated together at a table, paper

and pencil before them, engaged in a school-like task. In

this situation, word problems which involved calculating

with money and computation exercises (with no reference

to real objects or money) were given. *The success rates*

*were 73.4% for the word problems and 36.8% for the com -*

*putation exercises.* A Friedman 2-way ANOVA on ranks

showed the performance of the vendors to be significantly

different according to condition (p = 0.039).

No less dramatic than the quantitative differences were

the qualitative differences in performance depending upon

the condition. The following protocol is that of a 12-year-old

vendor who, in the market place shortly before, had correctly

figured out how much 4 coconuts cost at 35 cruzeiros

each.

*Interviewer* (in home situation): How much is

35 times 4?

Child writes: *Child* says:

2 «Four times five is 20;carry the

35 two (which is written above the

x4 three). Two plus three is

200 five . . . times four is 20».

(What happened is that the child added the two onto the

three before multiplying, rather than after.)

It is instructive to look at other contrasts of this sort. M,

aged 11 years, responded correctly and without any appreciable

pause when asked in the market place what 6 kilos

of watermelon would cost (at 50 cruzeiros per kilo).

Customer: «Let me see. How did you do that so fast?»

Child: «Counting one by one. Two kilos, one hundred.

Two hundred. Three hundred».

On the formal test, the child’s procedure is different:

Interviewer [reading test item aloud]: «A fisherman

caught 50 fish. The second one caught 6 times the

amount of fish the first fisherman had caught. How many

fish did the lucky fisherman catch?»

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Child [writes down 50 x 6, with 360 as the result. Then

answers]: «Thirty-six». [The examiner repeats the problem

and the child does the computation again, this time

recording 860 as the result. His oral response is 86.]

Examiner: «How did you calculate that?»

Child: «I did it like this. Six times six is thirty-six. Then I

put it there».

Examiner: « where did you put it?» [Child had not written

down the number to be carried.]

Child [pointing to the digit 5 in 50]: «That makes 86».

[Apparently adding 3 and 5 and placing this sum in the

result.]

Examiner [checking to see that the child had not forgotten

the original numbers]: How many fish did the first

fisherman catch?»

Child: «Fifty».

Another child, in the market place, is asked to give change

for a 500 cruzeiro bill on a 40 cruzeiro purchase. Before

reaching for the customer’s change he subtracts by adding

on: «Eighty, ninety, one hundred. Four hundred and twenty.

» In the formal test he must solve the problem «420 plus

80». He misaligns the 8 under the 4 and adds, getting 130

as the answer. Though the reasoning was not made explicit,

it appears that the child added the 8 to the 2, failed to

lower the zero, but carried the 1, then added the 8 again,

this time to the 4 carrying the 1. The child is once again

given the problem and proceeds mentally, getting the correct

answer.

In sum, then, we are faced with two basic facts: (1) the

performance of the vendors in the market place was substantially

superior to their performance on problems in the

formal setting; (2) the procedures were qualitatively different.

Procedures in the formal setting tended to involve

written, right-to-left computation. Procedures in the market

place were oral and used techniques which did not emerge

in the formal setting, such as chunked additions for

multiplication problems and subtraction by adding on.

Many issues were raised by these findings, questions such

as

- Are the differences in performance a matter of the

concreteness or abstractness involved?

- Is the important dimension the coral vs. written» continuum?

-How much can the results be explained on the basis of

poor teaching?

- How does school maths relate to this other knowledge of

computation displayed by the children?

Some of these issues are addressed by subsequent

research which is being reported in this Congress

(Carraher, 1984). Here we would like to address, in particular,

the issue of concreteness vs. abstractness.

It should be noted that there is nothing inherent to fruits

and vegetables which should make the calculations easier.

That is to say, there is nothing particularly mathematical

about produce. An inspection of the protocols does not give

much support to the idea that the vendors had memorised

the prices: their pacing and subtotals demonstrates that

they work out the problems as they go along. And it should

be recalled that they did the problems in their head, without

the benefit of pencil and paper for recording intermediary

steps.

Perhaps it is not so much a question of relative ease of

the market place problems as the relative difficulty of the

school problems. But one might ask: Why should arithmetic

be particularly difficult (except for the «weak», «dull», or

«deprived» child)?

An historical consideration of multiplication shows that

what schools teach today as Arithmetic is, in fact, one set

of concepts and procedures among several alternatives. In

modern Western societies, for example, we learn to multiply

by «column multiplication». Unknown to most users of

the system, column multiplication is really only one procedure

of several which have been invented throughout history.

In ancient Egypt, multiplication was performed by a

«halving and doubling» procedure (see Diagram 1). During

much of the Middle Ages in Europe counters and counting

boards were used for multiplying, as well as other computations

(Damerow/Lefevre,1981). Roman numerals were

used for recording answers but not for actually computing.

Even after the introduction of Indian or Arabic numerals

and new methods for computing, reckoning boards and

counters continued to be used for centuries and there was

resistance among many Europeans to learning to work out

numerical problems on paper. But even paper and pencil

methods varied considerably. In Venice hundreds of years

ago complicated lattices were used for multiplying (see

Diagram 2).

If we try to understand these systems today, we find them,

at least at first, awkward and strange. We begin to understand

what it means to say that numeric systems involve

arbitrary conventions for the manipulation of symbols.

When one uses a computational procedure not fully

understood one is likely to make errors through being out

of touch with what is going on. Even mathematics educators

may have an appreciation for this lack of touch when

they try to take square roots or to multiply determinants.

For most people, such procedures are not clear, and rote

memory must be relied upon. It should be recognised,

however, that some school procedures do begin to make

sense and one may begin to develop an understanding for

what were formerly strange conventions. How does this

happen? How is school mathematics related to the informal

types of maths described in the present report? What leads

to their integration? This remains an important theoretical

as well as a practical issue.

The present analysis strongly suggests that the errors

which the street vendors make when using school taught

procedures do not reflect a lack of understanding of addition,

subtraction, and multiplication but rather a difficulty

with the system of symbol manipulation conventionally

adopted in our societies for solving arithmetic problems.

Borrowing, for example, is a typical stumbling block for

maths as presently taught in schools. It should be recognised

that it is possible to subtract without borrowing, and the

vendors do subtract in this way, using regroupings to de-

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compose the problem into one with intermediate steps.

There appears to be a gulf between the rich intuitive

understanding which these vendors display and the

understanding which educators, with good reason, would

like to impart or develop. While one could argue that the

youngsters are out of touch with the formal systems of

notation and numerical operations, it could be argued that

the educational system is out of touch with its clientele.

Bridging this gap would require, it appears, a better knowledge

on the part of educators of the «spontaneous» procedures

and concepts which pupils bring into the classroom,

or perhaps, leave at the entranceway.

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Diagrarn l: The «Halving and Doubling» Method of

Multiplying

Explanation: The lesser of the multiplicands (13) is successively

halved, the result being written in the left-hand column. When there

is a remainder (always of one half), it is discarded. In the next

column, the multiplicand is successively doubled. Those members

of the column which stand opposite odd numbers in the left-hand

column are set aside and summed. 17 + 68 + 136 = 221, the correct

answer to 13 x 17.

Diagram 2: The Diagonal Lattice Method of

Multiplying 907 x 342

Explanation: All the possible multiplications, digit by digit, are made

and the results placed in the boxes. Summing proceeds from the bottom

right to the bottom left, then upwards to the top left. Each digit of

the final answer (310 194) is determined by adding the elements of

the corresponding diagonal.

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Example : 13 x 17 -> 17

6 34

3 68 -> + 68

1 136-> +136

Answer 221

**Can Mathematics Teachers Teach Proportions?**

Terezinha N. Carraher, David W. Carraher, and

Analucia D. Schliemann

When pupils learn new topics in mathematics in school —

say, ratio and proportions — one usually assumes that their

problem-solving ability has been expanded. Presumably,

they will be able to solve problems which they were previously

incapable of solving since they have at their disposal

the mathematical knowledge required for proper solution.

But how can pupils tell when this (or any) mathematical

knowledge is called for? And how do they choose which

information to plug into the mathematical routine when

there are many (irrelevant) facts available?

If mathematics is to be useful to everyone, mathematics

teachers must consider carefully issues related to the

transfer of knowledge acquired in the classroom to other

problem-solving situations. When pupils learn a general

problem-solving procedure in mathematics classes, it is

important that teachers concern themselves with ways of

turning these procedures into resources that their pupils

will, in fact, draw upon when actual problems arise.

Following the computational procedures appropriately in

the classroom in no way assures that they will be used

elsewhere when the lesson is over.

The Rule-of-Three is a simple procedure which mathematics

teachers present to pupils as a neat formal way of

solving problems involving ratio and proportions. When a is

to b as c is to *d,* one can find any unknown from the other

three values. The mathematics involved is quite straightforward.

However, the simplicity of the mathematics in

already-set-up problems may easily mislead one into treating

proportions as a topic which can be readily learned by

pupils in school. A problem must first be seen as one which

calls for a proportionality analysis before the Rule-of-Three

is considered a viable approach to its solution. Besides

general matters of transfer, cognitive development may be

another point to consider; several researchers

( P i a g e t / I n h e l d e r, 1951; Inhelder/Piaget, 1955;

Piaget/Grizel/Szeminska/Bang, 1968; Karplus/Peterson,

1970; Aguiar, 1980; Lima, 1982) have shown with different

contents that children adopt additive solutions to ratio problems

at earlier stages in development and that it is only

when the stage of formal operations is reached that proportionality

reasoning seems to appear.

In order to better understand how knowledge from mathematics

is deployed in other school-related subjects, we looked

in this study at how pupils solved three proportionality

problems from physics. We also investigated the tendency

to use the Rule-of-Three in two conditions: (1) when only

the essential information was given; and (2) when relevant

information was given along with information irrelevant for

solving the problem.

*Method*

Three problems involving proportions were presented in six

different forms each to 720 Brazilian pupils ranging in age

from 14 to 20 years and in level of schooling from 6th grade

to the last year of high school. This covered a range of six

years of schooling, with the lowest level corresponding to

the year at which proportions are taught in Brazil. Testing

was done collectively in written form and pupils were asked

to select one of six alternative answers to each problem

and justify their choice. Each pupil received a paper containing

one form of each of the three problems; order of problems

on the papers followed the Latin Square.

One problem involved judging the height of a building

from its shadow, when the height and a shadow of a pole

are known. The second involved determining the weight

necessary to balance a scale with unequal arm lengths.

The third problem type involved the size of shadows as a

function of the distance of an object from the light source

and the size of the object. Six different forms of presentation

of the three problems were designed to check the

influence of the following variations upon problem difficulty:

(1) verbal versus diagramatic presentation of the problem;

(2) specific, numeric versus general, algebraic response

form; (3) availability versus non-availability of a formula to

compute solution; and (4) applying a computation procedure

to given values versus identifying the relevant parameters

upon which the procedure should be applied. The

first two variations were orthogonal to the three ratio problems;

the last two were restricted to one or two specific

problems. In all, a total of 18 versions of the problems were

used.

*Results*

Even though the verbal presentation of the problems explicitly

mentioned that parameters were directly/inversely proportional

— which could have been used by pupils as a cue

to the choice of the Rule-of-Three — the form of presenta -

tion, verbal versus diagramatic, had no effect upon the difficulty

of the items. Further, computing with direct proportions

was consistently easier than computing with inverse

proportions; success rates varied between 30% and 46%

for the direct proportions problems and were around 12%

for inverse proportions problems. Success also varied

according to an interaction between response form - numeric

versus algebraic and problem type — direct versus

inverse proportions. Computing a response was easier

than indicating a formula when direct proportions were

involved while the reverse was true for inverse proportions.

The relative ease of inverse proportionality problems seemed

to result, however, more from a response bias in the

algebraic items than from a better understanding of the

problem.

Providing students with a formula for the solution rendered

solution significantly more likely but percentages of correct

responses still remained under 70.

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Identifying relevant parameters proved much more difficult

than computing a response for the same problem when the

necessary information had already been isolated. Pupils

were often unable to indicate the relevant parameters. This

type of error cannot usually be observed in mathematics

lessons.

The justifications provided by a sample of 220 pupils for

their answers (660 in total) were analyzed in order to identify

the distinct problem-solving routines used. The overwhelming

majority of responses was in fact not justified:

pupils either provided a linguistic account of their computations

(such as «I multiplied and then divided»), or claimed

not to know enough about the content of the problem (such

as «I have not yet studied this in physics»), or presented

rather vague justifications (e.g., «I followed the logic of the

problem»). When an explanation was clearly given, the

Rule-of-Three was observed with greater frequency (which

varied between 18.6% and 20.6% across the three problems)

than any other specific explanation. It was usually

associated with successful solution when direct proportions

were involved while the reverse was true with respect

to inverse proportions.

Several pupils (percentages varied between 7.1 and 32.2

across problems) used a functions approach to the problems

(e.g., «If the shadow of the pole is 3 times its height,

then the shadow of the building is 3 times its height, and

the building is one third the shadow»). This approach often

resulted in error because pupils used additive comparisons

between the measures (e. g., «The shadow is 6 m longer

than the pole, thus the building must be 6 m less than its

shadow»). This type of error can be related to trends observed

by Piaget and Inhelder (1951) and several others in

cognitive development. Some pupils observed still other

relations between the measures (e.g., «The shadow is the

square of the size of the pole. For the same reason, the

building is the square root of the size of its shadow»).

A scalar approach (e. g., «The shadow of the pole is one

fourth the shadow of the building. That means that the building

is 4 times the height of the pole») to the solution was

much less common (percentages varied between 3.9 and

7.1 across problems) than either a functions or a

Rule-of-Three approach; it tended to yield correct responses

with direct proportions and wrong ones with inverse

proportions. In summary, what both the functions and

the scalar approaches seem to reflect, in general, is an

attempt on the pupils’ part to relate one set of two numbers

in some way and then transfer this relationship to the

second set without much analysis of what type of relationship

may in fact hold.

*Conclusions*

Four main conclusions will be stressed here. First, teachers

seem to be somewhat successful in teaching pupils

how to use formulas to solve problems and significantly

less successful in teaching them how to use the

Rule-of-Three. The very low success rates in problems

with inverse proportions uncover the pupils’ difficulties with

this algorithm. Second, analyzing problems and rendering

them amenable to solution by using the Rule-of-Three is

even more difficult for students. This difficulty appears if

pupils must simply point out which information is crucial

and also if they must indicate an algebraic formula for solving

the problem. This result underlines the issues related

to the type of knowledge acquired by pupils in mathematics

classes and those related to transfer of training. Third,

pupils cannot be said to have truly learned proportions if

their competence is restricted to their performance in

mathematics lessons. Hart (1981), working with the daily

life problem of decreasing quantities in a recipe, found

even less indication of transfer than that which was observed

in our study. Finally, it is necessary to turn back to the

theme of this group’s work: If mathematics is to be useful

to everyone, issues related to the transfer of knowledge

from the classroom to other problem-solving situations

must receive a much more systematic treatment both by

researchers and teachers.

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**Mathematics Among Carpentry Apprentices:**

**Implications for School Teaching**

Analucia D. Schliemann

One possible source of children’s difficulties when dealing

with problem solving at school may lie in the discontinuity

between formal school methods and the natural strategies

they develop in their daily activities (see

Carraher/Carraher/Schliemann, 1985). Alternative proposals

to minimize this gap require a deeper analysis of how

problem-solving skills relate to specific experiences and

how arithmetic training contributes to an improvement in

problem-solving ability in and out of school. Scribner (in

press) has shown that, compared with students, dairy

employees show more variability and more effort-saving

strategies when solving problems related to their job activities.

Lave (in preparation, see Reed/Lave, 1979), working

with Liberian tailors, found that problem-solving procedures

are closely related to practical and school experiences:

tailors who learn arithmetic in the shop understand

the general principles of problem solving but have

difficulties with large numbers- those who have been to

school can easily deal with large numbers by means of

school-taught algorithms but more often make absurd

errors that are overlooked. These data provide evidence

for the context-specific approach (see The Laboratory of

Comparative Human Cognition, in press) and suggest that

cognitive skills are closely linked to specific experiences

and practice. However, concerning problem-solving abilities,

clearer data, such as those gathered by Scribner and

Cole (1981) on literacy, are still required. In the present

study, carried out in Recife, Brazil, data on problem solving

among a group of professional carpenters and a group of

carpentry apprentices, with different educational backgrounds,

are analysed.

The group of professional carpenters was made up of 12

adults who had had from none to five years of formal

schooling. They learned their profession while working as

assistants to the owner of the shop, in most cases their

own fathers. Their verbal reports suggest that this process

of instruction closely followed the pattern described by

Greenfield and Lave (1982) for informal education.

Naturalistic observation of the daily work of these professionals

revealed that arithmetical problem solving often

occurs when a customer brings to the carpenter a drawing

or a photo of a piece of furniture to be made. The carpenter

has then to calculate how much wood he needs to buy

and how much he will charge for the finished product. He

buys wood from large shops already cut into standard

pieces from which parts are to be cut.

The group of carpentry apprentices was composed of 18

adolescents from poor backgrounds, aged 13 to 18 years,

who attended a three-year course of instruction in carpentry.

All of them were also attending the formal school system

and had at least four years of school instruction in

mathematics. Naturalistic observation of the activities in

the carpentry school revealed that: (a) carpentry apprentices

start their practical training by performing simple tasks

such as cleaning and polishing; (b) teaching is mostly done

by demonstration with few verbal explanations that could

help to improve performance in more difficult tasks; (c)

after one year of the course apprentices begin to build

pieces of furniture; (d) instructions for building each piece

are accompanied by a drawing or a three-dimensional

model of the piece and by a list of all the parts needed,

each one specified in terms of length, width and thickness;

(e) wood is available in blocks, from which each part is cut

with the aid of powered tools; (f) only at the end of the

three-year course are apprentices trained how to make up

a list of the parts required for building a particular piece of

furniture; (g) parallel to practical training, formal classes on

language, arithmetic, geometry and drawing are regularly

offered with great emphasis laid from the outset on measurement

and how to calculate area and volume.

In this study, in order to analyse how the two groups differ

in the way they deal with a problem related to their daily

work, each of the carpenters and apprentices was asked to

find out how much wood he would need to buy if he were

to build five beds like the one shown in a drawing (see

Figure 1). They were told that they could use paper and

pencil, if they so wished. While they were trying to solve the

problem, the examiner talked to them and discussed

details of the drawing as well as the steps they followed in

order to find a solution. The sessions were tape-recorded

and were run in the shops during working hours or, for the

apprentices, in a school classroom. An observer took

notes, which were used in the analysis, together with tape

transcripts and written material produced by the subjects.

*Results*

Results were analysed in terms of arithmetical operations

performed, strategies used to perform operations, dimensions

taken into account, and final result.

Tables 1 to 4 show the answers of the first-year apprentices

compared to those in their second and third year, and

to those of the professional carpenters, in each of the

a b o v e-mentioned items. Only one apprentice did not

attempt to perform the task. Two professional carpenters

who had never been to school gave a final answer without

explaining how it was obtained. These three cases were

not included in the analysis that follows.

As shown in Table 1, more than half of the first-year

apprentices preferred to use addition even when multiplication

could have been applied as a short cut. Among

second- and third-year apprentices, multiplication was

used by 70% of the subjects and, among professionals by

90%. The correlation between the level of mastery of carpentry

(considering that professionals are at the highest

level) and the use of multiplication as opposed to addition,

although not very high (Kendall’s p= 0.37), was very significant

(z = 2.70, p = 0.0028).

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The strategies to solve addition and subtraction operations

were classified into three categories: (a) mental computation,

when the answer was immediately given without

the use of paper and pencil; (b) school algorithms, when

paper and pencil were used and the answer was found by

working initially with units, then with tens, followed by hundreds,

and so on, with carrying from one column to the

next, whenever needed; (c) mixed strategy, when mental

computation was used for the simpler operations and

school algorithms for the harder ones. Table 2 shows that

mental computation occurred more often-among professionals

and that school algorithms were preferred by apprentices:

9 out of 10 professionals used head computing in isolation

or combined with school algorithms, while only 4 out

of 17 apprentices did the same. The Fisher Exact

Probability Test shows that such distributions differ significantly

(p = 0.005).

Table 3 shows that more than half of the first-year apprentices

considered only the length of the parts in their

attempts to solve the problem. Second- and third-year

apprentices considered both length and width or, in most

cases, length, width, and thickness. Professional carpenters

always worked with the three dimensions. The correlation

between the number of dimensions considered and

the level of mastery of carpentry was very significant

(Kendall’s t = 0.59,Z = 4.13, p < 0.0001).

The final answers given to the question «How much wood

do you need to buy if you have to build five beds like the

one in the drawing?», were classified into four categories.

In the first kind of answer, where 9 apprentices were classified

(see Table 4), the dimensions considered were all

added up and a final result was inadequately given as the

number of meters, or square meters, or cubic meters

necessary to build the beds. A second sort of answer, given

by 6 apprentices, consisted of a specification of the length,

the width and, in some cases, the thickness of a huge block

of wood. The length of such a block was obtained by

adding up the length of each part of the bed, the width by

adding up the width of each part, and the thickness by

adding up the thicknesses. The third category of answers

consisted of a list of all the parts with a specification of how

many of each was needed to build one or five beds. Only 2

second-year apprentices gave such an answer. Finally, the

fourth kind of answer, where 8 professionals were classified,

consisted of the compilation of two lists, the first specifying

the parts as seen in the third category, and the

second listing the standard parts usually found in the market

from which the parts could be cut. Two professionals,

not included in Table 4, after computing the size of certain

pieces gave a final answer in terms of how much money

the five beds would cost. Correlation between the degree

of mastery of carpentry and the kind of answer, considering

that the fourth category was the best of all, was very high

(Kendall’s t = 0.85) and significant (z = 5.95, p < 0.0001)

An analysis of the relationship between the answers given

by professional carpenters and the number of years they

had been to school did not reveal any clear trend, the only

noteworthy feature being that the two subjects who declined

to explain how they arrived at a final answer were illiterate.

*Discussion*

The results obtained in this study suggest first of all that,

when faced with a problem-solving task, individuals try to

find an answer that is closely related to their daily experience:

while professional carpenters seek a list of standard

pieces to buy, apprentices try to find the measures of a

block of wood from which parts could be cut. What is most

striking in these attempts is the suitability of professional

carpenters’ strategies to find a solution when compared

with the unsuitability of the apprentices’ a p p r o a c h .

Although the apprentices had had formal teaching on how

to calculate volume, their attempts were unsuccessful and

the results obtained were absurd. However, they did not

seem to perceive the absurdity. It seems that the task was

approached by the apprentices as a school assignment

and they did not try to judge the suitability of the answers.

For the professionals it was taken as a practical assignment

and the solution sought was a feasible one. That difference

between a school approach and a practical

approach, as noted by Lave (personal communication),

seems to change the nature of the problem.

Computing strategies, although different, were equally

effective in both groups, for hardly any mistakes were

made. This is an unexpected result bearing in mind that formal

school attendance was very different between the two

groups and among the individuals in the professional

group.

Of special importance for education is the fact that, despite

receiving special teaching on how to calculate area and

volume, and how to solve formal problems involving these,

apprentices were not able to use this formal knowledge to

solve a practical problem. This fact is even more striking if

we consider that the elements of the problem were part of

their daily experience. It seems then that problem solving

at school has to be taught differently if it is to have any use

out of school. One possible suggestion arising from the

data presented here is to provide, in addition to formal teaching,

opportunities for problem solving in practical

contexts. This may improve comprehension and lead to the

discovery of new and more economical strategies and

solutions.

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Table 1: Number of Subjects in Each Sub-group

According to Operations Used While Trying to Solve the

Problem

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Sub-groups Addition Addition Total

and Multiplication

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1 st-year

Apprentices 4 3 7

2nd- and 3rd-year

Apprentices 3 7 10

Professional

Carpenters 1 9 10

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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Table 2: Number of Subjects in Each Sub-group According to Strategies

Used to Solve Arithmetical Operations

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Sub-groups Mental Mixed School Total

Computation Strategy Algorithms

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

lst-year

Apprentices 0 1 6 7

2nd - and 3rd - year

Apprentices 0 3 7 10

Professional

Carpenters 0 0 10 10

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Table 3: Number of Subjects in Each Sub -group According to Dimensions

Considered When Trying to Solve the Problem

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Sub-groups Length Length and Length, Width Total

Width and Thickness

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

lst-year

Apprentices 4 1 2 7

2nd- and 3rd-year

Apprentices 0 4 6 10

Professional

Carpenters 0 0 10 10

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Table 4: Number of Subjects in Each Sub -group According to

Kind of Final Answer Given

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Sub-group Addition of Block from List of List of

all dimensions adding up parts standard

considered each dimension parts

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

lst-year

Apprentices 6 1 0 0

2nd- and 3rd-year

Apprentices 3 5 2 0

Professional

Carpenters 0 0 0 8

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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**The Relevance of Primary School Mathematics in**

**Tribal Aboriginal Communities\***

Pam Harris

**Introduction**

Up until fairly recently (i. e. within the last ten years) it has

usually been the policy of Departments of Education to

expect all schools to use the same mathematics syllabus in

basically the same way, irrespective of the cultural, linguistic,

and mathematical background of the pupils. In

Aboriginal schools in the Northern Territory this led eventually

to a fairly strong backlash from teachers who claimed

that wit is not relevant to our situation», the Bite referring to

either mathematics in general, or to the particular syllabus

they were required to follow, and specific requirements of

that syllabus.

Relevance of curriculum content has long been an issue

in Aboriginal schools and is bound to remain so for as long

as Aboriginal people lack control over the education their

children receive, and non-Aboriginal teachers come from

outside to teach in a situation they do not understand.

Some education administrators are inclined to dismiss the

issue, saying that teachers are using irrelevance as an

excuse for their own laziness and incompetence, and that

good teaching of the set syllabus without adjustments is all

that is needed for Aboriginal pupils to succeed in maths.

Whilst it is no doubt true that some teachers may too easily

give up on teaching maths, rationalising that it is «not

relevant» to their Aboriginal pupils so they Won’t do its, the

question cannot be so easily dismissed. Many experienced

and hard-working teachers have questioned the relevance

of the mathematics curriculum in Aboriginal communities in

such comments as «You do your best, but you wonder

what it’s all for», «It’s not worthwhile to teach them what

they’re not going to use», and «What’s the use of teaching

topics that are not needed?».

The question of whether the same syllabus and set

of aims for teaching primary mathematics is equally

suitable for both Aboriginal and non-A b o r i g i n a l

schools—and whether it would even be practicable to

have different guidelines—is one which is decided

separately in each state department, and it is not proposed

to discuss it here. In this paper I will look at the

broader question of whether primary mathemat-

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\* This paper is part of a larger publication entitled Te a c h i n g

Mathematics in Tribal Aboriginal Schools, which is one of four

publications in the Mathematics in Aboriginal Schools Project

series. The Mathematics in Aboriginal Schools Project was a

national research project jointly funded by the Curriculum

Development Centre in Canberra and the Northern Territory

Department of Education during 1980-81.

ics is relevant for tribal Aboriginal children living in tradition-

oriented communities.

First I will outline some of the factors which discourage

people who are teaching maths in remote Aboriginal communities

and which often lead to the protests that it is not

relevant. Then I will consider factors that make mathematics

a particularly difficult subject, not just for one group,

such as Aborigines, but for many people in any population.

That should help to put the problems frequently encountered

in Aboriginal schools into perspective. Finally I will

give reasons why maths *is* relevant in Aboriginal communities,

though the aspects which are most relevant and useful

may be different from those which are most obviously

relevant in other types of communities.

**1. The Feeling that Mathematics is Not Relevant in**

**Aboriginal Communities — How it Arises**

The feeling which some teachers have that maths — or

most of it — is not relevant in the remote Aboriginal communities

where they are teaching seems to come from

three main sources:

1. Negative expectations passed on by other people.

2. The teacher’s own observations of lack of reinforcement

of maths in the pupils’ home life.

3. The cultural and linguistic bias of teaching materials.

4. Discouragement because of difficulties teaching maths

and the pupils’ generally low level of achievement.

If these influences on the classroom teacher’s attitude can

be appreciated by mathematics advisers and those controlling

curriculum decisions, and if the teachers themselves

can understand some of the forces at work, this should

help to put the question of relevance into perspective and

enable freer communication between all levels of those

concerned with primary maths education in Aboriginal

schools.

*1.1 Negative expectations of teachers*

It has been reported previously (P*.* Harris, 1980, p. 19) how

teachers often receive negative attitudes from other people

to the extent that they go to an Aboriginal community

expecting that their pupils will not be able to do mathematics.

A stereotype very common in the wider Australian

community is that «Aborigines can only count one, two,

three, many» followed by the conclusion, self-evident to the

speaker, that therefore they cannot do maths. This stereotype

is based on a mixture of fact, ignorance, and over

-generalisation.

The fact is that most Aboriginal languages do have very

few words for cardinal number. It is common to have separate

words only for one and two, and perhaps three, and

then words that refer to a few and many, or, as it is often

colloquially said «little mob» and «big mob». However,

there is ignorance about what this really means for

Aboriginal children learning to count (which they most often

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do in English), and it is a gross over-generalisation to assume

that some lack of counting vocabulary in their own language

could be taken as evidence of a general lack of ability

to cope with all areas of mathematics. (The lack of number

words in Aboriginal languages has been briefly discussed

in P. Harris, 1980, p. 13.)

Contrary to the popular myths, one prominent linguist who

is familiar with Australian languages has argued that the

gap in number vocabulary does not indicate that counting

itself is lacking in the culture. He suggests that it is there

«in the sense that the principle of addition which underlies

the activity of exact enumeration is everywhere present»

(Hale, n. d.).

Although new teachers going to Aboriginal schools may

not consciously espouse the common myths and stereotypes,

they are often confronted with them and, without any

information to the contrary, it is not surprising that many of

the teachers take up their new appointments with low

expectations of what their pupils might achieve in mathematics,

and a feeling that there is probably not a great deal

that they personally can do to change the situation.

*1.2 Lack of reinforcement of maths in the pupils’ home life*

On arriving in an Aboriginal community, the teacher is often

greatly impressed by the difference in lifestyle and living

conditions — differences which they see have implications

for their teaching of mathematics in that place.

Living conditions vary of course, and there are some communities

where the pupils are living in conditions not too

different from the teachers’, but in many of the more distant

tradition-oriented communities with which the Mathematics

in Aboriginal Schools Project has been particularly concerned,

the teacher will find that their pupils live in makeshift

humpies, beach camps, one-roomed tin huts, or maybe in

nothing at all, just sleeping behind a windbreak. This reality

comes as a shock to some.

As the teacher begins to see the many aspects of

mathematics which do not appear to be reinforced in

the pupils’ home life—aspects which are constantly

reinforced in the home lives of most A n g l o-A u s t r a lian

children—the word «irrelevant» soon comes to

mind. It does not seem relevant, for example, to

teach young children to tell the time on a clock when

the teacher knows that very few, if any, of the pupils

have a clock in their home, their parents rarely mention

clock time, and, in fact, about the only time they

see a clock or are expected to use one is in the

classroom. Telling the time seems to be a schoolbased

activity with neither reinforcement nor usefulness

in the child’s home life,\*\* in contrast to the situ-

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\*\* Notice that these examples are of things that *seem* irrelevant to many

teachers coming into Aboriginal communities — the validity of this

conclusion will be discussed later. To follow up the question of teaching

clock time in Aboriginal schools, the reader is referred to a publication in

the Mathematics in Aboriginal Schools Project series entitled *Teaching*

*about Time in Tribal Aboriginal Communities,* Pam Harris, 1984 published

by the Northern Territory Department of Education, Darwin.

ation of most children brought up in the Western-European

tradition where the child has usually been surrounded by

clocks and had its daily activities regulated by the clock

almost since it was born, and where the parents often

consciously encourage the child or infants of lower primary

age to learn to tell the time, and assist it in its efforts,

thus reinforcing what the teacher does at school.

Many more examples could be given of areas of mathematics

where the non-Aboriginal teacher is often frustrated

in attempts to teach skills and knowledge in contexts that

will be meaningful to the child and will be used and reinforced

outside of the classroom. Fractions, for example, are

difficult to teach, and appear to be rarely used, even in

employment; and how meaningful and motivating is it to

teach measurement of mass (weighing) and measurement

of capacity through cooking activities using written recipes,

when the child never sees a recipe being followed or a

standard measuring instrument being used at home? How

does the teacher teach division in a meaningful context

when the pupils and their families customarily divide and

share in unequal portions according to kinship obligations?

These and many more such questions daily confront the

teacher and require decisions which the newcomer may

not feel qualified to make, not yet having had a chance to

think through his or her own philosophy of education in a

bicultural situation.

Apart from the lack of home reinforcement during the child’s

schooling, there is often also an absence of the motivation

which some people have to learn mathematics

because of its uses in employment. In many communities,

both teachers and pupils are well aware that school leavers

have little chance of finding a good job — or any kind of job

at all. For example, the magazine of one large Aboriginal

community, in reporting that a certain young man had started

work with the Housing Association, noted that this was

his first job since leaving school five years before, and that

out of all the young men who had left school over the past

six years, only six were employed at the moment (Junga

Yimi 2 :3).

*1.3 Cultural and linguistic bias of teaching materials*

Some teachers, sensitive to the different lifestyle, interest,

and aspirations of their Aboriginal pupils, also consider

many of the commercially available teaching aids unsuitable.

The pictures and examples given in work-books

often seem to portray very little that is a familiar part of the

Aboriginal child’s daily life. And of the hundreds of rhymes

and songs available to introduce number and counting to

preschoolers and infants, the majority seem to talk about

subjects that are rather meaningless to Aboriginal children.

While it is possible for individual teachers to overcome this

problem to some extent by making their own worksheets,

using materials in the environment, and adapting the wording

of the number rhymes, the fact that these adaptations

are necessary can be a nagging reminder that the maths

materials were not intended for Aboriginal pupils.

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The fact that all materials are presented only in the

English language seems to be another indication that

mathematics is strictly «whitefella» business, not a part of

Aboriginal culture or current lifestyle.

*1.4 Beaching difficulties and low level of pupil performance*

Whether the teacher comes with low or high expectations

of their pupils, they are often discouraged by the apparent

low level of performance of many of their pupils in mathematics,

and the reports they have heard seem to be confirmed.

The pupils often do not seem to understand or

remember certain things no matter how many times they

are taught or how clear the explanation seems to be.

Some teachers who have previously taught Englishspeaking

children may find that the methods that worked

before do not seem to work with their Aboriginal pupils, and

so may tend to think that the problem lies with the pupils,

perhaps with some difference in thinking processes which

prevents Aborigines from «catching on» to maths. Other

teachers may have tried hard to bridge the gap between

the home experiences of their Aboriginal pupils and the

background experiences that seem to be assumed in the

maths syllabus, to «make mathematics more relevant in

the home and community» as suggested in the Northern

Territory’s 1974 Infants Curriculum, but find they are fighting

an uphill battle with little support. In such circumstances

it is easy to come to a general conclusion,

consciously or unconsciously, that the children in that and

similar Aboriginal communities «can’t do maths» mainly

because «it’s irrelevant» — the mismatch between the syllabus

requirements and the community’s requirements

seems to be too great.

Before looking at the counter arguments that the study of

mathematics *is* relevant in Aboriginal communities, we

should first question the extent to which the difficulties in

teaching and learning mathematics which are often experienced

in Aboriginal schools are actually peculiar to those

schools.

**2. The Problem in Perspective**

*2.1 Mathematics is a difficult subject*

The fact is that a great many people everywhere, including

many living in sophisticated Western societies, find mathematics

much more difficult than other subjects, question its

relevance for themselves, dislike it, and feel that they

«can’t do it».

This frequent rejection of mathematics by otherwise well

educated people has been pointed out (and accepted) by

leading mathematics writers and mathematicians. The first

sentence in Skemp’s *Psychology of Learning Mathematics*

(1971) talks about «Readers for whom mathematics at

school was a collection of unintelligible rules (. . .)», and

Kline (1962) begins his *Mathematics: A Cultural Approach*

with the words «One can wisely doubt whether the study of

mathematics is worth-while (...)». And the great French

mathematician René Descartes (1596 — 1650) told how,

after doing some study in Arithmetic and Geometry, he

found the «hows» and «whys» of the subjects not sufficiently

clear, and consequently «was not surprised that many

people, even of talent and scholarship, should (...) have either given

them up as being empty and childish or, taking them to be difficult

and intricate, been deterred at the very outset from learning them (.

. .)» (from leading quotation in Kline,1953, *Mathematics in Western*

*Culture).*

There are a number of factors which make mathematics

more difficult than other subjects for both school children

and adults, and these factors apply just as much for

Anglo-Australians and other Westerners as they do for

Aborigines.

(1) Mathematics is very abstract, much more abstract than

any other subject introduced in the primary school.

(2) Mathematics is more sequential than other subjects.

(3) Mathematics learning is more teacher dependent than

other subjects — there is not so much that can be «discovered

» by the student working alone.

(4) Mathematics is often taught in a dull, uninteresting way

without any meaningful context or examples.

(5) In some areas of mathematics, especially number

work, it is possible to perform well without the understanding

that will enable the learning to be used later;

thus problems are often not detected by the teacher.

(6) There is less support for remedial work in mathematics

(e.g. compared to the facilities provided for remedial

reading).

(7) There are more teachers who lack confidence in their

own grasp of the subject and their ability to teach it than

there are in the other basic subjects. (For example, an

article in the Arithmetic Teacher, May 1981, states that

in one teacher training program in Ohio, two-thirds of the

students counted over a nine-year period have named

mathematics as their least favourite and most feared

subject.)

In addition to these, there are two more major factors

which affect the teaching and learning of mathematics in

traditional Aboriginal communities.

(8) Learning mathematics and adopting a mathematical

way of thinking is like learning and adopting a second

culture, and,

(9) when this is done in English, then the second culture

has to be learned in a second or foreign language.

These last two points need some further explanation.

*2.2 Learning Mathematics is like learning another culture*

When an immigrant child, whose family speaks for example,

only Greek at home, enters an Australian primary

school and is required to learn mathematics in English, this

is not as difficult a task as when a vernacular-speaking

Aboriginal child is required to learn mathematics in English.

The Greek child is already part of a culture which has a

rich tradition of mathematics going back for hundreds and

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thousands of years. It is part of that child’s way of life and

way of thinking, and the child’s task on entering the

English-speaking program is merely to transfer what he or

she *already knows* from his or her own language into another

language which is closely related.

The Aboriginal child’s task is different, and much more difficult.

He also has a rich cultural heritage, but it does not

include much of the Western mathematics which is taught

in primary schools. Western mathematics is a new way of

thinking, a new way of ordering the world which is in many

respects at variance with Aboriginal ways. In a sense, it is

another culture. For the Aboriginal student, learning mathematics

in English is not a case of transferring ideas from

one language to another; old ideas must be reorganised

and a whole range of new ideas must be learned and

appreciated.

*2.3 Aboriginal children often have to learn the*

*«second culture « of mathematics in a second or*

*foreign language*

Many Aboriginal children have to learn the second culture

in a second or foreign language - English. A N. T.

Department of Education linguist, who is herself bilingual in

English and French, is of the opinion that number and

mathematics are among the most difficult areas of learning

in a second language. This applies even to «those of us

who have all the conceptual knowledge at our fingertips»

(Mary Laughren, pers. comm.). Ways of expressing mathematical

ideas such as comparison are very language specific

and the differences between languages are great,

even between closely related languages such as French

and English.

If these language differences and learning difficulties are

so significant for highly educated people who have a similar

mathematical background and speak a closely related

language, how much more significant and potentially

constraining must they be for Aboriginal people whose

mathematical background is quite different and whose own

language is quite unrelated to English or any of the other

languages which have contributed to the growth of formal

mathematics, such as Greek, Hindu, and Arabic?

Having looked at some of the differences which impress

and often discourage those involved in mathematics education

in tradition-oriented Aboriginal communities, and

tried to put them into clearer perspective, we now turn to

look at the positive side — the assertion that mathematics

is relevant in Aboriginal communities.

**3. The Assertion that Mathematics is Relevant in**

**Aboriginal Schools**

*3.1 Mathematics is relevant because . . .*

Mathematics is relevant and necessary in tradition-oriented

Aboriginal communities as it is in other Australian communities,

for the following reasons —

*3.1.1 Mathematics is needed in everyday life, in*

*employment, and in the conduct of community affairs*

People tend to think that the more Aborigines move back to

their homelands and assert their right to live in an

Aboriginal way, as many are doing these days, the less

they will need or want what Western-style education provides,

including mathematics. The practical reality is exactly

the opposite. In order for an Aboriginal community to

exist independently and run its own affairs according to the

wishes of its people, there must be at least some in the

group who are fluent in English and competent in mathematics

and thus able to communicate confidently with

government officials and other white Australians in the

wider community. The greater the desire for independence,

the more urgent is the need for Aboriginal people to acquire

for themselves skills in English literacy and Western

mathematics.

(By stressing the need for skills in Western mathematics

and literacy in English in this context, I am not at all questioning

the value of bilingual/bicultural education programs

in which the early emphasis is on acquiring literacy in the

vernacular and understanding mathematics concepts

which are a part of the traditional tribal way of life. These

vernacular programs, apart from their other advantages,

provide a sound basis for improved performance when the

student must later tackle English literacy and primary

school mathematics taught through the medium of English

as a second language.)

*3.1.2 Aboriginal people have requested mathematics*

Whenever tribal Aborigines have stated what they want

from education, they have always (in my experience) included

high on the list of priorities (a) ability to speak, read,

and write English, and (b) Knowing numbers The reasons

given for wanting to «know numbers» are very practically

oriented to managing their own affairs. See, for example,

comments recorded by H. H. Penny in the report of his

investigation into the training of Pitjantjatjara teachers in

South Australia (1976, p. 18).

*3.1.3 Mathematics is necessary for secondary and*

*most tertiary education*

If Aborigines are to achieve their aims of being teachers,

doctors, mechanics, etc. then they must have a good basic

understanding of maths, and the option to choose to do it

at higher levels beyond primary schooling.

*3.1.4 Mathematics is a major clue to understanding*

*the way Anglo-Australians think (in line with*

*their Western-European cultural heritage)*

Morris Kline begins his important book *Mathematics in*

*Western Culture* thus:

«(.. .) mathematics has been a major cultural force in Western civilisation.

Almost everyone knows that mathematics services the very practical

purpose of dictating engineering design (v.) It is (...) less widely

known that mathematics has determined the direction and content of

much philosophic thought, has destroyed and rebuilt religious doctrines,

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has supplied substance and economic and political theories, has

fashioned major painting, musical, architectural, and literary styles,

has fathered our logic, and has furnished the best answers we have

to fundamental questions about the nature of man and his universes

(Kline,1953)

To understand the Anglo-Australian culture by which they

are surrounded, Aboriginal people need to have some

understanding of mathematical thinking.

*3.2 Some mathematics topics are more relevant*

*than others*

Nevertheless, although mathematics as an area of study is

relevant in both Aboriginal and non-Aboriginal societies, it

soon becomes evident to the teacher in an outback

Aboriginal community that some maths topics (like money,

for example) are much *more relevant* than others, some

which are accepted without question in other schools

appear to have *very little relevance* (e. g. fractions, division),

and yet others appear to be *relevant and motivating*

*only if they are introduced at a different stage from that*

*recommended in the syllabus* and with different emphasis

(examples are the introduction of standard units of measure

and learning how to tell the time on a clock).

The teacher soon sees a need to adjust the syllabus to

meet local requirements. This does not imply that the aims

for the endpoints to be reached by the end of primary

school should be changed, but that in Aboriginal schools

these endpoints may be more effectively reached through

a primary maths program that has different emphases, different

sequencing, and different teaching methods from

those recommended for Anglo-Australian children.

*3.3 Some mathematics topics are more useful than*

*they at first appear*

Adjustment to the syllabus are necessary to meet the

needs of differing local conditions, but newcomers especially

should be wary of making changes, particularly any

which involve not treating a topic because it seems irrelevant.

Such decisions need to be made only after careful

consideration of the future needs of the child and the relation

of that topic to other parts of the syllabus, and are best

made in consultation with an adviser, where one is available.

*3.3.1 For example, why teach fractions?*

One Northern Territory teacher, an experienced and

conscientious person, once wrote and asked the mathematics

curriculum unit in Darwin to give her some good

reasons why she should teach fractions, because she said

she could not see the use of them and there seemed to her

to be more important things on which to spend one’s teaching

time.

The questions «Why teach fractions?» and «Should we

teach fractions at all?» are often asked in Aboriginal

schools, so I will give here some reasons for teaching fractions

and these will serve as an example of the various

aspects to be considered in regard to topics which at first

seem irrelevant.

Aboriginal children, like any others, need an elementary

understanding of fractions because:

(1) Fraction terms such as «half» and «quarter» are an

integral part of everyday English speech.

(2) Decimal fractions cannot be properly understood if the

idea of fractions (equal parts) is not understood.

(3) Fractions are used in employment, for example in the

hospital (half dose of medicine for a child), when making

out time sheets (time-and-a-half pay for working after

hours), and when stock-taking in the store.

(4) Common fractions cannot be entirely replaced by decimals

— not all situations involving fractions can be

handled in decimal form.

In addition, work on equivalence of fractions and simple

addition and subtraction of fractions provides older students

with extra practice in the four operations which does

not look like the same old «sums» being dished up yet

again. That is important for slower students who have not

achieved in the four operations but are not motivated by

the methods used with younger pupils. One principal in a

large Aboriginal secondary boys’ school reported that he

had found work on fractions very helpful in increasing students’

skills in the four operations. This idea is also supported

in some teachers’ guides, see for example page 152

of *Mathematics - A Way of Thinking* by Robert

Baratta-Lorton. Here I have presented linguistic, cultural,

practical, mathematical and motivational reasons for retaining

some work on fractions. These are just some of the

aspects which will have to be considered in every question

of adjusting the content and sequencing of the mathematics

syllabus to suit a particular situation or group.

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**Bicultural Teacher Training in Mathematics**

**Education for Aboriginal Trainees from**

**Traditional Communities\***

Kathryn Crawford

**Introduction**

This paper will discuss the challenges facing an educator

in the development of a bicultural, bilingual teacher training

programme in mathematics curriculum for Aborigines from

traditional communities in Central Australia.

Although many of these challenges stem in particular

from the characteristics of the communities involved and

their particular culture, there are also many aspects of this

educational task that are paralled in any country where

efforts are made to cater for indigenous groups of people

in an education system that has been derived from

Anglo-European cultures.

The course described forms part of the Anangu Teacher

Education Programme (ANTEP) an accredited teacher

training course intended for traditionally oriented Aboriginal

people currently residing in the Anangu communities who

wish to take on greater teaching responsibilities in South

Australian Anangu schools. The course will be directed

from the South Australian C.A.E. but most teaching will be

carried out on site by a lecturer residing within the

communities. Pukatja (Ernabella) will be the host community

for the project.

The programme as a whole represents a significant

departure from conventional teacher education in a number

of ways. Perhaps the most striking difference between

this teacher training course and many others is that from

the beginning, development of the curriculum has been a

co-operative venture between lecturers and educators on

the one hand, and community leaders and prospective students

on the other.

The extent of this co-operation is indicated by

French-Kennedy’s (1984) description of the aims of the

curriculum design workshop held in April 1984:

«The general aim (...) was to bring together prospective ANTEP students;

interested Anangu; non-Anangu with demonstrated expertise

in the area; the relevant ANTEP lecturers in charge and the on-site

lecturer for the purpose of considering, in detail, the initial offering of

units. « (p. 3)

The first group of students will commence the course in

August 1984.

The tone of the early negotiations with the A n a n g u

communities indicated that an interactionist perspective

on bicultural education and on mathematics

education and curriculum development in particular

would be most appropriate. The rationale for the design

of the two units Teaching Mathematics I and II

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\* This is a revised version of a paper presented at the 1984

Conference of the South Pacific Association of Teacher

Education.

that form the mathematical component of the course has

evolved from the urgent need to provide experiences that

will enable students to negotiate the complex interacting

factors from the known in their own culture to a competence

in the use of mathematical ideas from Anglo-European

cultures. The perceived community needs in Western

mathematics were eloquently stated by one member of the

community as follows:

«Our children need to know enough maths so they don’t get ripped

off.»

Initial discussions with community leaders and prospective

students suggested the following general aims for the course:

1. Development of student awareness of their cultural

expertise in

a) the Anangu ways of thinking about relationships and

patterns to do with the locations, qualities and quantities

of objects and people in the environment;

b) the needs of the Anangu community to

- affirm Anangu culture and Pitjantjatjara language;

- develop new strategies and mathematical knowledge

to meet the need for dealing with Anglo-Europeans

and their culture;

- explore traditional ways of teaching young children and

the modification of these methods as necessary to

accommodate new knowledge.

2. Widen student awareness of and ability to apply elementary

mathematical knowledge (S.A. Curriculum K—

8) to solve community problems.

To enable students to develop a rationale for teaching

behaviour and methods that are appropriate to the

needs of the children of the community.

**Negotiating Meanings Between Two Cultures**

Gay and Cole (1967) examine the teaching of mathematics

in a cross-cultural situation. They suggest:

«(...) in order to teach mathematics effectively, we must know more

about our students. In particular we must know about the indigenous

mathematics so that we can build effective bridges to the new mathematics

we are trying to introduce.» (p. 1)

The need to build conceptual bridges from the known to the

unknown is not of course an educational problem restricted

to the context of bicultural education. The mathematics curricula

in most primary and secondary schools are notably

dissociated from the everyday concerns of the student

population. This has been a cause for considerable

concern among mathematics educators in an increasingly

technological society. In a bicultural context the situation is

made more serious by the fact that different cultures

emphasise different conceptual schema. Thus, temporal

sequences and quantitative measurement are dominant

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themes in industrialised Western cultures but largely irrelevant

in traditional Aboriginal cultures. Scientific and technological

thought, the appropriate registers of language

and mathematics as an abstract discipline have developed

over hundreds of years within Anglo-European culture and

reflect these dominant themes particularly at the elementary

level. Experience suggests that for many Aboriginals

from traditional communities the content of the elementary

mathematics curriculum is perceived as incomprehensible

and often irrelevant.

There has been a great deal of valuable descriptive

research by J. Harris (1979), Dasen (1970), Kearins

(1976), Rudder (1983) and others about the kinds of classification

systems arid the rate and order of development of

concepts related to mathematics in differing Aboriginal cultures.

The work of Parm Harris1 provides an account of the

effect of these differing cultural perspectives on mathematics

learning. Her detailed and historical accounts of

Aboriginal attitudes and beliefs about such topics as

money, measurement and number provide a valuable

resource for teachers. Her work is important in combatting

the negative expectations expressed by teachers in such

statements as: Aboriginal children do not generalised

The clear accounts of how Aboriginal children *do* generalise

and why they find school mathematics difficult are instructive.

Her explanations of how these difficulties may be

overcome redirect the focus of the problem from the «failings

» of Aboriginals and Aboriginal culture to the inappropriateness

of many teaching practices for children from traditionally

oriented communities.

Most recently published curriculum materials such as

those devised by Western (1979), Northern Te r r i t o r y

Department of Education (1982) and Guy (1982), intended

for use by Aboriginal teacher trainees or as resources for

teachers in Aboriginal schools have acknowledged the

implications of J. Harris’s (1979) statement:

«As the child matures learning to label and order his experiences it

is inevitable that his cognitive development will be very strongly

influenced by Aboriginal systems of knowledge.» (p. 143)

Thus, these materials take care to acknowledge language

difficulties, use materials from a familiar context for illustration

and take particular care in topics such as time and

measurement to provide experiences to facilitate conceptual

development. However, a closer analysis of such

materials suggests that the teaching procedures and the

content are still culturally biased to the extent that

Aboriginal people are likely to have difficulty relating school

experiences in mathematics to community needs and problems.

In a community-based teacher training course it seems

that it is possible for the first time to develop procedures for

negotiating meanings between the two cultures. With this

in mind the lecturer’s notes at the beginning of the first

module in the course state:

«A co-operative exchange of knowledge is particularly important in

the tutorial sessions because mathematics by its

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1Personal correspondence and CDC Mathematics in Aboriginal

Schools Project Series (in press).

nature involves the use of higher order cognitive skills and problem

solving requires confidence. There is considerable research evidence

to suggest that egalitarian relationships foster these skins better

than authoritarian directions. It is important that student/teacher interaction

and role play used in the sessions provide a suitable model for

interaction with children.» (Teaching Mathematics I, Module 1)

The course has been developed based on a model designed

to maximise the possibility of interaction between

the world view expressed by Anangu culture and that of

Anglo-European culture as evidenced in school mathematics.

This is achieved by placing an emphasis on the student

expertise *and* contribution in providing information

about Anangu world views *as a necessary part of the cour -*

*se.* The course is constructed in such a way that students

are invited to participate in co-operative decision-making

about appropriate methods for negotiating meanings from

one culture to another.

Group interaction and co-operation in arriving at solutions

to the problems set by tasks in each module are an essential

component of course experiences. This is a necessary

process if an understood social concensus about the multiple

realities, perceived by Anangu and non-Anangu tutorial

group members is to be achieved. Such cognitive interaction

is the basis for the development of a synthesis of

world views and of the clearly understood and generalised

universal premises that will form the basis for future group

decision-making. The outcomes of such a synthesis are in

the changed perspectives that the participants take away

with them.

Figure I below illustrates some of the contexts in which

this approach will be used.

It is expected that in Teaching Mathematics I the mathematical

content will be that of the South A u s t r a l i a n

Department of Education Syllabus’s Early Childhood section

(Modules 1 — 10). In the second year of the course the

emphasis will shift to the mathematical content of the upper

grades of primary school. The community has expressed

the desire that teacher trainees should function as a

resource of useful mathematical knowledge within the

community. To meet this need there will be a mathematics

component in the Work Skills Unit that is also included in

the ANTEP programme.

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A system of *clustered* modules has been used in the

course so that conceptual construction links between related

topics are made as explicit as possible.

Figure 2

The experiences provided for students during the course

have been chosen so that the links between mathematical

ideas as expressed in traditional Anangu culture, the settlement

community, Anglo-European culture and the school

curriculum are emphasised.

The diagram below illustrates the first 20 modules of

Teaching Mathematics I. See appendix for a more detailed

description of the form of a particular module.

More modules are constructed to promote an exchange of

ideas between the distinctly different conceptual frameworks

of the two cultures. The diagram below shows some

of the ways in which this occurs.

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Figure 3

At all points emphasis is placed on practical experimentation

and application of ideas in the community and

the local school. The teaching staff at Ernabella school has

been most supportive of the programme and is enthusiastic

about allowing students to expand their practical experience

within the school as their skills increase.

**Communication Between Cultures**

During the development of this course it has been necessary

to pay considerable attention to the communication

difficulties of a bilingual programme. For some of the reasons

expressed in the previous section, language difficulties

are *particularly evident* in the mathematics area.

Barbara Sayers (1983) suggests that language difficulties

are a particular problem in the teaching of mathematics.

She goes on to describe her experience at the

Wik-Mungkan community at Aurukan:

«I have understood what was said in terms of understanding the linguistic

aspects of the language, but I have not understood the message

they encoded. Such messages are incomprehensible because

I did not understand the presuppositions on which they were built,

nor the Aboriginal concepts which were involved. To sum up, I could

understand what was said but not what was meant.» (p.3)

The reverse situation occurs all too often when mathematics

is taught in Aboriginal schools.

Some proficiency in English has been required for selection

to the ANTEP course. However, on-site experience at

Ernabella suggests that:

1. Bilingual presentation of materials is essential.

2. Articles and workbooks should include English and

Pitjantjatjara translations.

3. Participant responses were always in Pitjantjatjara with

the exception of one person.

4. Difficulties will be experienced in translating some

English words into Pitjantjatjara particularly in the field of

mathematics.

The course has been designed with considerable emphasis

on an activity-based process/discovery learning design

to maximise the possibility of concensus about the *mea -*

*ning* of language generated by students (in either language).

It has also seemed appropriate to maximise the superior

visual/spatial skills of the Anangu people, and the precision

of the Pitjantjatjara language in this respect, by illustrating

concepts by role play or diagram as far as possible. For

example, in Module 7 of Teaching Mathematics I, posture,

which is an effective and important means of communication

within the Anangu communities, is used extensively in

an action-based activity to convey ideas about seriation.

During the development of the course students will be

encouraged to develop techniques for using graphic displays

to convey relationships.

The Ernabella school has an Apple computer. Work has

already been done by Klich2 in converting spatial

games known to community elders into a form suitable

for presentation on a video screen. The prospective

students have already expressed much in te r -

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2 Personal correspondence.

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est in learning to use this computer. The development of

cultural symbolism through the use of Apple Logo software

seems an extremely promising medium for the introduction

of Euclidean notions of geometry and sequenced procedural

skills.

It has become evident that there is insufficient information

available about the use of Pitjantjatjara language in relation

to mathematical concepts. In any case there has been

some suggestion from members of the community that language

development in the vernacular is somewhat depressed

among settlement children. It seemed advisable to

include in the course some action-based research for students

directed at the collection of information about the

ways in which Pitjantjatjara language is used to describe

mathematical ideas.

This seemed an important aspect of their learning because:

- On-location experience with Aboriginal teacher aids suggests

that their assistance of children even when

expressed in the vernacular consists of, often incomplete,

attempts to translate Anglo-European ideas rather

than the out-of-school modes of expression. Confusion

results.

- The procedure seems likely to provide experiences that

will heighten student awareness of mathematical ideas

and concepts within their own culture.

- Eagleson et al. (1982) suggest that Aboriginal English is

a restricted code. It is restricted, not in the sense that it

is inferior but because it has been developed as a

means of communicating ideas derived from Aboriginal

world views. The syntax of this fomm of English often

follows that of the vernacular. Without further development

and clarification it is usually not an appropriate

means of communicating mathematical ideas from

AngloEuropean cultures.

- The collection of information of this kind seems likely to

heighten the awareness of both students and educators

of points where confusions about mathematical

concepts is likely to arise.

The inclusion of these types of exercises in the course

was confirmed as a useful innovation by David Wilkins, linguist,

of Yipirinya school in Alice Springs. The teachers at

that school have found a similar approach most useful,

especially for mathematics. Prospective students were

enthusiastic about the idea since it affirms their cultural

expertise and provides opportunities for consultation with

community leaders about precise vocabulary.

The procedure as it has currently been developed

involves collecting taped information of children and adult

language usage to describe certain situations. For

example, a child may be blindfolded and directed through

a maze of carefully placed obstacles by the rest of the

group to elicit information about vocabulary and syntax

connected with location and direction. The school linguist,

on-site lecturers and students then use the collected information

as a basis for language development in the vernacular

and as a source of information about conceptual differences

between cultures. For example, Anglo-Europeans

tend to describe direction in terms of Left and Right,

many Aboriginal groups use the four points of the compass.

There does not appear to be a set policy for bilingual teaching

in the Ernabella school. In general, the linguistic

resources available and the language competence of children

in either language govern the level of verbal discourse.

In a bicultural context it is necessary to actively affirm

language registers that are appropriate for discourse about

science, technology and mathematics. This register of

English language is not normally evident in the language-

arts curricula of Australian primary schools. The teacher

training experience provided in the courses described

above, emphasises the use of small group co-operative

tasks and elaborated ideas as logical and meaningful communication.

It is hoped that these will provide students with

the necessary skills to allow for needed curriculum change

and a rationale for the appropriate use of first the vernacular

and then English as a useful medium for instruction in

mathematics.

**Conclusion**

The emphasis in the teacher training course described

above has been in the development of a rationale for

connecting the conceptual frameworks (with respect to

mathematics) of two very different cultures.

Experiences have been provided to increase student awareness

of the mathematical ideas within Anangu culture.

Strategies have been suggested and opportunities provided

for the development of a rationale for teaching procedures

that will assist children as they move from one cultural

context to another.

To this end, the course has been constructed on a process

model where information is collected and students are

encouraged to play an active role in the decision-making

about outcomes. Language development in both English

and the vernacular will be an important factor in this process.

The visual/spatial knowledge and the heightened awareness

of relationships that are characteristic outcomes of

Anangu culture will be used in teaching procedures that

emphasise these aspects of mathematics rather than

demanding prior mastery of incomprehensible algorithmic

procedures.

It is to be expected that there will be some problems to be

negotiated as the course proceeds. At this stage, however,

it seems that success, in terms of student competence as

teachers and a more appropriate learning environment for

the mathematics curriculum in Anangu schools, may well

depend on the extent to which students are enabled to actively

participate in building bridges between the two cultures

for themselves. Only then will they become truly competent

as teachers in a bicultural context.

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*Appendix*

**Module 7**

Comparison II — (comparing people)

*Lecturer’s Notes*

The purpose of this exercise is to expand direct comparison

according to one attribute to seriation strategies.

The use of people appears appropriate since limits on

behaviour and location between different members of the

community are likely to be familiar ideas.

*You will need:* large sheets of paper and felt pens.

*Student activity:* Explain to students that the following activity

is one way of *ordering* by comparison of an attribute

using personal orientation to show a *relationship.*

*Tutorial Session*

1. a) All students to compare heights in the following way.

As two people approach, the taller person turns side on

and put hand on hip, the shorter person turns side on

and puts hand in air

e.g.

b) Practice this until all possible pairs have been tried.

c) After two students have approached each other, as in

the above diagram, a third approaches. If he is shorter

than A and B he stands next to B side on with hand in

air. B places hand on hip facing C.

e.g.

If he is taller than A the following arrangement is made:

d) Students should try different patterns using the rule that

one can only have one person on either side, e. g. a person

with both hands on hips can only be approached by

two shorter people, and a person with both hands in the

air can only be approached by two taller people.

e) Change the rule so that when both hands are used one

should be up in the air and one on the hip.

e.g.

The resulting arrangement should be as above. A fifth

person approaching the group may:

- join at the tall end if he/she is the tallest;

- join at the short end if he/she is the shortest.

Find a position between two people in the line so that

he/she stands appropriately with one hand in the air and

one hand on hip.

2. Discuss the activity. Note the similarity of hand on his

shape to > symbol used in mathematics. Seek suggestions

from students for other actions or postures that

may be suitable. How else might people be compared/

ordered? Age?, kinship?, totem?, weight?

3. Devise suitable ways of recording results or ordering

activities. How can we show the relationship between

people?

4. Discuss ways of using this activity and this type of experience

when teaching children to order objects according

to attributes such as smoothness, length, weight.

How can the relationship between objects be shown?

*Practicum*

The group should devise an ordering lesson suitable for

young children (seek modification and improvements). This

activity should be carried out with a small group of children

(< 10). The results should be reported next session.

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